WAVE OPTICS
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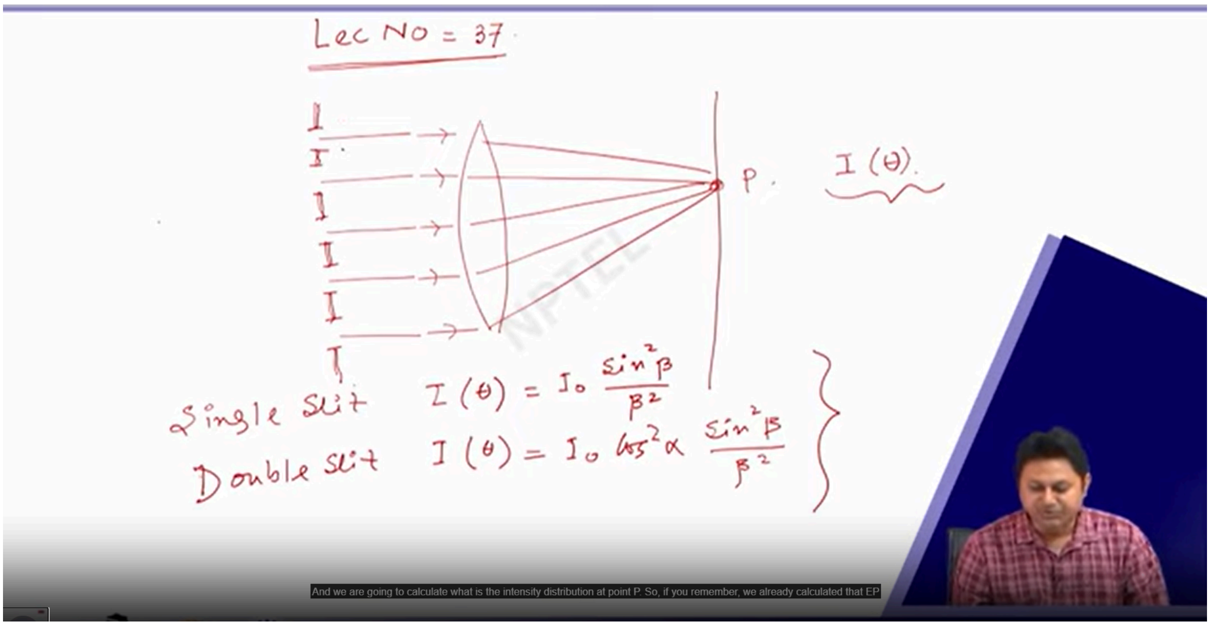
Lecture - 37: Multi-slit Diffraction (Cont.)

Hello, students in our wave optics course. Today we will have lecture number 37 and in this lecture, we will discuss more about the multi-slit diffraction phenomena that we started in the last class. So we have lecture number 37 today and we are going to discuss the multi-slit diffraction. The formalism I already mentioned in the last class is that instead of one slit, we have multiple slits here. The interference pattern and diffraction pattern both are going to take place together here. Let me draw in a different place that I have these lines here and light will come from this aperture. There are n number of apertures and if I place a lens here at some point, the light will come. Not necessarily it will come in a straight path. I am just showing you the crude setup and it will focus at some point. And this is the P where you are supposed to get the pattern. We want to calculate what is the intensity distribution that is I as a function of theta. For single slit and double slit, we have calculated that. And for a single slit, I theta was I naught sine square beta divided by beta square, and for a double slit, I theta was I naught cos square alpha, sine square beta divided by beta square. That was the structure we calculated in the last few classes. Now, we are going to extend this idea and try to find out what should be the structure of I theta, what is the mathematical form of the I theta or the intensity distribution for not only one slit but multiple slits. So, these are the opaque spaces, and in between the opaque spaces, we have slits. These are the slits, not one slit, but n number of slits. And we are going to calculate the intensity distribution at point P. So, if you remember, we already calculated that EP was E0 then sum over j, which goes to 1 to n by 2, where n is a number of slits and then two terms one is i1 should be a function of j theta and another term I2 (Refer slide time: 04:13)



These two integrals we are supposed to calculate should be a function of j. And then we need to sum over that thing, where I1 was the integral it was 2j minus 1 multiplied by a minus b, whole divided by 2 and the upper limit was 2j minus 1 a plus b, divided by 2, and integral was e to the power of i then k y sin theta dy that was i1 and i2 was integral, only the limit will going to change and now we have a minus sign here, so minus of 2 j minus 1 a and then minus of b divided by 2 and this term was minus of 2 j minus 1 a plus b, divided by 2, e to the power of i k y sine theta d y that was i 2. So, we just need to calculate these two integrals and that is our task today. So, i1, I can calculate because the integral is in front of us is simply 1 divided by i k sine of theta, and then this value will be e to the power ik sine theta and it should be executed in these two upper limits, I am writing at a stage it should be e to the power of i k sine theta, it should be common and then the bracket, I have 2j minus 1 and then a plus b bracket close divided by 2, that is one term. Another term is minus e to the power of ik sin theta and the lower limit lower limit is 2j minus 1 multiplied by a minus b bracket close divided by 2. So, we need to execute this quantity and that is like i1 similarly i2 is something similar, so it is 1 divided by i k sine theta usual term and then this integral will be e to the power i ky sine theta will be executed in these two limits. So I just need to put these limits. So it should be e to the power of i k sine theta and then put this limit. So first the upper limit should be minus 2 j minus 1 a plus b divided by 2 and another term minus a to the power of i k sine theta and it should be minus 2j minus 1, a minus b bracket close divided by 2. So these two terms I execute here now, I need to just rearrange a few things and then I will get alpha by definition is half K a sine theta and beta was half k b sine theta. So, I like to introduce it here because in the integral you can see that k sine theta divided by 2 this term is present. If I put it then the integral looks simpler. So, if I put this alpha-beta then i1 and i2 will be simply this i1 will be 1 divided by ik sine theta and then in terms of alpha and beta if I write, then it looks simple, it will be e to the power of i, then alpha 2 j minus 1 that term will be there plus i of beta. That is the first term and second term minus e to the power of i alpha to j minus 1 and then it should be minus of i beta, that will be i1 and i2 will be 1 divided by i k sine theta and then e to the power i alpha 2j minus 1 and then I have plus,

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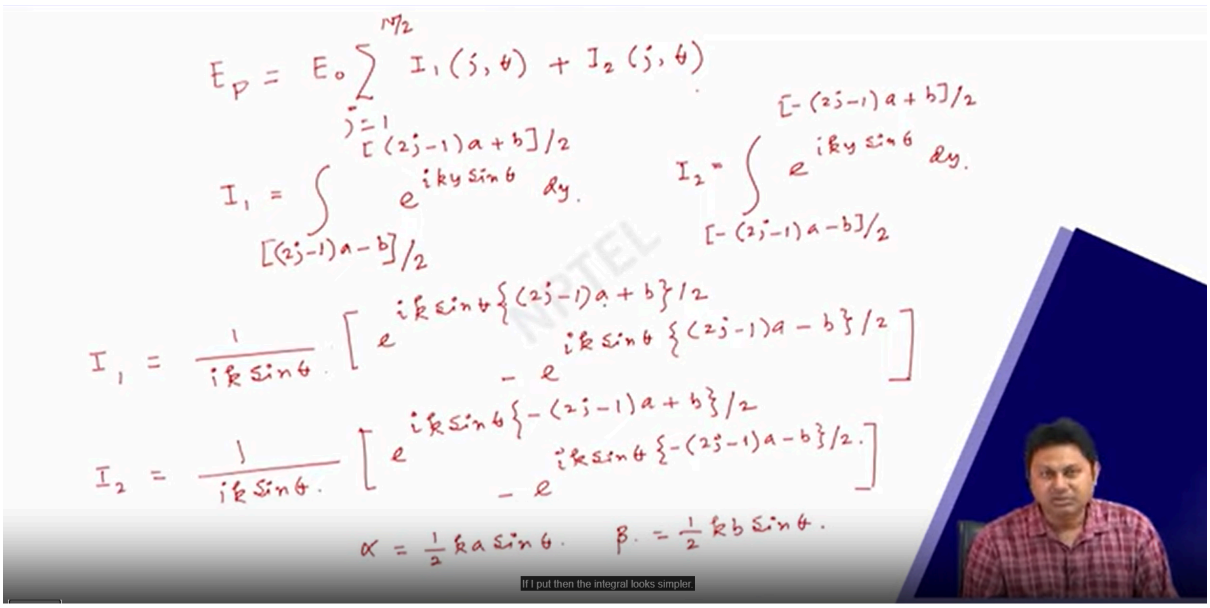

there should be a minus sign, minus i beta, this will be plus i beta actually. Other terms will be minus of e to the power of i with a negative sign, alpha 2 j minus 1, and minus of i beta. Okay so i1, i2 I calculate but inside that summation, we have i1 plus i2. So i1 plus i2 if I calculate, then it should be ik sin theta. And then from these two, you can see that e to the power i beta, I can take, in the first case, I can take common e to the power i alpha 2j minus 1, so that I can have e to the power i beta minus e to the power minus i beta. From the second case, we can take common e to the power i minus alpha 2j minus 1. And in that case, again, we get e to the power i beta minus e to the power of minus i beta. So, that thing again, we can have something in common. And if we do so, then essentially, what we get is this, e to the power i beta, minus e to the power of minus i beta, multiplied by e to the power of i alpha 2j minus 1, plus e to the power of i alpha, with a negative sign 2j minus 1, this term will get and now we can simplify a bit because now we are in a position to write in a more compact form. So ik sine theta, this term I write 2 of i of sine beta because it is e to the power i beta minus of i beta. So, we are going to get 2i sine beta which is one term and this term I can have as 2 of cos alpha 2j minus 1. So, this term contains the j over which we have the sum that we need to remember. Again, these I can write, this i, i can cancel, this 2, I can put like k by 2 and 1 b multiplier I can do and I can write it as b sine of beta divided by beta, to get this sine beta divided by beta form and rest of the term is 2 of cos of alpha 2 j minus 1, that term is there and that is my i1 plus i2 , this I will write because I need to make a sum, that over j so that's why I'll put this term in this way b sine beta divided by beta and this term we put 2, I will take outside and this term I put that it is the real part of quantity e to the power i alpha, 2 j minus 1 since we need to do the sum over j it is better, that I can put the entire term in terms of exponential rather putting it as a cos. So, cos alpha 2 j minus 1, I put in terms of exponential function by writing that real part of e to the power i alpha 2 j minus 1, which will help us a lot in the calculation. So, then once we have this, my E p that I need to calculate is essentially E0 and sum over, I1 plus I2. So it is sum over j, goes from 1 to n by 2 and we have 2 of b. Then, sin beta is divided by beta, and the real part of e to the power of i alpha 2 j minus 1, that is the expression we have.

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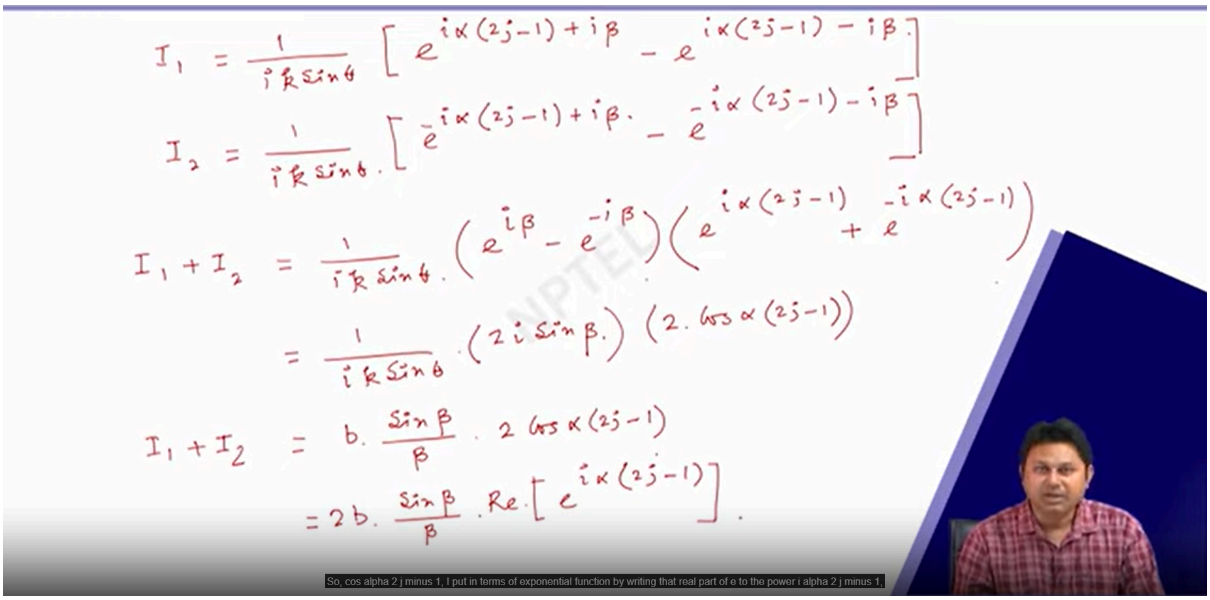

Well, you can see that we need to make a sum over J, and let us do that. So this quantity is essentially 2B, so EP turns out to be 2b E naught, sine of beta divided by beta, I take it outside and then i sum and that will be a real part of when you put z equal to 1, z equal to 2, then I am going to get this i alpha plus 3i, alpha plus 5i, alpha up to n minus 1 alpha. I just put the value of the j to make this sum. Now, this is an interesting-looking sum. So, I need to first execute that. So, let us see how to do that. So, here if I take e to the power i alpha common, then the rest of the term we have is 1 plus e to the power of 2 i, alpha plus e to the power of 4 i alpha, and so on and the last term will be e to the power of i because I am taking e to the power i alpha common, then this term will be n minus 2 alpha. Now this is further e to the power i alpha into 1 plus, if I write x to be e to the power of 2 i alpha, then this term is x, then x square, and so on. And this term, e to the power 2i alpha, I consider x. So this term will be x to the power n minus 2 divided by 2, that will be our last term. So, this is a geometric series and you can note that there are total n minus n by 2 terms. So, this sum we can do easily and it will be e to the power of i alpha and x to the power n by 2 because the number of terms is there minus 1 divided by x minus 1, that will be the sum of this geometric series, which we know. Now let us execute this quickly. So that thing is essentially e to the power of i alpha and then e to the power of 2 i alpha, which is x multiplied by n by 2, minus 1 whole divided by e to the power of 2 i alpha minus 1. So this thing is as I mentioned it is a bit lengthy calculation but straightforward. We have e to the power i n alpha minus 1 all divided by e to the power of i, if i just multiply e to the power, okay, so let me do this step and then 2 alpha minus 1. So that we have. So i n alpha, now if i divide these things with e to the power i alpha. So this will be simply e to the power of i n alpha, minus 1, whole divided by if I divide that it should be e to the power of i alpha, minus e to the power of minus i alpha. So these things are essential if I divide that, so it should be cos of n alpha, minus 1, plus i of sine of n alpha, that is in the top numerator and denominator. We simply have 2 of i of sine alpha, so, essentially, this term is this. If I multiply minus i, both the side upper and lower then it should be minus of i cos n alpha minus 1 plus sine N alpha whole divided by 2 of sin alpha. Now, note that I do not require the entire part of that. I require only the real part of that.

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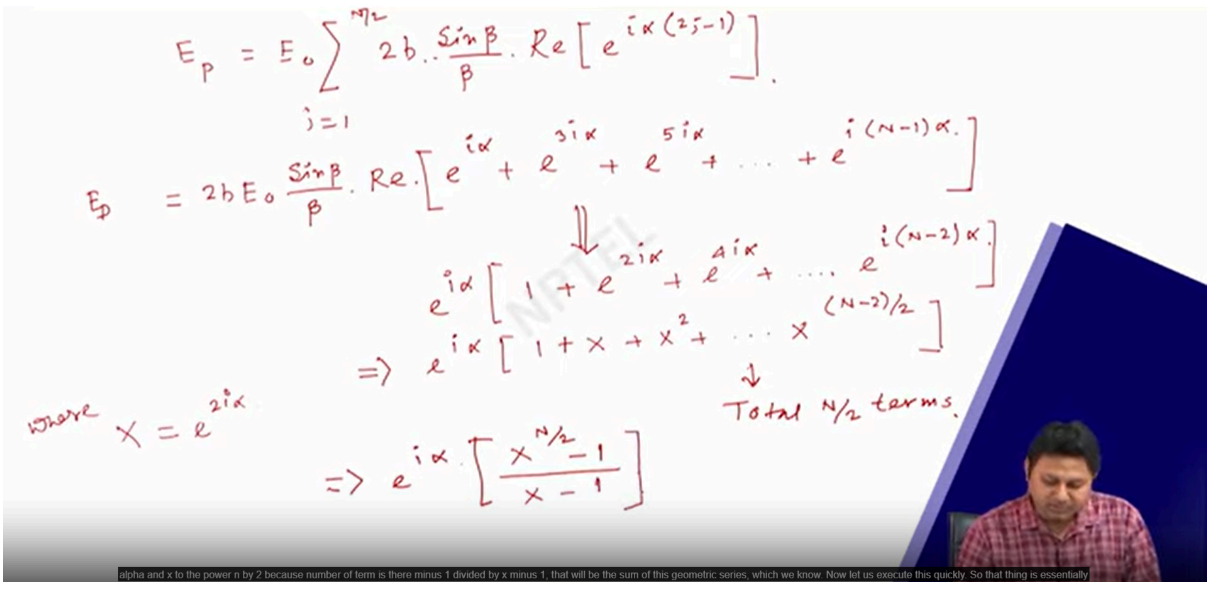

If I go back to that quantity I require the real here. I require only the real part of this sum. So if I calculate the real part of these things, so real part of them, whatever, we have, say, I put a name here, say, this is m. So I required the real part of m, so the real part of m is simply equivalent to, from the expression, it is straightforward, it is sine N alpha divided by 2 of sine alpha. So, once we have the real part, we are almost there. So, the next thing is to calculate E p, because E p was equal to 2 of B of E naught and then sine of beta divided by beta multiplied by that quantity, that sum and that we already execute and that value is sine n alpha divided by 2 of sin alpha. These two terms will cancel out from here to here and we get an even simpler expression, which is E naught b multiplied by sin beta, by beta into sin n alpha, divided by sin alpha. Now, the intensity at point P is proportional to the mod square of E p. And with that, I can write down the intensity straight away, which is a function of theta, by the way, at point P is equal to I naught sine square beta divided by beta square into sine square, n alpha, divided by sine square alpha, this is the expression I have and that expression contains two terms and here I will mention that this term is the contribution of the diffraction of the interference of n number of slits. And the values of beta and alpha I defined several times. So, let me define it is k by 2 b sin theta and k by 2 a sine theta, that is the expression we have and when, what is a and b, let me define that if there are multiple slits like this and so on. So, A is the difference between two successive slits, measured from the center point to the center point and B is the individual slit with these two, we define and based on that we get these results. So, today I do not have much time to discuss more about how the spectrum will look like, and how the pattern will look like. In the next class, we will do that, we will start from this point and try to understand what happens if we plot this function, it will be the same as the two-slit problem but here the number of slits is n. So there will be a slight difference in the pattern but we will try to trace the pattern based on the intensity distribution expression that we derived today and try to understand more about in the practical field how it will look like. So, with that note I would like to conclude here. Thank you very much for your attention and see you in the next class.

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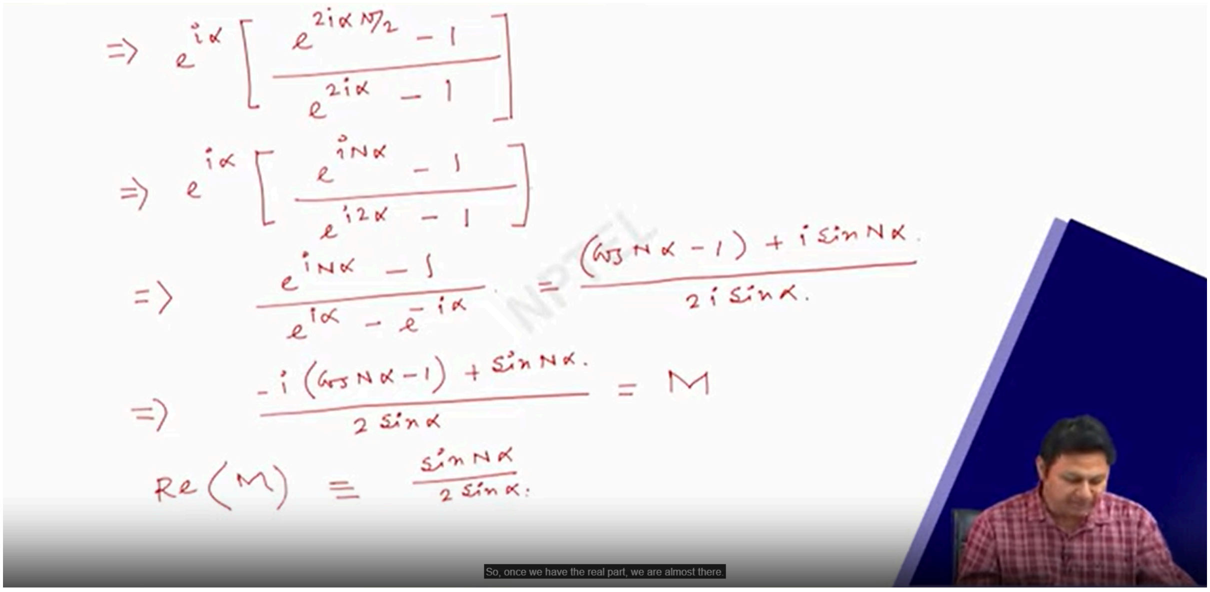

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