WAVE OPTICS
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur
Lecture - 36: Multi–Slit Diffraction

Hello, student, welcome to the wave optics course in today's lecture, which is lecture number 36, we are going to discuss multi-slit diffraction. So we have lecture number 36 today. I am going to discuss the multi-slit diffraction. But before that, we need to discuss a few things regarding the double slit problem. So, let me draw once again what was the double slit problem. In the double slit problem, we had these two slits, this is the opaque region, the light was coming here and then want to see the distribution here and the light was coming here as a plane wave like this and then from these two points the diffract circular wavefronts are generated and they will interfere here at some point P. The intensity distribution for this system at any angle theta is I naught cos square alpha, sin square beta divided by beta square that we derived. Where alpha was k divided by 2 a sin theta, where theta is this angle. If I draw a dotted line from the center to this, that is the angle we are talking about, this is my theta which is pi a divided by 2 pi a so pi a divided by lambda and sine theta and beta was k by 2 b sin theta, which is pi b by lambda sin theta. So, these are two definitions, by definition these are alpha and beta. So, today we will discuss this missing order because when we have cos square alpha and sin square beta multiplier then we have the individual 0 and individual maxima for cos alpha and sin alpha. So, we need to first calculate that diffraction, so cos theta, cos alpha is coming due to interference, and sin beta divided by beta square term is coming due to diffraction. So, first, we calculate the diffraction maximum for this. So, diffraction maximum. or rather diffraction minima because we need to calculate the missing order.
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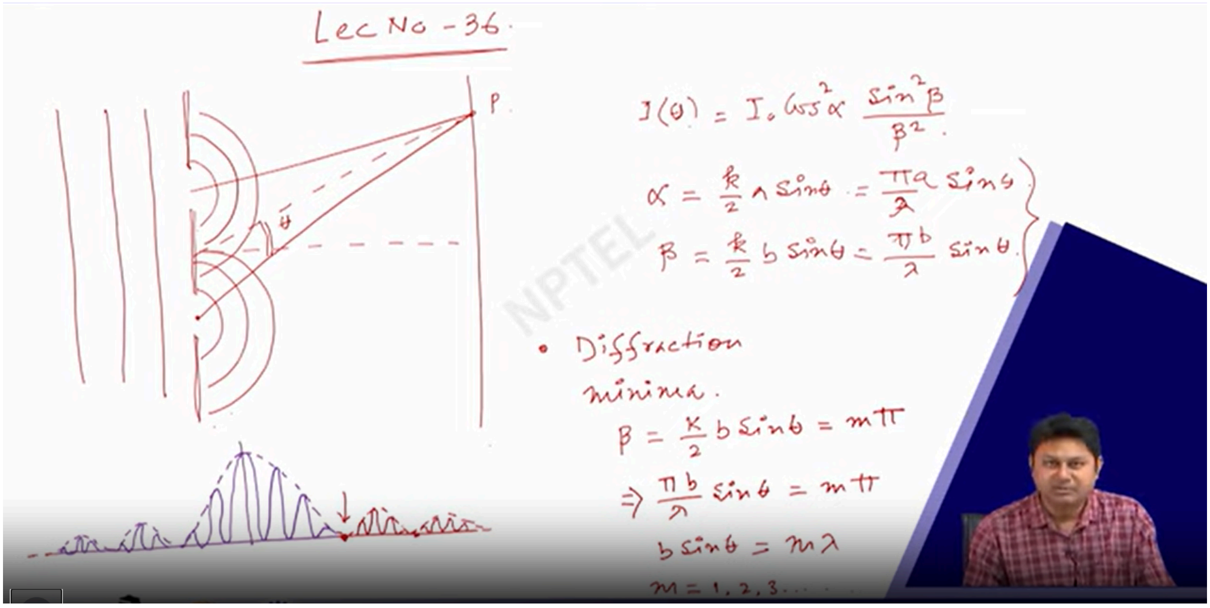

So better we write because the diffraction maxima we already calculated and this transcendental equation is required. So, diffraction minima one can find when the value of the beta will is equal to k by 2, beta is k by 2 into b sine theta that is equal to m pi, where m is not equal to 0 but 1, 2, 3, 4 etcetera because when beta m is 0 then beta should be 0 and for that particular beta will get the maximum. So let me draw here what was the resultant distribution of the intensity, then it will be easy to understand that. So this envelope that I am drawing here with a dotted line is due to the diffraction, we have the distribution of cos and that is due to interference distribution. So when we are talking about the diffraction minima, actually we are talking about these points. Here this is diffraction minima but when we are talking about interference maxima next we do so, the interference maxima, let me first find out what is the condition here. So k is 2 pi by, so I can write it as pi b divided by lambda, sine theta is equal to m pi, and then from here we can write that b of sine theta, that is the condition we have b of sin theta is equal to m of lambda. That is the condition for minima and m value is simply 1, 2, 3, etcetera okay. So, B sine theta equal to m lambda is the condition for diffraction minima that is having the minima here in this location. The value of the theta should be this. T Well, then what do we do? We will try to find out what is the condition for interference maxima. So, interference maxima will be when cos alpha will have the maximum value or alpha will have n pi that is the condition when the cos alpha has the maximum value rather cos square alpha has the maximum value. So, that value includes 0 also so, n will have 0 1 plus minus 1 2 3. So, I am just putting the plus sign, for the timing minus will also work, that is the other side of the diffraction pattern. So, then this condition leads us to k by 2 a sin theta or pi A divided by lambda sine theta is equal to n pi. Or the simple condition is A sine theta will be n lambda, previous condition was b sin theta was m lambda now it is n lambda. So, the condition of the missing order means, what is the missing order? The missing order means the minima of the interference, the diffraction pattern if it coincides with the maximum interference pattern then we should have a minima. So, let me draw here to make these things clear.

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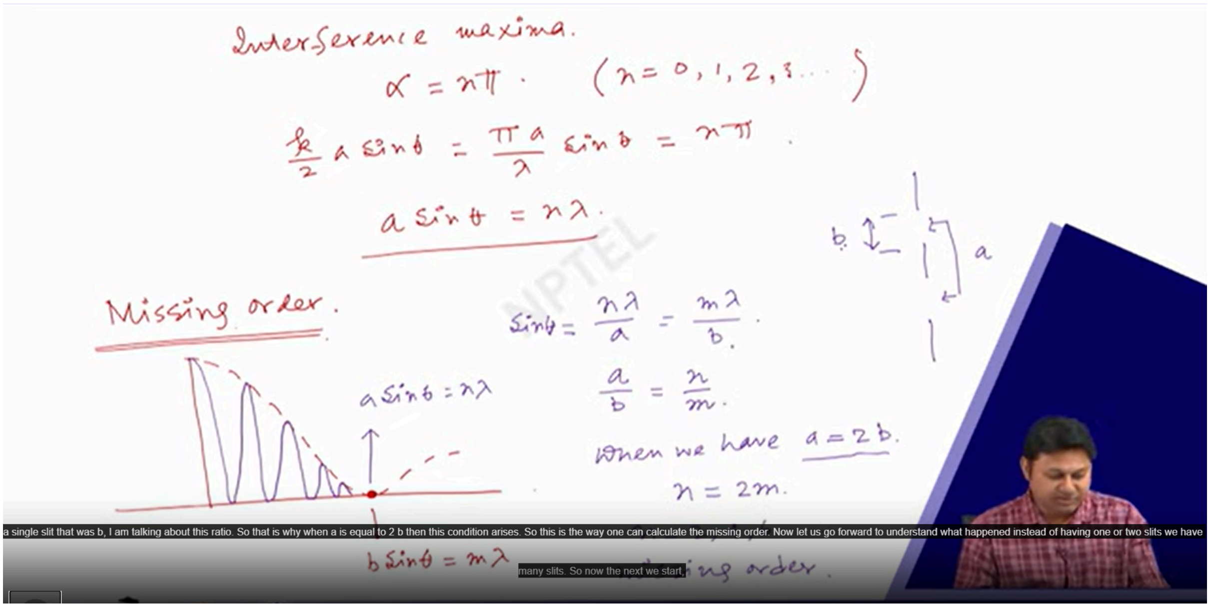

Suppose this is the dotted line I am first drawing and the condition suggests that at this point there will be a, this is the point where we have a diffraction minima. At this point, we have a diffraction minimum. So diffraction minima means at this point my condition is B sine theta is equal to M lambda. Now for the same wave, we can have an intensity distribution like this, and this intensity distribution suggests that maybe there is a maxima at this point, which becomes 0 because of the lowest value, minima of the diffraction but so interference maxima was supposed to be there and that is vanished due to this. So the condition here for interference maxima is A sine theta is equal to n lambda but this is the same sine theta. So that means n of lambda divided by a is my sine theta and I am calculating everything in the same sine theta or same theta that is equivalent to m of lambda divided by b or in other words we have a divided by b, is equal to n divided by m. So that means if I when we have, for example, a equal to 2b. Suppose we have a slit, two slits such that this a equal to 2b condition is fulfilled, then n will be equal to a equal to 2b that is n is equal to 2m that means, for if that is the case then n is the maxima n define the maxima for the interference. So, n 2, 4, 6 these are the maximum order for due to this interference will no longer be there. So, these are the missing orders. So, what is A and B? If you forget then the A was the separation between the two slits that was A and B, the width of a single slit that was b, I am talking about this ratio. So that is why when a is equal to 2 b then this condition arises. So this is the way one can calculate the missing order. Now let us go forward to understand what happened instead of having one or two slits we have many slits. So now the next we start, so, most generally, there is a multi-slit diffraction and formally it is called the grating. So, in multi-slit what happens? Instead of one slit, we have many numbers of slits and the first thing that we are going to calculate, the way we calculate for single slit and double slit is what is the intensity distribution. It should be a lengthy calculation, but very straightforward since the number of slits is large here the number of slits will be n. So, we will do the same integral and we will get a sum of this integral n times and that is why it will be a lengthy calculation, but the formalism is the same. So, pictorially if I draw then this is the multi-slits pattern and if I draw here the distribution, so instead of having one slit here we have many numbers of slits 2, for example, 3, then 4, and then this side 1, 2, 3, 4. So, this is 1 slit, this is 2 slit, this is 3 slit, this is 4 slit on one side and this is 1 slit, 2 slit, 3 slit, and 4 slit on the other side. So, these are the opaque regions and these arrows swing. So, light will go through these slits and they will interfere here at some point P. I need to calculate the intensity distribution at this point. So, as I mentioned it will be a lengthy calculation, but very much doable.

So, let us now define the coordinates as we did in the early class, early cases with this. Suppose this is my central and from here to here the separation between two consecutive slits is A and the width of this is b usual definition, then if that is the case then we need to find out what are the coordinates of these points, for example, the first coordinate it will be the straightforward first coordinate of this point it is a minus b by 2 like earlier case. What should be the next one? It is a plus b by 2 like double slits. Now we go further to this level and it should be a plus b by 2 plus something and plus a and that is simply 3 a minus b by 2 and so on. And here this one will be 5a, so let me draw in this side in a magnified version then it will be clear. So I have one slit, maybe I erase this then I have another okay. Now suppose this is my central point, so from here to here this is A and from here to here this is b true for all the cases. Now if that is the case, so this is the point here this is a by 2 minus b. So this is a minus b by 2 which is the first coordinate of the first point, then I go up to this slit, this one is a plus b by 2. So, I know this coordinate now I need to go here and this distance is I need to calculate this distance basically. So, from here to here it is a. So, from here to here it is a by 2 and a by 2 minus b. So, that is the distance we have from here to here. So, a by 2 minus b multiplied by 2 should be the distance. So, that needs to be added here and if I do we are going to get this 3 b minus b by 2, this will be similarly, sorry, 3 a minus b, I made a mistake here, this is 3 a plus b by 2, this point will be similarly 5 a minus b by 2, this point will be 5 a plus b by 2 and so on. I believe, you just follow the geometry and you can readily find out what are the coordinates of this point. So I am just doing a few cases. So for the jth slit in the upper side, we have the expression of 2j, and the coordinate is 2j minus 1 multiplied by a plus b divided by 2, this upper side means I am talking about this and lower is this. So this is for example the first slit, this is the second slit, this is the third slit, and so on, and for the lower side coordinate that will be 2j minus 1 multiplied by a minus b whole divided by 2. Okay, so we can check that actually by putting j equal to 1, you can see this is a plus b divided by 2. I am talking about this upper of 1 and this is lower of 1 and it is a minus b by 2. Similarly, for the second slit, if I put j equal to 2, then it should be 3a plus b by 2 and it is 3a minus b by 2. For the third slit, it is 5, for the sixth, the fourth slit is 7, and so on. So, I suggest that you please check once by yourself how these things are calculated. This is on the positive side and this is the negative side. On the negative side, the value will be just related to a negative number. For example, if it is symmetric, then this is minus of a by 2, minus of a plus a by 2, and so on. So, the problem is we are now having n number of slits, and if I want to calculate what is the total field the form will be something like this. So, let me just write down the form that E p is the field at point P will be E naught and sum over all the slits n number of slits, I need to put a sum here, sum j can go from slit 1 to slit n by 2 one side and lower limit is 2 j minus 1 multiplied by a minus b by 2 and upper limit is 2j minus 1 a plus b by 2 and then e to the power of i same thing k y sin theta dy. This is the contribution of the dy and I add all the contributions, plus also the lower side will be under this integral everything is the same except a minus sign. So minus of 2j minus 1 a minus b divided by 2 and it should be minus of 2j minus 1 a plus b divided by 2 and the same integral e to the power of i k y sin theta dy. So, that is the form of the integral we have.

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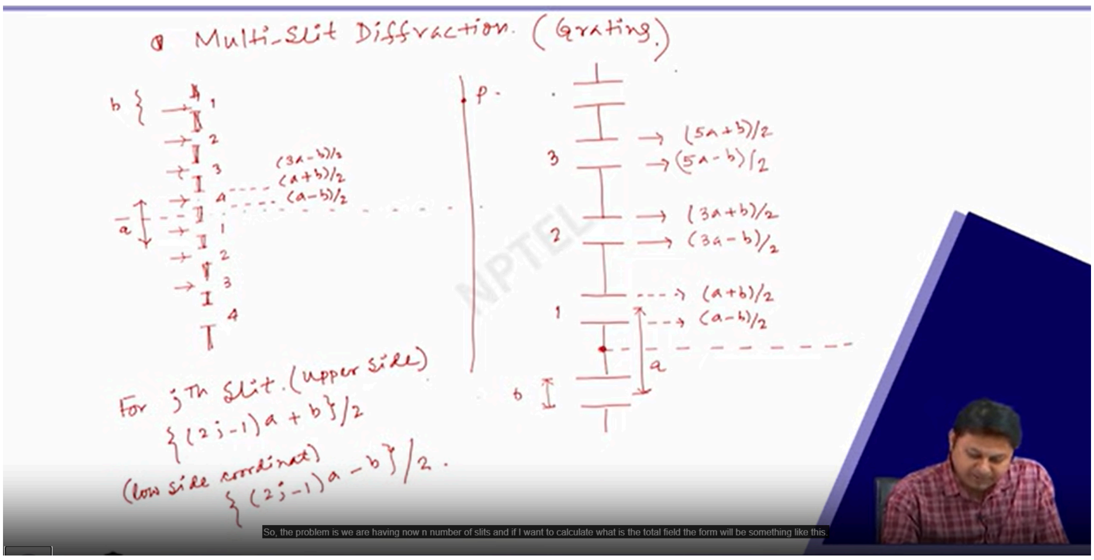

We need to solve this and essentially I can write this as the summation of, so, let me do that two contributions one is e naught and then j equal to 1 to n by 2 positive sides and a negative side and then two sums one is i 1, which should be a function of j and theta what is the value I calculate then it should be the sum over j and another is the lowest side is i 2, it should be also a function of j and theta and then by calculating i and j we can do the sum. So, today I do not have much time to discuss the calculation in detail. In the next class, we start from this point and go to calculate What should be the total field at point P due to n number of slits. So far we have done single slit, the calculation was easy, double slit also the calculation was not that much difficult, but here the number of slits is n. So, that is why I need to do a sum over all the integrals. So that we are going to do this in the next class and show what should be the final expression, the general expression for n number of slits or formally it is called the grating and how the spectra will look like because of this interference and diffraction combination of both two phenomena, then what should be the form of this diffraction pattern. So with that note, I would like to conclude here. Thank you very much for your attention. See you in the next class.

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