WAVE OPTICS

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Lecture - 34: Single-slit Diffraction (Cont.)

Hello, students in our wave optics course. Today we have lecture number 34 and in this lecture, we will continue with the single-slit diffraction problem. In the single-slit diffraction problem, we already derived that we have a single-slit pattern. This is a single-slit aperture and when the light is falling here under the Fraunhofer diffraction category, we have a diffraction pattern that is like this. It is essentially a sinc function. So, the intensity is a function of theta will be I naught sine square beta whole divided by beta square. That was the intensity distribution. We find what is beta, beta is pi D divided by lambda and sine theta is the angular separation along this direction and D is the slit width this is d and lambda is a wavelength that is applied here. So, this is overall the structure that we calculate and then we try to understand what is the condition of maxima and minima that means at which theta is going to get maxima and minima. So, let me write down the structure once again. Several times I have drawn this structure. So hope you realize how these things are happening here. So this is a symmetric structure, sin function and along this direction, we have a beta which is pi multiplied by d divided by lambda sine theta. And this is intensity as a function of theta that I plot. This is the maximum intensity, which is I naught. And if I move in this direction, I find the intensity is rapidly falling and here it goes to 0 in principle. Then again move up and then go to 0, again move up, and go to 0 like this. And this 0 point is the minima. and the condition of the minima we figure out the last day minima when a beta is equal to n pi where n is one two three and so on this is minima and from that condition, one can calculate that when n pi is equal to pi of d divided by lambda and this is the nth 0. So, I have sin theta n. So, one can calculate what is sin theta n.

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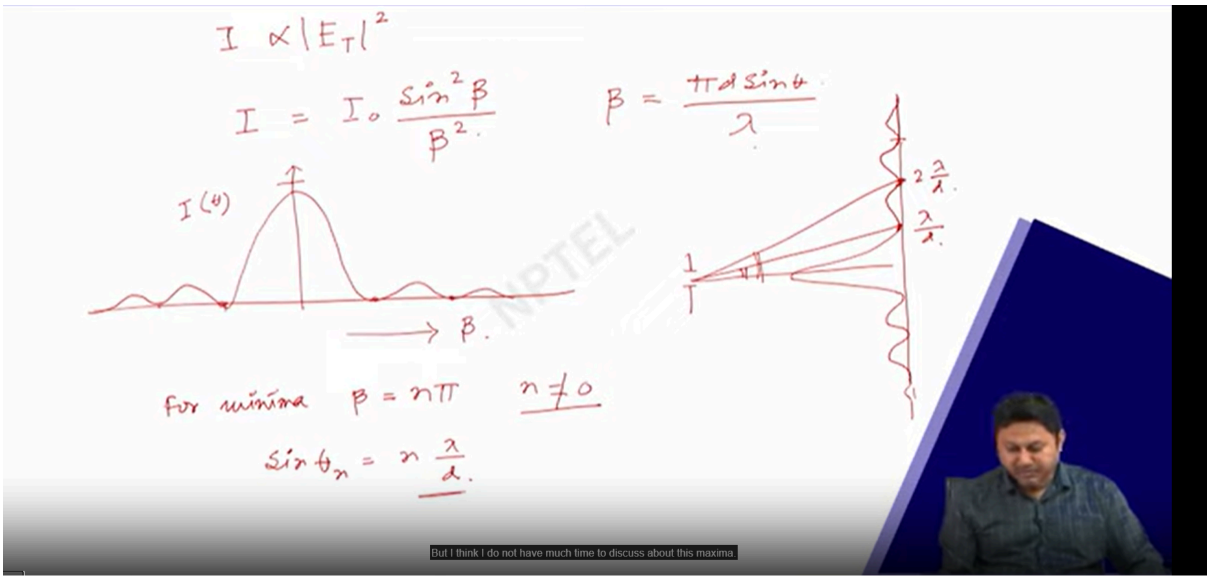

So, Pi Pi is going to cancel out. So, we have n lambda divided by d is equal to sine theta n so from this value one can calculate this is the angle at which we have the minima if we increase the value of n we can go first-second third minima, and so on now we will going to extend ah this calculation for maxima so what I am trying to get is this is the structure this is the central or principal maxima and this is the secondary maxima this is the first secondary maxima and we want to find out what is the location of the beta for that value so my ah aim is to find out what is the value of beta for this or what is the value of beta for this and so on. So, it is not straightforward because here I have I which is a function of theta or beta is I0 sin square of beta divided by beta square where beta is pi d divided by lambda and sine theta that was the thing we have so far maxima what we have that this intensity which is a function of beta with respect to beta if I make a derivative that is essentially 0 that is the condition for the maxima that the intensity of the intensity which is a function of beta the derivative with respect to beta should be 0 so that means I have I not d d beta and sin square beta divided by beta square that is 0 that is the condition for minima and now if I expand this condition it is 2 of sin beta then cos beta then beta square minus 2 of beta sin square beta whole divided by beta to the power 4. That is essentially 0. So from that it is easy to show that I can have an expression: beta cos of beta is equal to sin of beta. This is the condition we have and that condition simply leads to beta being equal to tan beta. So, this is the condition if I manage to solve this equation, then this equation gives me the value of beta for which these structures have a maxima. So, obviously, 0 is a solution you can see if you put beta equal to 0 on the left-hand side and on the right-hand side they are the solution, and the 0 solution is a solution for principal maxima which is this point. So, here we have a trivial kind of solution beta equal to 0, but I am interested in finding out the solution here at this point and so on so that means not only one solution we have many other solutions so beta here several times I am writing this is pi d divided by lambda then sine theta that is the value of the beta Well in order to solve this, so this equation cannot be solved analytically. So, we need to use the graphical method of that and that means I need to plot tan beta and beta together and their cutting point will be the solution we are looking for.

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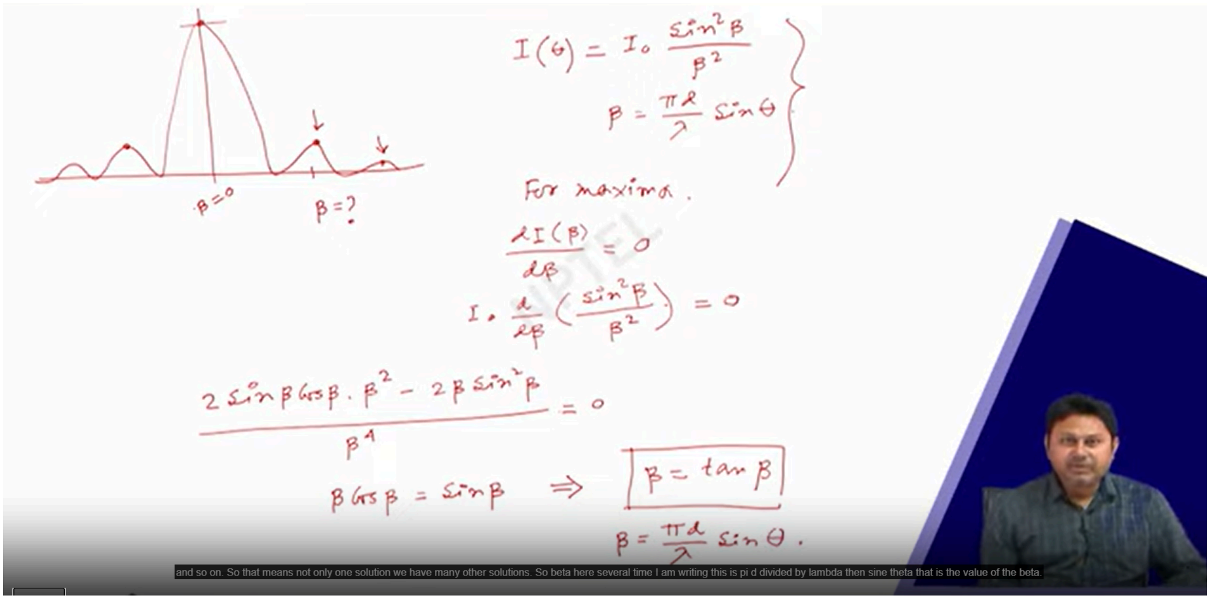

So, this is the plot, we have said this point is pi by 2 and then this is pi and then this is 3 pi by

2 then this is 2 pi and so on so we know that the tan how the tan curve behaves, so the tan 0 is 0 and when it is pi by 2 it reaches to infinity so I have a curve something like this and then again at minus pi by 2 it goes to 0 and then it come back and come here and again it goes to like this so this is the way the tan curve will go and now I need to draw the beta curve also so this is the green line is basically these are tan beta curve y equal to tan beta curve I am drawing essentially. So, I draw here the beta y-axis this y equal to beta and y equal to tan beta both I am and here in this axis I draw x equal to beta. So, beta is a function. So, x y I am plotting as a function of beta. So, this is tan beta. Now, I draw the y equal to say beta I which is a 45-degree line roughly. So, you can see that it cuts to a point. So, the first point is 0, and the second cuts at some point. So, and also it cuts to another point and so on. So, the solution the first solution of beta is equal to roughly around 1.43 pi this is the point when once if you do this calculation using a computer then that should be the first solution the second solution will be when beta is equal to 2.46 pi the third solution when beta equal to 3.46 7 pi and so on so these are first three solutions so this first three solution means if I draw the structure draw the diffraction pattern and if this is my beta so this point is my beta 1 first solution and it is 1.43 pi this is my second and the value of beta which is four six pi this is my third one and this value is beta equal to three point four seven pi and so on so this is the points where the values of beta I calculate So, now I need to calculate also what should be the amount of intensity. Here we know that is the intensity and that intensity is I naught and the intensity as a function of beta is known which is I naught sin square beta divided by beta square. Now once we know the value of the beta as the maxima so when beta tends to 0 that is this point then I theta I as a function of beta is simply I 0 which is the maximum intensity. So now I want to find out the first I max so the first secondary maxima what is the intensity of that? So of the first secondary maxima if I write first that is the value of i at beta equal to that point 1.43 pi so whatever the value you have on the right-hand side if you put this value then so essentially this is i naught I have sine square beta divided by beta square that is calculated at the value beta equal to 1.43 pi.

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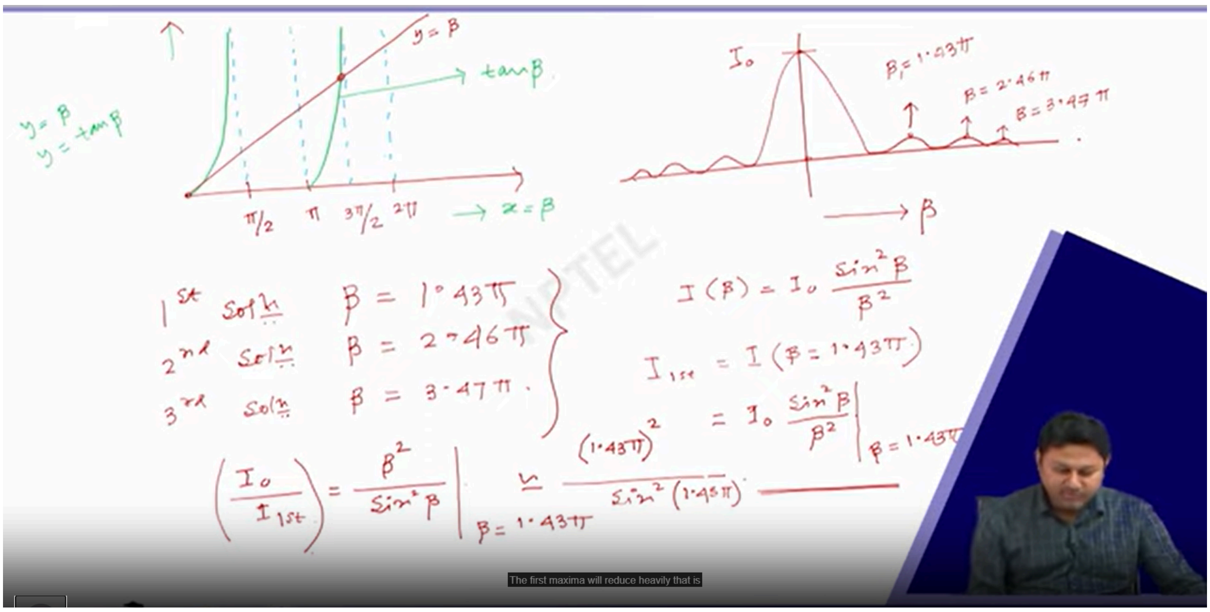

So, this is the value where this is the amount of intensity of the first maximum, the first secondary maximum. So, this value if I now want to find out what the ratio should be. So, I first divided by I naught. That is the ratio between this intensity and this intensity just to find out how small the first minima is compared to the principal maxima intensity-wise. So that ratio is essentially beta square divided by sine squared of beta. at the point beta equal to 1.435 and if I calculate if somebody calculates this amount it will be nearly equal to 221.2 which should be the value. So, that means no this is not actually this value yeah so so this actually points to this value I am okay let me erase this actually if so beta square okay I need to calculate that what is the value of the beta because I need to calculate sine and then so this value is okay let me write it I want the student to calculate these things so beta square means it will be 1 point something like 1.43 and pi whole square of that and divided by divided by by this whole divided by sine of okay it is not erasing properly yeah here we have so whole divided by sine square and that value actually we need to calculate 1.43 pi square of that. So, I get a value here and that value basically tells us what should yeah I first okay so here I made a mistake, so that's why it creates some problem this is I not, and this is I first. So whatever the value you get here after calculating, you will see that that value will be the ratio of I zero divided by I first. And you will find that this value should be of the order of, as I mentioned, it should be of the order of 20. So that means there will be a 20, there will be a, so the maxima, So, the principal maxima is nothing but the 20 percent roughly the 20 percent of the central maxima. So, the first principal maxima is roughly about 20 percent if you calculate and find this value you will find it is roughly 20 percent of the minima. In the similar way you can also calculate what is the secondary maxima. and then the secondary maxima, the second secondary maxima the third secondary maxima, and so on. And you will find that there is a rapid, very rapid decrease of the intensity if you go more and more in order of this maxima. The first maxima will reduce heavily that is the that is like this if it is 1 if it is 100 then this value is merely 20 and then you will find it will be much less and so on so if I have this is i 0 so this value is roughly 20 of the maxima I 0 and the rest of the value is even ah even smaller.

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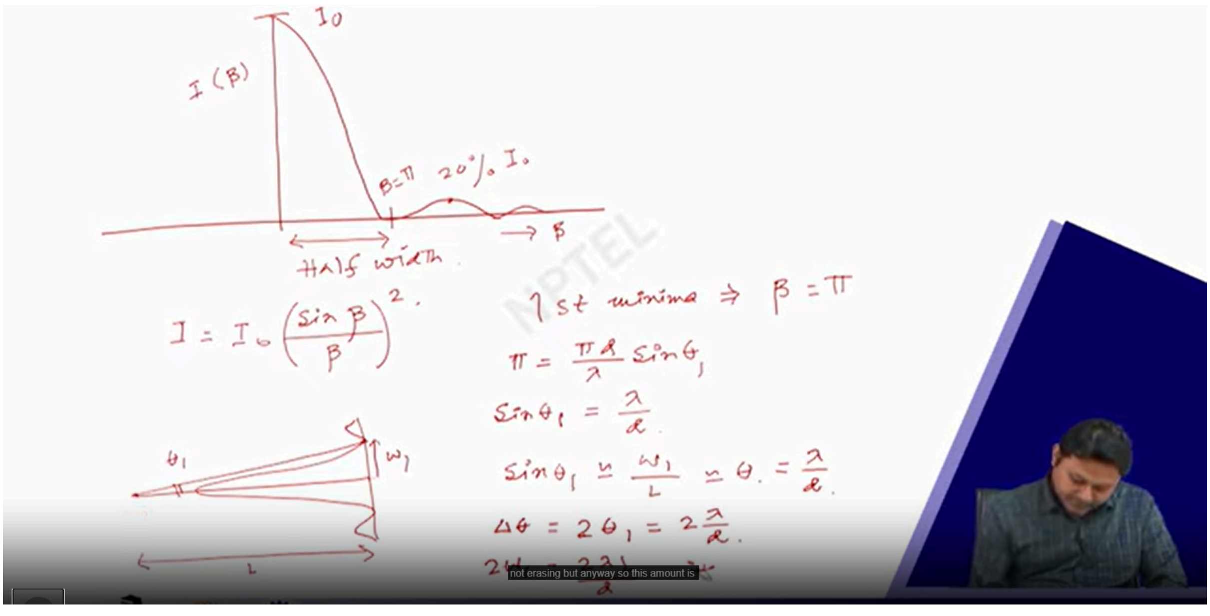

Next one thing I need to calculate is the width of this principal maxima. So, the width of the point from here to here is called the half-width. So, what we calculate is the beam with whatever the beam we have that we calculate. So, I which is a function of beta is distributed in this way along this direction we have a beta. So, I is I naught sin beta divided by beta, and the whole square of that is a, and the first minima is when beta equals pi. So, at this point, beta is equal to pi. At this point, we have beta is equal to pi. So, that means, we have pi equal to pi d divided by lambda and sine theta first minima and I write theta 1 so theta 1 is the angular separation from this point to this point so that gives me simple sine theta 1 is equal to lambda divided by d So what I am calculating this is the structure let me draw it. This is the principal maxima and then we have secondary maxima and from this point here to here this angle is my theta 1. this angle is my theta 1 so if we have this length as L and from here to here from the central point to this point if this is my say W say W 1 for first so then I have sine theta for small theta sine theta 1 is nearly equal to W 1 divided by W 1 W1 divided by L and that value is nearly equal to theta 1 or this is lambda divided by D so my angular separation delta theta will be 2 of theta 1 It should be 2 of lambda d. That is the angular width. And if I want to find out what is the spatial width, that is 2 of w1. That is double the half-width and that is simply 2 of lambda of multiplied by L divided by D. So, that is the width of not erasing but anyway, so this amount is so So that is my delta theta angular width and that is equal to if from this point it is L from here to here and here to here if it is W. Then delta theta equal to 2 of theta 1 and that is 2 of W divided by L and width is the width of the central maxima. will be simply W P that is 2 of W will be 2 of lambda of L divided by D so obviously depends on L the point is if I move the screen so this is my source double slit source and initially I have a screen here and I have to erase this So I have a source here and this is the structure we have for one length and for another length I have the structure like this. So the width of the central maxima will increase over the distance. So this is my theta 1. If this is my distance the first L1 and if this is the distance L2 the width here w1 width w1 so the width here w1 will be 2 lambda divided by d which is constant multiplied by l1 and here w2 will be 2 . multiplied by L2. So, these are the two that one can have and this width depends on how far the screen is placed compared to the source. So, today I do not have much time to discuss more regarding slits mostly this is the expression we had this is the calculation we have for single slit diffraction in the coming class we will extend this idea and calculate how the intensity distribution will going to change if I introduce another slit that is instead of having single slit if we have a double slit then what should be the distribution of the intensity and then calculate all the things fringe width, etc for that case so with that note i like to conclude here thank you very much and see you in the next class.

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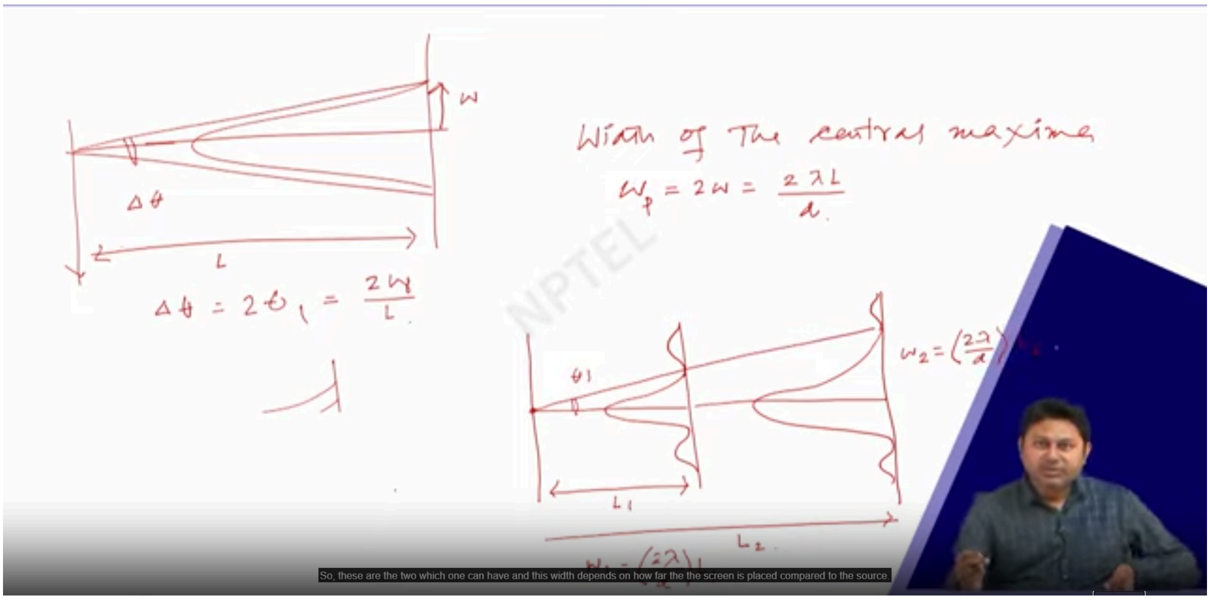

W
Δθ
L
Δθ = 2θ1 = 2W/L
Width of The central maxima
Wp = 2W = 2λL/d
NPTEL
θ1
L1
L2
W2 = (2λ/d) L2
So, these are the two which one can have and this width depends on how far the the screen is placed compared to the source.