WAVE OPTICS
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Lecture - 33: Single-slit Diffraction

Hello. student, welcome to the wave optics course. Today we have lecture number 33 and in today's lecture we are going to discuss the single slit diffraction problem. So already we have started this problem in earlier class. So we have lecture number 33 and in the last class we mentioned that we try to calculate the Fraunhofer diffraction pattern of an aperture like this. So we have an aperture here which is essentially a single slit and then the light is falling here. The plane wave that is coming here. And then we have an aperture that then hits this aperture and we introduce a linear oscillator model here that when it hits the aperture then this portion of this aperture allows the light to pass through. But here at this portion we have the location of linear oscillators and then it waves are emitted from these oscillators and if I place a lens here then at some point P this hits here and then it converges to a point p and we want to find out the intensity distribution at p, that is the overall structure. So this is the light waves that are passing through this aperture and then they are allowed to merge at some point p on the screen and this is the lens l. So what we find is that if I magnify this portion, these are the individual oscillators which emit the waves. These are the light waves that are moving like this and then it converges to a point P. So, what we were doing is we add all the contributions of this wave and in order to do that we name the waves as E 1, E 2, E 3 and E n. If I consider n number of oscillators then these are the field of the waves that is coming out where E in general E j was E 0 amplitude e to the power of i k rj minus omega t, that was the form of the wave and the total field Et for example it is sum over all the field Ej, j goes to 1 to n and we did this sum and when we did this sum we get some value and let me write down that value.
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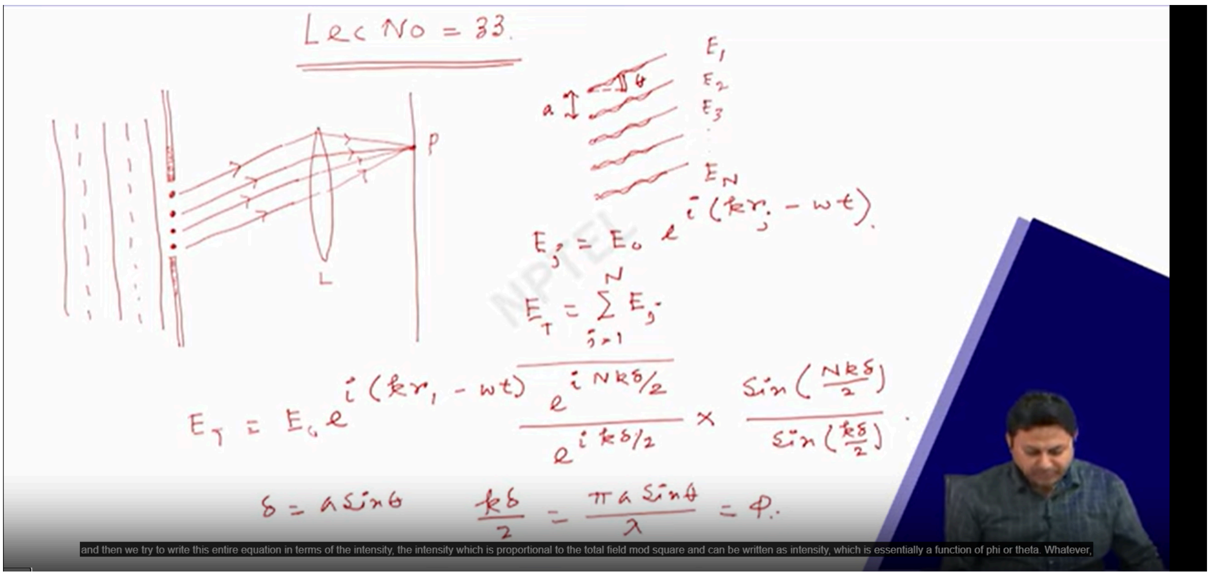

So, my total electric field Et was E naught e to the power of i krj sorry kri, kr1, rather omega t that was the term that we take common and then we have something like e to the power of n k delta by 2 whole divided by e to the power of i k delta by 2, that was the term and then multiplied by one important term and that was sine of n k delta divided by 2, wholly divided by sin of k delta divided by 2. Where delta is the important term and that is. So, delta here is A of sin theta where A is the distance between two consecutive source points and theta is this angle theta. So, k delta divided by 2 that term was essentially pi A sin theta, whole divided by lambda that was the value. Also, this k delta is this quantity we write as phi and then we try to write this entire equation in terms of the intensity, the intensity which is proportional to the total field mod square and can be written as intensity, which is essentially a function of phi or theta. Whatever, it is some intensity I naught then sin square n phi whole divided by sin square phi, where phi is equal to k by 2 delta, which is pi a sine theta divided by lambda. Okay that we derived the last class also one thing d, if the separation of the aperture is the width of the aperture rather is d then d is equivalent to n my plus 1 if a is a number of linear oscillators and a is a separation between two consecutive linear oscillators, then that is the relationship between d and a and that is constant, that is n, the number will going to increase if a is decreased but their multiplication will remain conserved. So, when n tends to infinity then a tends to 0 that is the condition. So, for large n, d is nearly equal to Na, and essentially my I, is I naught sine square and phi whole divided by, sine square phi, that is original expression and then n phi is Na pi divided by lambda sine theta, that is equal to d pi divided by lambda sine theta and phi is that quantity. So for small phi sin phi is nearly equal to phi, that is another thing. So essentially, I, which is a function of phi, we can write down as I naught tilde sine square n phi, by phi square which is the thing we derived in the last class. So, I re-do that where phi is that quantity, pi a is all divided by lambda and sin theta is the value of the phi. So, now I am modifying it. Today I would like to modify this thing slightly. So, i theta which is a function of phi or theta that I can write i naught tilde, then I write n square. I will write an extra n square here, sin square n phi, and then in the denominator I have n square phi square or n phi whole square. So, I add one n square here in the numerator and then add another n square.

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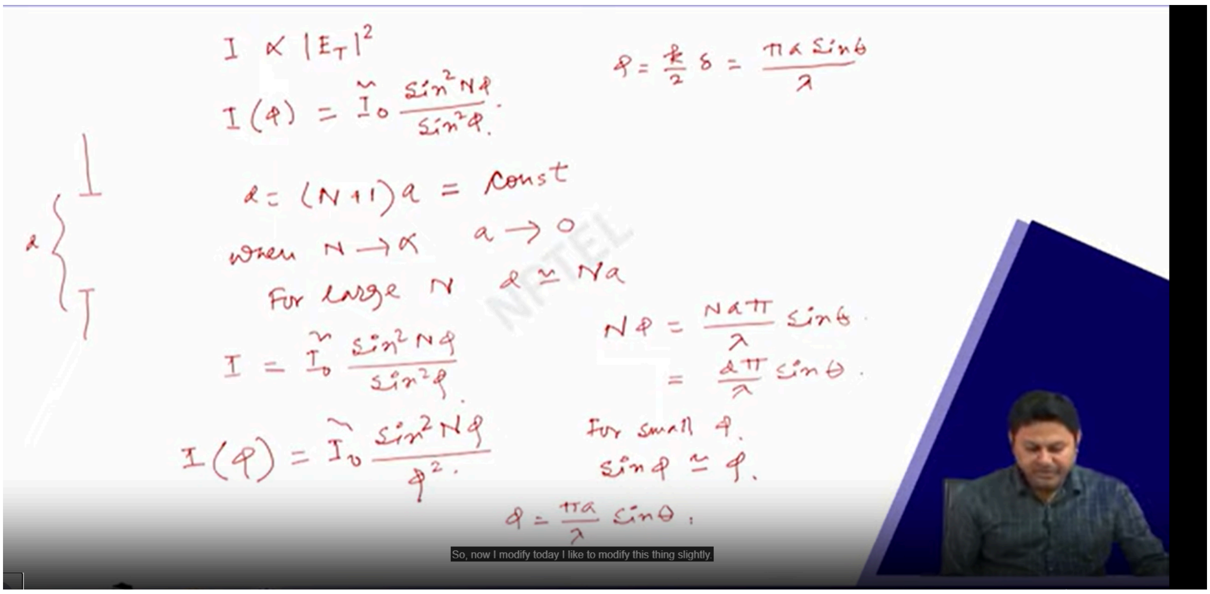

So, this n square will cancel out and I go back to the original expression by writing. So, this expression now has a specific form and it is I 0 then sin square beta whole divided by beta square, where my beta which is N phi, which is pi d sin theta whole divided by lambda. So I write here and that is the intensity distribution expression and now if somebody wants to plot it, this is a thing that is called a sinc function then, the intensity will look like, so the overall picture will be like this. So I have an aperture here and we have a sinc function. So it will be distributed this way. The intensity will be distributed in this way. This is the square of the sinc function. So the intensity of this distribution will be like this. So if I draw here this function, that will be a central maxima and we have a symmetric kind of distribution of the intensity like this. So that is the intensity pattern one can expect. So this value is the peak value, I naught and this is the direction, this is the change of beta and beta is this quantity. So, a change of beta means essentially for a fixed wavelength it is a change of theta as well. So, here you can see that the intensity is I 0 and when D is very much greater than lambda. So, D is very very greater than lambda means when beta is a relatively high value. Then with the slight change of theta, there is a very large drop of this amplitude peak. So we are going to discuss more about this structure now. So we have a minima on these points that one can readily figure out with this expression that, when the beta value is n pi then in all the cases this will be 0 and we also have some maxima and we are going to calculate what is the condition to get this maxima. But before that once we know this is the structure of the intensity distribution that one can get from the single slit diffraction pattern as this gets as a single slit diffraction pattern. So, this calculation now we will do this in a more general way without assuming the linear oscillator model which is a straightforward single-slit diffraction calculation also we proceed with this calculation for double-slit and multi-slit. So, let me first develop this calculation. So, this is a single-slit diffraction problem and this single-slit diffraction problem. Instead of doing a discrete summation, now we will do a constant summation in this way. Let me draw that. So this is the single slit we have. So this is the slit width and let me draw the coordinate. This is the middle point of the slit and suppose this is my x-axis. So this is my x-axis and along this direction, I should have a y-axis.

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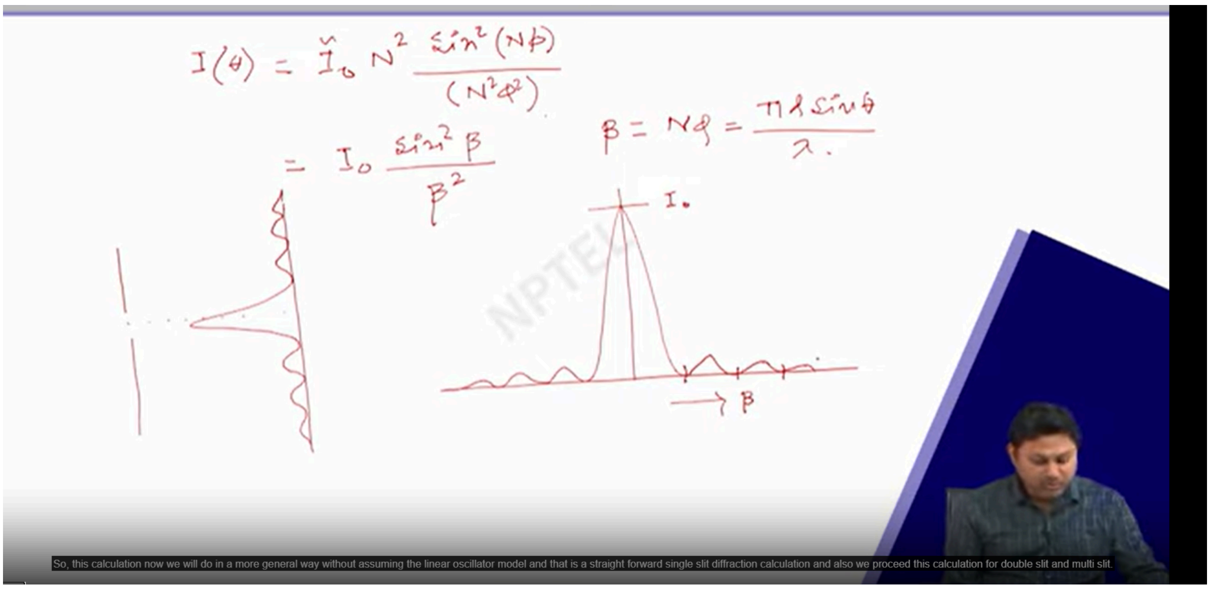

Okay so, this is the point I call the central point as 0 0 point, and if I move if I take a small section here this small section dy is at the point of y. And this is my central point. So, coordinate-wise if this entire width is d then this point is d by 2 and this point is minus d by 2. This is the coordinate of these two points, y coordinate and x coordinate is 0 because this line I consider x equal to 0 over x equal to 0. Now the ray will come from two rays that come from these small sections and another will be from zero. So the electric field that is coming out from this small section at some point p is dE so dE is an electric field that is the contribution of the small section that is coming at some point p. So, that value dE will be essentially amplitude E naught and then e to the power of i delta and dy. So, the field at any point p due to dy is this. So, this is a field at any point P due to the aperture dy, that is the amount of the field one can have and delta. What is delta here that is important is the phase difference between the point y 0 and y. So, this is simply, I write the phase difference between the point y and y plus dy. So the phase difference between these two is essential. So that means if one ray is coming from this line and another ray is coming from this line or another way, the phase difference can be calculated by joining this. If this angle is theta this will be theta. So, the path difference one can calculate the path difference delta is y sin theta because this is the extra path difference. So this is the extra path difference we have. This value is y sine theta and the phase difference comes out to be 2 pi by lambda y sine theta or it is simply k naught multiplied by path difference delta. So, that is the phase difference. So, I calculate the phase difference and that is this. So, this I can write also as b k multiplied by y where b k is 2 pi by lambda multiplied by sin theta, and for a fixed angle this is constant okay. So, now the total is the contribution of d. So, the total field that is the goal of our problem, the total field at some point P will be the superposition of all the contributions, and the superposition of all the contributions means, I need to integrate the field over the range of d. So, E t the total field will be E t and that is integration running from minus d by 2 to d by 2 and dE. So, essentially I am using the same principle that we have done earlier, that summing over all the contributions of the linear oscillator, individual linear of the field that is emitting from the linear oscillator, we combine everything as a sum here we are doing, but in a continuous manner. In the previous approach, we calculated as a discrete sum, but now we are doing this as a continuous sum by integrating these things.

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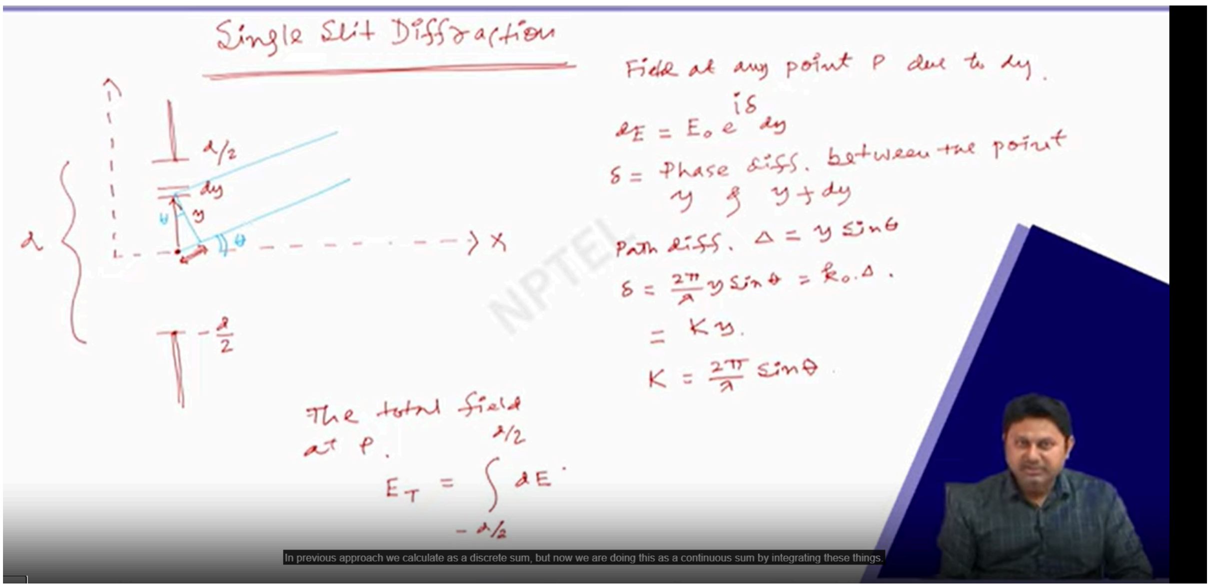

So, dE I know what dE is. So, this is E naught then integration minus d by 2 to d by 2 that is the limit, and then e to the power of i delta, and then we have dy. Note that inside the delta, I have y delta is ky, so let me do this integration here so ET total field at some point P is E naught integration minus d by 2 to d by 2 and then I have e to the power of i big ky dy where big k is 2 pi by lambda into sine theta and that is constant here this integration is very straightforward. So we have e naught, e to the power of i of k y whole divided by i k sorry i big k and it will be d by 2 and minus k by 2 and that is simply E naught and when you put this d by 2 plus d by 2 and minus d by 2 we simply have, 2 i sin of big k d by 2 k d by 2 whole divided by the denominator we have k d by 2 and also 1, i will be there and this d by 2 I put because we in the in here we have a d by 2. So, what I do I will multiply a d by 2 altogether to make it a sinc function. So, this is essentially E naught d and then sin of k d divided by 2 big k d divided by 2 and here we have big k d divided by 2. So, this is E naught d sine of pi d divided by lambda because I am now putting the value of the k. It is pi d divided by lambda and then we have sine whole theta divided by pi d lambda. Now this we write as, I mean this quantity I can write at E naught d sin of beta divided by beta. So, you may note that this beta we already defined I mean in this calculation let me go back. So, the beta was pi d sin theta divided by the lambda that we calculated and the same notation I will use here that pi d divided by lambda sin theta is my beta. So, after having that I can write that my intensity should be proportional to the total electric field square of that. So, intensity can be written simply as I naught sin square beta divided by beta square the same expression that we derived in the linear oscillator model but here we did the continuous integral. So where beta is pi d sine theta divided by lambda the intensity will be distributed in this way, This is the distribution of the intensity. This is the peak intensity along this direction. We have I as a function of theta or beta and on this side we have beta, which is this quantity. Now, if I want to find out what is the maxima and what is the minima then it will be easy. So, quickly let us find out. So, this is the point we have minima. Let us find out the minima first. This is the point we have minima. So, for minima beta will be simply n pi, where n is integer, but note that n should not be equal to 0. So, n is 1 2 etcetera. So, sin so, when the beta is n pi. So, sin theta n will be n of lambda divided by d because beta is pi d sin theta by lambda. So, instead of beta if I write n pi is pi d sin theta divided by lambda. So, theta n should be n lambda divided by d. d. So, these are the thetas at which we have minima. So, how will these things look? So, the structure will be like this. So, we have a structure like this and it goes to infinity with reducing amplitude and if I calculate from this is the slit width and if I calculate this is the theta, this is my 1 theta for which I have a minima then we have another theta here for which we have another minima and all these things. So, this theta is lambda divided by d. This theta is 2 of lambda divided by d. This theta is 3 3 lambda divided by d and so on. So, this is the point we have where we have minima. Now, quickly understand where we get to which point we get the maxima. But I think I do not have much time to discuss this maxima. So, the time is up. So, in the next class what we try to describe for this single slit pattern is how one can calculate that as which theta we will get at the maximum points. So that is a bit tricky, but we will try to figure out by using something called a transcendental equation how one can find out the location of the maximum peaks when light is diffracted by a single slit under the Fraunhofer diffraction pattern. So with that note, I would like to conclude here. Thank you for your attention and see you in the next class.

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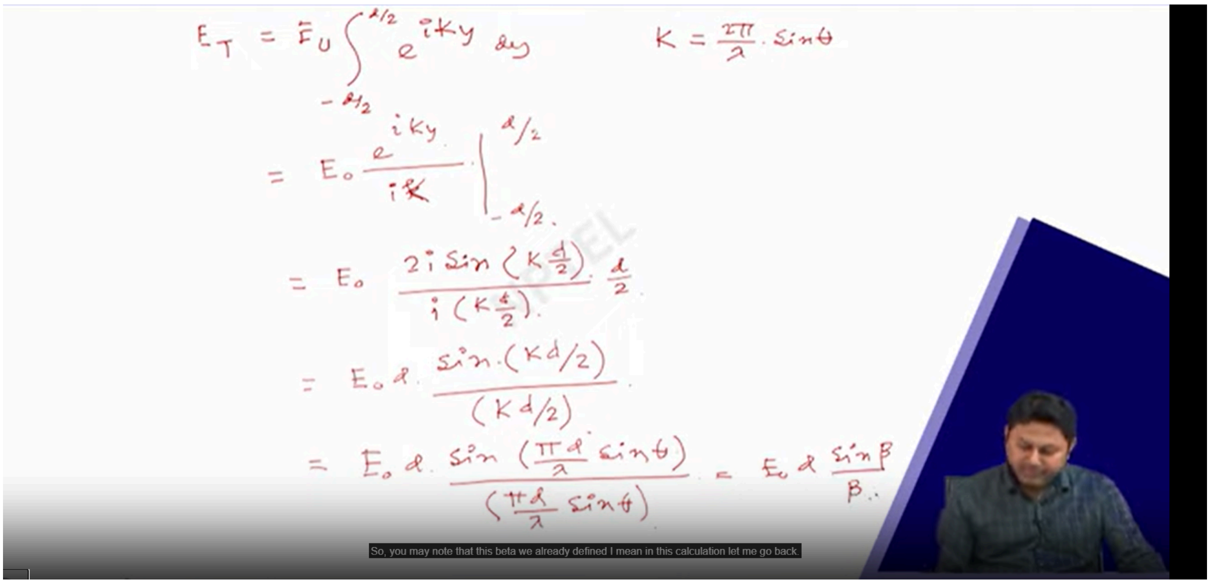

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