

**WAVE OPTICS**  
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**Lecture - 31: Huygen's Theory**

Hello, student to wave optics course today we have lecture number 31, where we will discuss more about the Huygens theory. So, we have lecture number 31. So in the last class, we discussed the Huygens theory. We suggest that if we have a wavefront that is moving at each point of the wavefront, for example, this is a wavefront, a part of the wavefront we will produce a secondary spherical source and then the tangent of these things is going to propagate. So for a propagating wave, this is the wavefront and each point on the wavefront is a secondary source. It also produces this kind of spherical wavefronts. And if I join the tangent line, then that should be the next wavefront, and so on. So this is the way in Huygens principle the light is propagated, the light will go to propagate and now in today's class what we try to understand, is that this is the direction of the propagation, and if it is at  $\Delta t$  time then if the velocity is  $v$  and then  $v$  multiplied by  $\Delta t$  with this distance and this is the direction of the light propagation. So if the light is propagated in a particular direction then the corresponding wavefront that we draw is this dotted line and you can see that this dotted line is the wavefront of this propagating light assuming that the source is present at infinite then this wavefront is basically perpendicular to the direction of the light. So this is the wavefront and this is the light ray. It starts with a point, even though this starts with a point and this is the way the wavefronts are going to emerge or propagate and when it goes to a very large distance it essentially looks like a straight wavefront. So initially it is circular and then for sufficient length after propagating a sufficient distance, this is the form of the wave front one can have. This is called the plane wave front. So we have a plane wave front for a large distance. So assuming the light is coming from a source that is placed at a very large distance, then this is the way the wave front will move and that should be perpendicular to the light rays.

Now let us try to understand the laws of reflection and refraction in terms of Huygens's theories. So that should be one of today's topics and so are laws of reflection from the wave theory. So already introduced wave theory through Huygens theory, that light is propagating in a particular direction and then this wavefront on all the points on the wavefront are the secondary source of the secondary sources etcetera. So based on that, first, we try to understand the laws of reflection. So let us draw first. So this is the interface or the reflecting surface and so the light ray will fall here. These are the three light rays that are falling over the surface. And let me draw the corresponding wavefront here, which should be maybe I can

put a different colour. So this blue line is a corresponding wavefront and the light rays are those that are falling on the surface. If there is no such boundary then light should propagate and this wavefront should move without any obstacle let me draw a wavefront here and the light ray will essentially reach this point. Okay so, let me now put some names here. Suppose this point is A, this is A1, this point is B, this is B1 and this point is C. Also, because of the presence of this AC plane which is a boundary through which the light is reflecting back then what happened, that the ray that is coming through A in the absence of this boundary should go from A to A prime but because of that there will be reflection of light. So let me draw this reflected light here and this is the reflected light for the first ray. So this is, let us put O, this is the reflected light for O and this is the second ray and third ray that is hitting C. Now according to this Huygens theory what happens is that there will be a secondary wavelet and if I join the tangent of this curvature, that should be the wavefront of this reflected ray which is again perpendicular. So, let me draw it carefully. So that dotted line that I put here is the line that is joining this envelope, that is created by this reflected ray and that should be the wavefront of this reflected ray. So now let us designate the names. So this is AB, this is A1, B1, C and now let us put this as B, this point as B, the wavefront let us put the name and this point as C and this point is C1, I should not write this as B, this is B, this is B1, so this B should not be there. Instead of that let us put a different name B and the reflected ray that is sitting here is A2 and this is B2 okay so that is the structure we have. And this angle, so this angle whatever I am drawing here is the incident angle A, and this angle OAK or angle A is designated by i. This angle however is making the reflected ray I call it r. Okay now let us start. So this incident wavefront I defined as ABC this line and the reflected wavefront I defined as A2 B2 and C1 this is the reflected wavefront. If the light velocity is V0, so V0 is the velocity of light, then A if you note carefully A A1 that is the length the light should propagate in the absence of any boundary at some time t will be V0t. But because of the presence of this boundary AC 1, the light reaches from A to A2. So AA2 is equal to V0t as well. So, essentially we have AA2 will be equal to AA1, that is the condition we have first. Now, let us consider these triangles. There are many triangles that are formed here. So, let us consider one by one. So, from triangles A A 2 and C 1 and A A 1, C1 these two that are AA2 C1 and AA1 C1 we get you can see that the angle AA2 C1 is 90 degrees. So, let me write it here: angle AA2 C1 is equal to 90 degrees and AA1 C1 is also 90 degrees and they have a common line AC1 and also AA2 and AA1 are the same. So, these two triangles are similar triangles.

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⊙ Laws of Reflection from the "wave theory"

Incident wavefront ABC  
 Reflected wavefront A<sub>2</sub>B<sub>2</sub>C<sub>1</sub>  
 v<sub>0</sub> → vel of light.

$$\left. \begin{aligned} AA_1 &= v_0 t \\ AA_2 &= v_0 t \end{aligned} \right\}$$

$\Delta AA_2C_1$  and  $\Delta AA_1C_1$   
 From  $\Delta AA_2C_1$  &  $\Delta AA_1C_1$   
 $\angle AA_2C_1 = 90^\circ$   
 $\angle AA_1C_1 = 90^\circ$

From  $\Delta AA_2C_1$  &  $\Delta DB_2C_1$   
 $\Delta AA_1C_1$  &  $\Delta DB_1C_1$  } Similar  $\Delta$ s.

$$\frac{AA_2}{AC_1} = \frac{DB_2}{DC_1} \quad (1) \quad \frac{AA_1}{AC_1} = \frac{DB_1}{DC_1} \quad (2)$$

and the angle here are 90 degree to each other and if that is the case then I can write that AA2C1 which I defined as i and AA1C1 the angle is also i

On the other hand, another angle is there, these are similar in the same way we have DB to C1 and so this and this are identical triangles also from triangles AA2C1 and DB2C1 they are similar angles, and from that these two and AA1 and triangle DB1C1 these two triangles are similar. Since these two are similar triangles, then we are allowed to write AA2 divided by AC1 is equal to this and this ratio is DB2 divided by DC1. This is one equation. Another equation also I write is AA1 divided by AC1 which is equal to DB1 divided by DC1 this is equation 2. So from equations 1 and 2 go to the next page as AA2 is AA1. So we can have DB2 equal to DB1. What is the meaning of that? The meaning is if I look at it, DB2 is equal to DB1, that is DB1 is this and DB2 is this. So they are the same, that is, when the ray is reflected from A to A2 during this time B will reach D and then reflect and go to point B2 exactly where it should reach the geometry suggesting that it should reach exactly at this point over this line. Once this is ensured then the rest part is straightforward let me write here that as I mentioned already angle AA2 C1 which is equal to angle AA1 C1 both are 90 degrees. So let me check which angle I am talking about. So AA2 C1, so this angle I am talking about and this angle are 90 degrees to each other and if that is the case then I can write that A2AC which I defined as r, and A1AC1 this angle is also r. Okay so, these two angles should be r. So, I have angle A2AC1 is equal to angle A1AC1 this is r also one important expression I have is that i plus r is 90 degrees. So if I go back and check that this is 90 degrees, this angle is r and if this is r then this angle, let me draw it, then this angle must be r. So, you can see that i plus r should be perpendicular because KA is perpendicular to the plane of this reflection. So, this angle r and i they are, i and r, if I add these two they should be 90 degrees. So if that is the case then we can have OAK and also this angle KA. So let me check here if that is the case then I already have, so, angel OAK then has to be equal to angle KAA2. Which angle am I talking about? Let me go back, OAK means OAK, that is this angle is nothing but i as per what we define and that is equivalent to KAA2. What is KAA2? KAA2 is basically the reflected angle. So, that is nothing but the reflected angle. So, this angle is the same. So, I should have this as i. So, that means the incident angle is equal to the reflected angle and that is the law of reflection.

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$AS AA_2 = AA_1$  we can have  
 $DB_2 = DB_1$   
 $\angle AA_2C_1 = \angle AA_1C_1 = 90^\circ$   
 $\angle A_2AC_1 = \angle A_1AC_1 = r$   
 $\therefore i + r = 90^\circ$   
 So  $\angle OAK = \angle KAA_2$   
 $\parallel \quad \parallel$   
 $i \quad r$   
 Incident Angle = Reflected Angle.

So, exploiting Huygens's theory of how the light will propagate in terms of this wavefront one can show that the laws of reflection basically hold. So, this is one law. So, another law I need to prove quickly is the law of refraction from the wave theory. By exploiting the wave theory we will do that. So let me draw this is the interface I am drawing and I draw this is perpendicular and then light will fall here as before three rays will fall here and let me draw the wavefront, the way we did earlier, which is perpendicular to the ray. This is the wavefront I am drawing and if this interface is not there then the light should reach somewhere here. Okay, and the light I draw is this here at this point but the refractive index here in the upper and lower region is not the same. So here the velocity of the light is  $v_1$  and the refractive index is  $n_1$ . In this region, the velocity is  $v_2$  and the refractive index is  $n_2$ . Since there is a change in the refractive index and velocity, so the light will move in a different direction suppose it goes in this direction after refraction this moves in this direction and this is the wavelets that is generated and if I draw the tangent then this line is basically the tangent of these things. Okay, I need to engage a bit in this, now I define say this is the ray O hitting the point at A and this is A, say, MN this is the wavefront that is moving this point and this point and the rays are coming this direction and then it moves to here and here this point say B and this point is C. So this is A1 this is say B1 this point is B2 and this point is A2. Okay, this is the way the light is moving here and say this point is  $k_1$  and this is k. So what is the incident angle? Let us define this first the incident angle, here is this  $i$ , this is the incident angle and this angle is the refracted angle  $r$ , okay so now let us start with our treatment. So here  $n_1$  with the refractive index which is  $C$  divided by  $V_1$  and  $n_2$  should be  $C$  divided by  $V_2$ . So,  $AA_1$  will be  $V_2$  multiplied by  $t$ . This is the time taken for the light to go to  $AA_1$  but if there is no boundary then the light should go  $A_2 A_1$ . So,  $AA_2$  will be  $v_1$  into  $t$  which is the light propagating in the original medium that is  $n_1$ . So, again from triangle  $AA_1C$  and triangle  $BB_1C$ , which are similar triangles that we already described in the last calculation, I can have important expressions like  $AA_1$  divided by  $AC$  is equal to  $BB_1$  divided by  $BC$  and also from triangle  $AA_2C$  and triangle  $BB_2C$  in a similar way we can write  $AA_2$  divided by  $AC$  is equal to  $BB_2$  divided by  $BC$ , just taking these two angles like before, I can have two important expressions and these expressions are one is this and another is this.

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Laws of Refraction from the wave theory.

$v_1, n_1$   
 $v_2, n_2$

$n_1 = \frac{c}{v_1}$      $n_2 = \frac{c}{v_2}$

$AA_1 = v_2 t$   
 $AA_2 = v_1 t$

$\angle AA_1C = 90^\circ - r$   
 $\angle AA_2C = 90^\circ$   
 So  $\angle ACA_1 = r$   
 $r = \angle BCB_1$

From  $\triangle AA_1C$   
 &  $\triangle BB_1C$

$$\frac{AA_1}{AC} = \frac{BB_1}{BC}$$

Also from  $\triangle AA_2C$  &  
 $\triangle BB_2C$ .

$$\frac{AA_2}{AC} = \frac{BB_2}{BC}$$

now I need to go to next page, also in the same way angle A2AC that is 90 degree

Now once we have this then the rest of the part is straightforward. Here I should put that little information that angle  $AA_1C$ ,  $AA_1C$  that angle is 90 degree minus  $r$  right. Similarly, angle  $AA_2C$  which is 90 degrees  $AA_2C$ , so if that is the case then I can have angle  $ACA_1$  that I am talking about, this angle should be equal to  $r$  and that is equal to  $B$  angle  $BCB_1$ . Okay now I need to go to the next page, also in the same way angle  $A_2AC$  is 90 degrees minus  $i$  and angle  $AA_2C$  is 90 degrees which means I am talking about angle  $A_2A_2AC$  this angle so  $A_2A_2C$  is 90 degrees. So this angle is 90 degrees and then  $AA_2C$   $AA_2AC$   $A_2AC$  is 90 degrees minus this so this is  $i$  because this entire angle is  $i$ . So I have this temperature of 90 degrees. So,  $ACA_2$  is  $i$  which is equal to angle  $BCB_2$ . so from that we find an interesting thing that this angle  $ACA_2$ , that is what I am talking about, this angle is  $i$ . This small one is  $r$  and this big one is  $i$ . So once we have this then the rest of the things are simpler. So  $AA_1$  which we already got is  $AC \cdot \sin r$ .  $BB_1$  divided by  $BC$  is nothing but the sine of  $r$  from simple triangle  $BCB_1$ . Similarly,  $AA_2$  divided by  $AC$  is  $BB_2$  by  $BC$  it should be  $\sin i$  from triangle  $BCB_2$ . So, if I combine these two  $AA_1$  divided by  $AA_2$ , it should be  $\sin r$  by  $\sin i$  and that is  $v_2$  divided by  $v_1$  and that is  $n_1$  divided by  $n_2$ . So, we have  $n_1 \sin i$  is equal to  $n_2 \sin r$ . This is nothing but Snell's law or law of refraction, refraction. Okay so I do not have much time today to discuss more about Huygens theory but mostly we try to show that using the Huygens principle we can find out the general law of reflection and refraction. Further in the next class, I will try to introduce two different kinds of diffraction phenomena. One is called the Fraunhofer diffraction and another is called the Fresnel's kind of diffraction phenomena. In the Fraunhofer kind of diffraction phenomena, the light will go from infinite and the source point is infinite and also the screen will be placed at infinite. But in Fresnel's case, light is a finite distance and so is the source. And then in two cases, how the diffraction pattern is going to differ and different conditions and different apertures. If I put a different aperture or different disturbance to the light wave, then what kind of pattern as a diffraction phenomenon one can obtain in the screen we are going to discuss? So with that note, I would like to conclude here. Thank you very much for your attention. See you in the next class.

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$$\text{Also } \angle A_2AC = 90^\circ - i$$

$$\angle AA_2C = 90^\circ$$

$$\text{So } \angle ACA_2 = i = \angle BCB_2$$

$$\frac{AA_1}{AC} = \frac{BB_1}{BC} = \sin r \quad (\text{From } \triangle BCB_1)$$

$$\frac{AA_2}{AC} = \frac{BB_2}{BC} = \sin i \quad (\text{From } \triangle BCB_2)$$

$$\frac{AA_1}{AA_2} = \frac{\sin r}{\sin i} = \frac{v_2}{v_1} = \frac{n_1}{n_2}$$

So, we have  $n_1 \sin i$

