WAVE OPTICS
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur
Lecture-03

Hello, student to lecture number three and of wave optics. So today we will study the concept of wave and wave equation. So in the last class in fact we have already discussed the wave equation and discussed the form of the wave. Let me do that once again .So today we have lecture number three and let me draw what you had the last day. Suppose for a fixed coordinate system I have a disturbance like this which we call a wave in o prime frame, along this direction. So this is x and the disturbance amplitude is along this direction y direction and we write y as a function of x .Now if this wave moves over the time then at some point if this is a fixed frame and o is a o prime is a frame that is moving with the wave. So the wave now will be at this position whatever the point we have, here from here to here this length, if it is x prime since the frame o prime frame is moving at the same velocity that of the wave this length will remain x prime. However, if the wave is moving at some velocity v then after time t this length will be simply v into t and from this fixed coordinate , I can write this particular point the coordinate as x. So we have a relationship between x d and x prime something like this; x prime will be equal to x minus vt. In the last class we did this.

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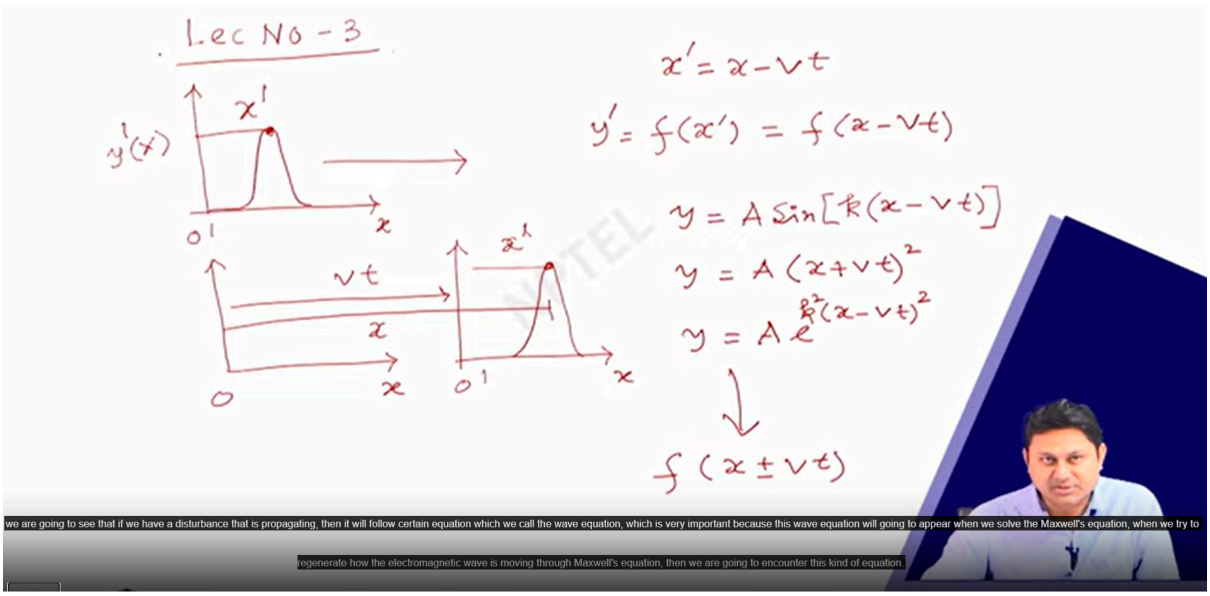

If the wave is moving in the forward direction it will be like this,if it is moving in the opposite direction then this sign will be this minus sign will be plus sign before v. So here we have a plus sign but the thing is the function y prime which was initially said this is in the prime frame. I should write y prime so y prime this is essentially a function of x prime and I can write this as a function of x minus vt the shape of the wave remains conserved during this process. Then we always have a functional form whose argument is x minus vt that gives us the feeling that this disturbance or wave is moving along x direction with a velocity v. So we give some examples like y equal to a sign of this is a very standard form we're going to use later k x minus v t or y equal to something like an amplitude x plus vt square or y equal to some amplitude e to the power of x minus vt with a factor k here square also. So all these are functions I can write in general as a function of x plus minus vt ,so all of them will in principle represent a wave that is propagating along x direction. Now since we have this expression from here we are going to see that if we have a disturbance that is propagating, then it will follow certain equation which we call the wave equation, which is very important because this wave equation will going to appear when we solve the Maxwell's equation, when we try to regenerate how the electromagnetic wave is moving through Maxwell's equation, then we are going to encounter this kind of equation. So, beforehand we need to know what the wave equation is .So let me derive that first wave equation. So y we already see it is a function of x prime in the previous example where x prime is x minus plus vt it can be minus vt it can be plus ,vt depending on whether the wave is moving in the forward direction or in the backward direction .Now from that we can have few information like if I make a derivative with respect to partial derivative, with respect to x of this parameter x prime we will going to get 1 and if we do that x prime as a function of derivative as a function of t with a function t then we will going to get plus minus v because x prime is a function of in general so I can write x prime here is a function of x and t okay. So, now I have a functional form like y should be a function of x prime in this particular form which is given here.
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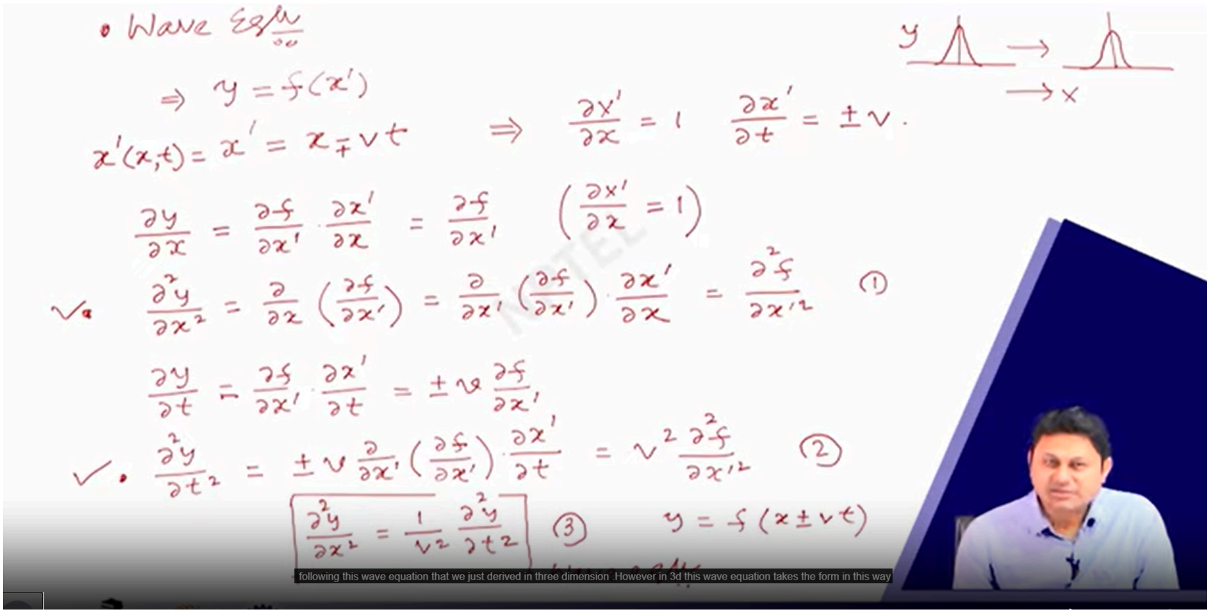

Then I can write del y del x is equal to del f del x prime del x prime del x y is a function of x and t but it is equal to f of function of x prime so using simply a chain rule I can have this so this is essentially del f del x prime because del x prime del x is equal to 1, with this transformation whatever that transformation we have.Similarly if we go a second order derivative of this quantity with respect to x we can have del del x and then d y d x I can replace as del f del x prime which is del del x prime del f del x prime and del x prime del x again this quantity is one so we will have something like del 2 f del x prime square which is equal to d 2 y dx square. Now we go so this is my one result. Another thing I can do by making a time derivative because another variable time is there so d y d t is essentially d d x prime sorry So, I need to erase this. So, yeah, it was okay actually. d d x prime of f and then d x prime d t. So dx prime dt already I figured out it is plus or minus v so it is plus minus v which is a constant quantity and then del f del x prime. If I go a second order derivative like we did in the previous case it should be a similar kind of thing where I can write plus minus v del del x prime of this quantity del f del x prime and then using the chain rule del x prime del t del x prime del again plus minus v. So, plus minus v multiplied by plus minus v, we have v square del 2 f del x prime. So, I can also have the value of the second order derivative, but if I compare equation 1 and equation 2. Then from this equation and one equation two we can readily have one expression which is one differential equation which is y dx square is equal to 1 by v square d2 y dt square. So this is the expression we have, this is the differential equation we have and if I write this is 3. So this equation 3 must be satisfied by this particular functional form that we have and this basically so you can see that the y having the form f x plus minus v a general solution of this differential equation that is derived. So whatever the form you have, if you have whatever the functional form you have but if x plus minus vt is there in the argument then always this kind of function should be a solution of this differential equation that is shown in equation 3. So that means that equation 3 is a governing equation is a governing equation of this propagating wave and that's why this equation 3 is called the wave equation. But note that this wave equation is done only for 1D. That means if the wave is moving in one-dimensional way, in a one-dimensional coordinate system, suppose it is moving in one dimension along x direction, then this wave defined by the variable y should be following this wave equation that we just derived in three dimensions .

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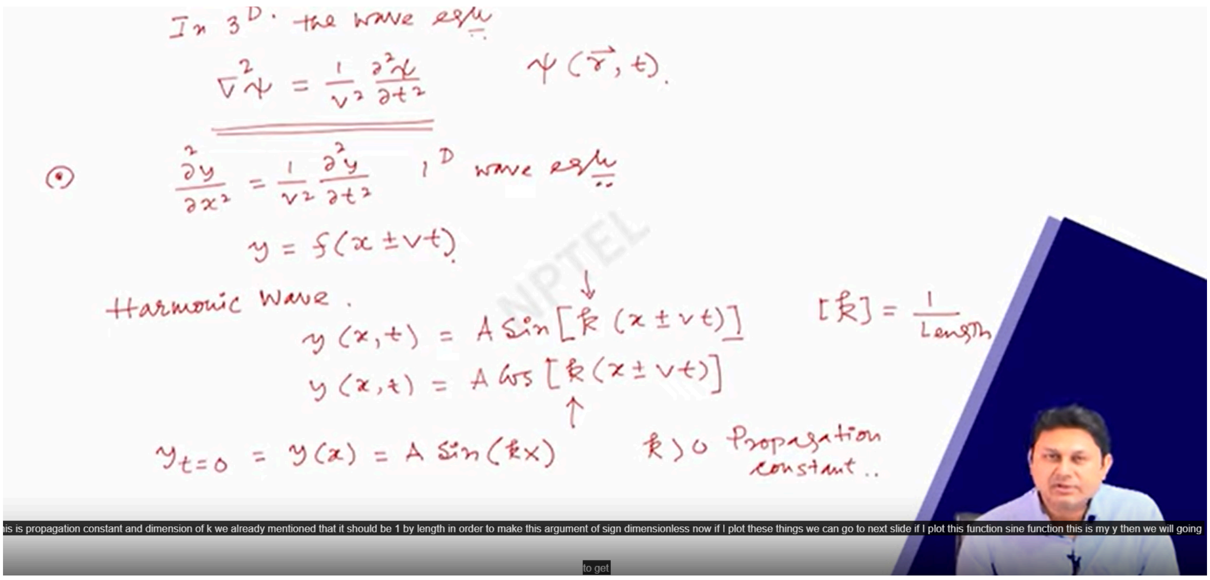

However in 3d this wave equation takes the form in this way where phi represents the wave and in general that is a function of r vector r and t and del square is a Laplacian operator depending on the system. We can write a del square operator and this is the expression for this is the wave equation for a three-dimensional wave.With this note now we like to show in the next case what is the most general kind of solution one can expect for example in the 1D wave equation. In 1D wave equation we have ,let me write it once again d2 dx square is equal to 1 by v square d 2 y dt square that is my 1d wave equation and this 1d wave equation have a solution of the form as I mentioned that it should have a solution of the form like this. So let us take a particular form which is a very general very well known form and that is called the harmonic wave harmonic function that represents the harmonic wave. So what is the harmonic wave x should be a function of x and t and i should have the form of the wave or the solution of this wave equation, as sine or cosine function .So let us consider this as a sine function k x plus minus vt and this or we can have a solution as a harmonic wave as a amplitude cos of k x plus minus vt okay .So if we look carefully to this equation we can find that we have a multiplier every time k here and that we have to do because to match the dimensionality because sine and we have some argument and this argument has to be dimensionless x plus minus v t is a quantity and that has a dimension of length. We should have a multiplier here whose dimension has to be 1 by length. So, that k multiplied by this quantity should have dimensionless, this thing should be dimensionless.So, the dimension of k is 1 by length that we can check easily from here later we will going to discuss what is k and all these things here only now at y if I find what happened at t equal to 0 then we can see that y is a function of x only and we have something like a of sine of kx because t is 0 so we have a sign of kx so k here generally greater than 0 and it has a name we call this as propagation constant. This is the propagation constant and dimension of k. We already mentioned that it should be 1 by length in order to make this argument of sign dimensionless. Now if I plot these things we can go to the next slide. If I plot this function sine function this is my y then we will get a function like this if I go beyond it should be like this okay. So over the t if I plot it will be a sine function for a constant x and for a constant t also.

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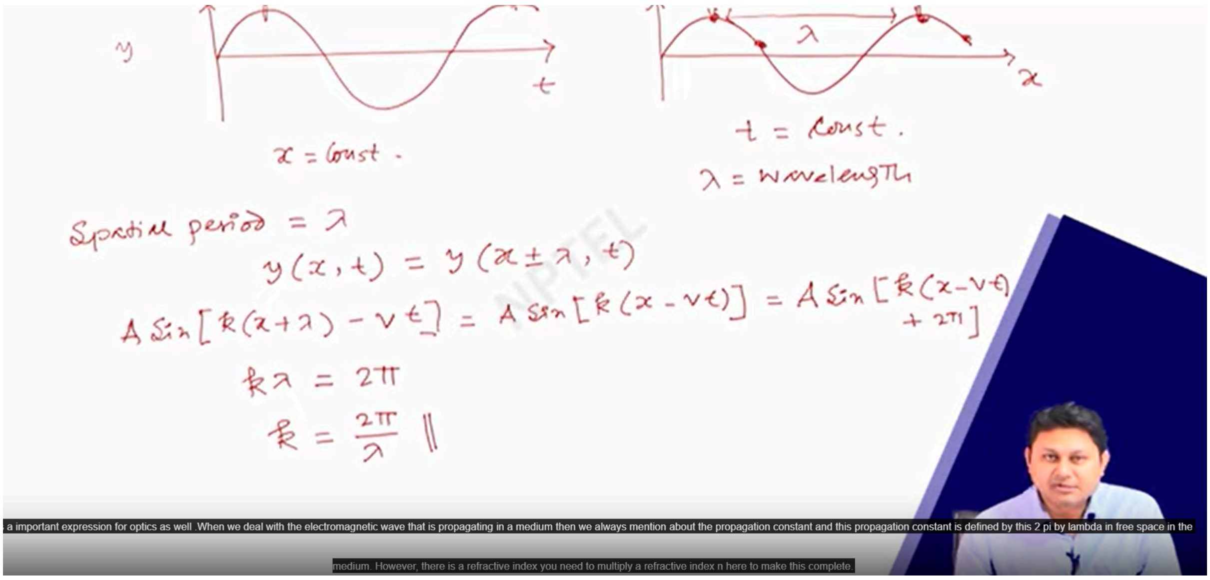

We will be going to get a similar kind of expression ,similar kind of form, similar kind of figure .Now for constant x I have two points here to here and for constant t also I have two points here to here we have which are in the same phase. So the distance between two points having the same phase is known as lambda ,where lambda is the wavelength .So this is the spatial period we have.So essentially the spatial period is lambda which is the distance between the points having the same phase and we already mentioned this we call wavelength .So that means if we have the function xt some value then this value I will get equal. If I move this spatial period that means if I move from x to x plus or minus lambda then I'm going to get the same point, for example here from here to here I'm going to get the same point also I have a same phase point here and here the distance here is also lambda .So if I have the amplitude here then if I move by the length of the spatial period lambda which is the wavelength here, then I'm going to get the same point. So if that is the case I can write the equation like a sine of K and X plus lambda say minus VT that is the equation of a propagating wave that is equal to a sine kx minus vt the same point I'll get. So if I tally these two equation then readily we can see that this value is k multiplied by lambda should be 2 pi because I'm going to get the same .So this is equal to a of sine of k x minus vt plus 2 pi you can have also 2 pi multiplied by some integer but let us take with this the smallest increment. So that I will going to get the same point same consecutive points from here one can find the amplitude of the K K is 2 pi divided by the spatial period lambda and that is a important expression for optics as well .When we deal with the electromagnetic wave that is propagating in a medium then we always mention about the propagation constant and this propagation constant is defined by this 2 pi by lambda in free space in the medium. However, there is a refractive index you need to multiply a refractive index n here to make this complete. Well like the spatial period we can also have a temporal period .So let me draw it once again so I have y function of x and t this is harmonic wave this is a property of the harmonic wave and over the time for x equal to constant. That means for a given X is vibrating and again 2's we will get two points that are having the same phase. When the wave is moving in the time domain then we call these two consecutive points having the same phase difference;

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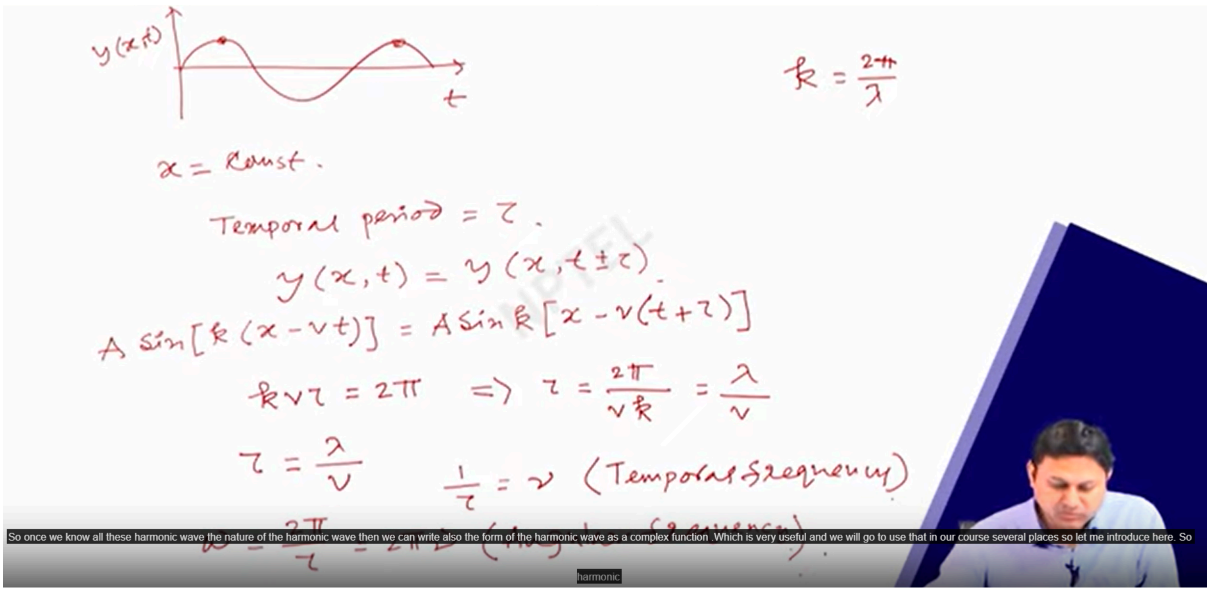

The distance between the two points in the time domain is called the temporal period. Write it as tau like in the previous case I have x t that is equal to y x t plus minus tau I'm going to get the same function. So I can have a sine k x minus vt and then that is equal to a sine k x minus v t say plus tau. So from these two equations again I can have an equation k multiplied by v multiplied by tau has to be 2 pi like in the previous case and from here I can have tau to be 2 pi divided by v multiplied by k. But we already know that the value of the k is 2 pi divided by lambda.So, here if I put this value, I am going to get simply lambda divided by v. So, tau is lambda divided by v, that is the amount of temporal period one have if the spatial period is known then the spatial period .From the spatial period we can have the temporal period through the velocity of this wave now 1 by tau is equal to mu and that quantity is called temporal frequency. T That is a quantity. which is also important. Temporal frequency, there is also one important quantity that we are going to use. In our calculations, that is omega. Which is 2 pi divided by tau or 2 pi multiplied by the temporal frequency mu this quantity is known as angular frequency. So once we know all these harmonic waves and the nature of the harmonic waves then we can also write the form of the harmonic wave as a complex function .Which is very useful and we will go to use that in our course several places so let me introduce myself here. So the harmonic wave has a complex function. So we have y tilde that is the notation normally we use .So let me erase this part so ,we have y tilde that is equal to a e to the power of I kx minus omega t .Now we know all the values what is k what is omega and now we are allowed to route the expression like this allowed to write the expression like this .So y is basically I can write as a real part of Y tilde and that is A cos kx minus omega t. If we want to write the sine, this is the cosine form of the harmonic wave. If I write the harmonic wave in sine form, then I can also write this. and then you need to write that is the imaginary part of y tilde and that is a sine of kx minus omega t where k is a propagation constant defined by 2 pi by lambda and omega is an angular frequency, defined by 2 pi by tau and omega divided by k that quantity is 2 pi divided by tau multiplied by lambda ,divided by 2 pi or it is lambda divided by tau, which essentially represents the velocity of this wave. So with this notation we can write down the harmonic wave in a complex fashion and from that we can extract the real and imaginary part as cosine and sine function and inside the cosine and sine function I have two important term one is k which is the propagation constant defined by 2 pi by lambda and also the angular frequency which is defined by 2 pi by tau lambda is a special period and tau is a temporal period of a harmonic wave and omega and k are also related in this way so that at the end of the day we find that omega divided by k is the velocity at which the wave is moving. So with this note I would like to conclude here in the next class we will learn more about the wave nature ,different kinds of waves in different system plane waves then spherical waves etc and then we do all the related mathematics in detail. So that you can understand the physical concept as well as the background mathematics. With this note I would like to conclude here. Thank you very much for your attention. See you in the next class.

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