WAVE OPTICS

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Lecture - 28: Fabry-Perot Interferometer (Cont.)

Hello, student, welcome to the wave optics course. Today we have lecture number 28, where we are going to discuss more about the Fabry-Perot interferometer that we started in our last class. So, lecture number 28 and let us discuss more about the Fabry-Perot interferometer. So, the Fabry-Perot interferometer is what we have quickly. Let me describe that we have these two quartz plates or glass plates with very polished surfaces and then we have a source point, the light will come here and then experience multiple reflections inside this cavity and these rays will be transmitted and we have a lens system here, which make this parallel ray to confine in a plane and this is called the point P. So rays are coming and we have multiple interference, multiple reflections and these are the transmitted rays that will converge here at point P and based on the path difference one can have a pattern here. This is the source point and if this cavity length is d and this angle is theta t then the corresponding phase will be calculated accordingly and the phase delta was 2d cos theta t, considering that the medium whatever we have here is air. And then we calculated the general form of the intensity of transmitted right and that is IT is equal to t square divided by 1 minus R square of that into 1 divided by, 1 plus F sin square, delta by 2, where delta is the path difference and incident in density and F was 4R divided by 1 minus R square of this and we call this F as the coefficient of finesse, which determines the sharpness of the fringes. So, that is what we are going to describe today. So, once we have this then the next thing is let me write down the expression once again, IT divided by Ii is essentially t square divided by 1 minus R whole square of that 1 divided by 1 plus F, sin square delta by 2.

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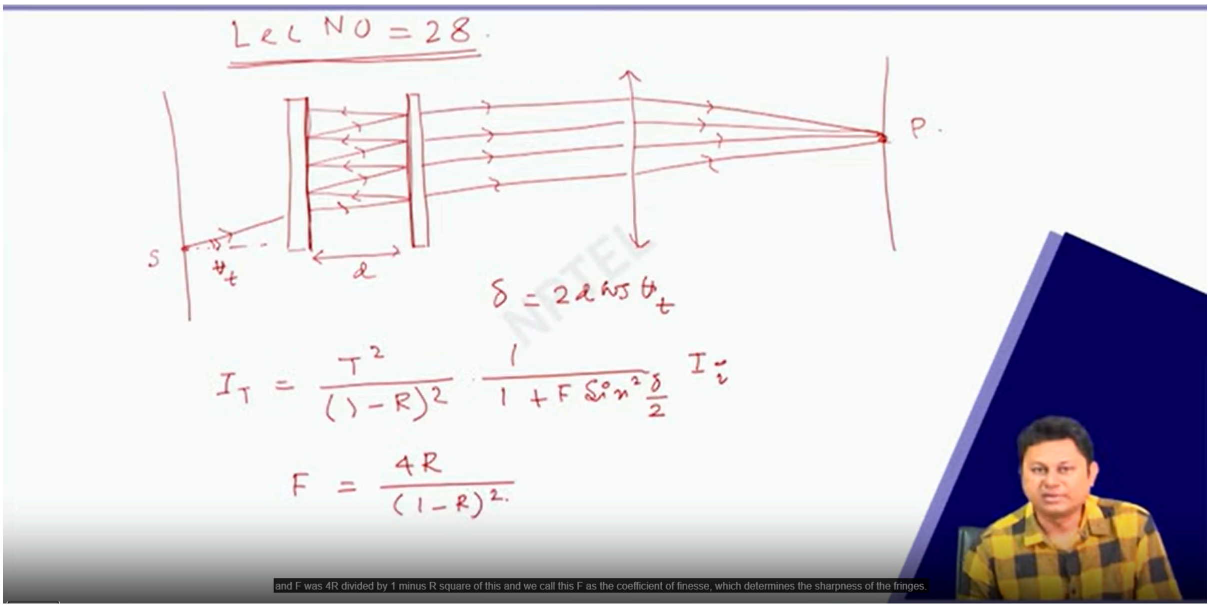

Now we have T plus R transmittance and reflectivity; this is 1 according to the Stokes relation. So IT divided by Ii is simply equal to 1 divided by 1 plus F sin square delta by 2. Because in that case t will be simply 1 minus R, if there is no absorption then t square divided by 1 by R square should be canceled out and we will get this is the value where F is 4R divided by 1 minus R whole square of that. Now, R which is a reflectivity can have the range from 0 to 1 and that makes the value of F, F less than infinity to greater than 0. So, this is the range of F one can have because when r tends to 0. So, when R tends to, R is nearly equal to 1. So, it goes to say infinity, because R tends to 1, when R tends to 0 it should be F tends to 0 because in the numerator we have r sitting. So, the range of F should be between 0 to infinity as R ranges from 0 to 1 the reflectivity is 0 to 1. Now let us consider the fringe contrast and the fringe contrast one can calculate by just drawing a single fringe. Let me do that on the next page because it takes some space. Ok so, let us put the maximum intensity as a function of delta. So this is the way the fringe will look as a function of Delta if I plot a single. So that value is I max and this is the value where it picks and I max let me write down the expression so, IT divided by Ii was 1 divided by 1 plus F sin square delta by 2 that we had and now I max is the value where this delta by 2 should be such that sin square value, sin square delta by 2 vanishes. So, that is the value one can have. So, that will put. So, here I can write I. So, this is the I max and this value says I max divided by 2. So, I is I max from this expression, I can write I max whole divided by 1 plus F. I am writing this expression only for whatever is written here I am writing, but I am writing Ii as I max F sin square delta by 2. So, I will be equivalent to I max when delta by 2 goes to m pi or delta goes to 2m pi where delta is a phase difference and I can write it as delta max. So, this is the value at which I become max. Now if I want to find out the fringe contrast. So, essentially I want to find out the fringe sharpness and how I get that. I want to find out the width of this fringe, if it is very sharp then the fringe contrast will be very high. So, let I become I max by 2 when the delta shifts from delta max plus some value delta half because if I increase the delta. So, this is the value, say some value 2m pi, and then if I go move further if I increase further then the intensity will fall and this is the point where we have roughly I max by 2.

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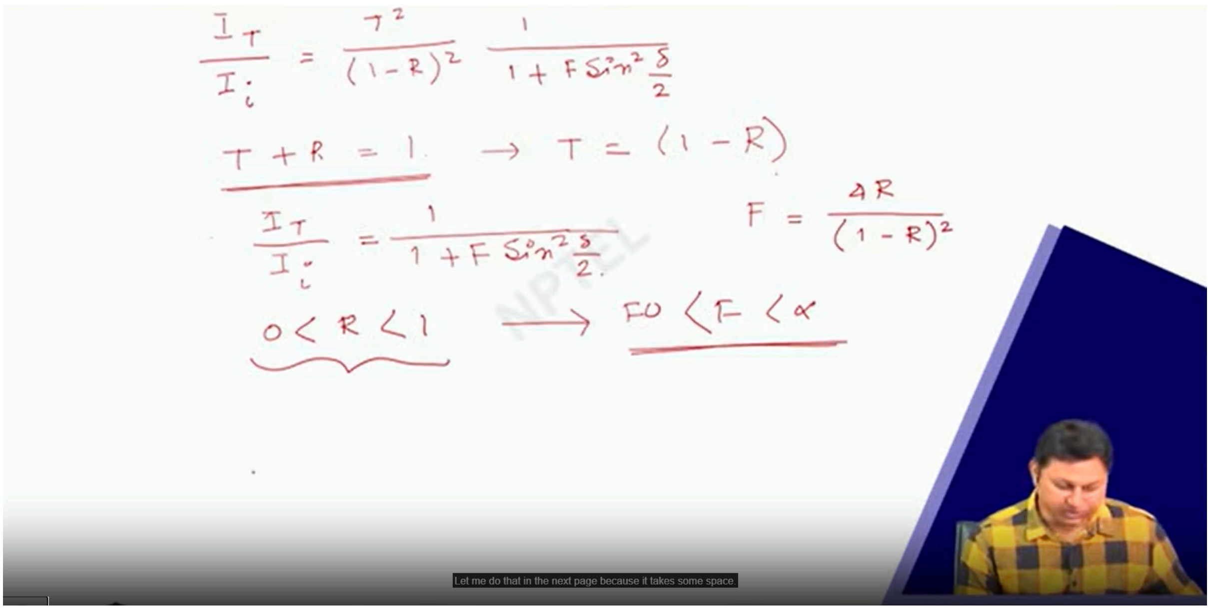

So, I want to find out what the value of the delta for which the intensity falls down from I max to I max by 2 and this is the condition I have delta max that is where it peaks and then I move forward to another delta half such that the intensity falls to I max by 2 very standard way to define this kind of things. Now, if that is the case, this is the fringe, this is my delta max and if I move to another point this is my delta max plus delta half. So, that is I max by 2 and this point is I max that is the structure we have. Now I max by 2 is equal to I max, I am using the general expression divided by 1 plus F sin square. Now here it should be this because I move to delta max plus delta half. So, delta max plus delta half then is equivalent to delta half by 2 because it is delta by 2 and that condition tells us that if the left-hand side is I max by 2 right-hand side has to be I max by 2 and it happens when F of sin square delta half by 2 is equivalent to 1, that is the condition that need to be fulfilled to satisfy this expression. Whatever the expression we have, from this we can write because our aim is to find out delta half in terms of F. So delta half by 2 is equal to sin inverse 1 by root over of F that will be the expression. Now one can see that, okay, from here delta half is I can write 2 of sin inverse root over of F. So, essentially if delta half is one side and another side is also one can have delta half. So, essentially what we are calculating? We are calculating the fringe half-width. So, if this is the fringe, this is half the width we are calculating. So this is delta half and this side is also delta half and this is the fringe half width rather this is one side is half width if I calculate the full width then I need to multiply it by 2 and that is calculated at I by I max 2. So, the fringe width is simply 2 of delta half, which is essentially 4 sin inverse of 1 by root F. So, generally, F is a large quantity. So, w which is a width is nearly equal to 4 by root over of F because if F is large 1 by root over of s it will be small and we know that when the angle is small sin inverse of that argument will be equivalent to the argument itself. So, for sharp fringes what we have is W to be small. Because if it is small then let me draw that in one case, we have this fringe and in another case, we have this fringe. So, W is both the cases if it is W 1 it is W 2, W 2 is less than W 1 and this fringe is much more sharp compared to the given one. So, W 1 when W is small then we have more sharpness. So, that means, if w is small that means, F needs to be F needs to be large.

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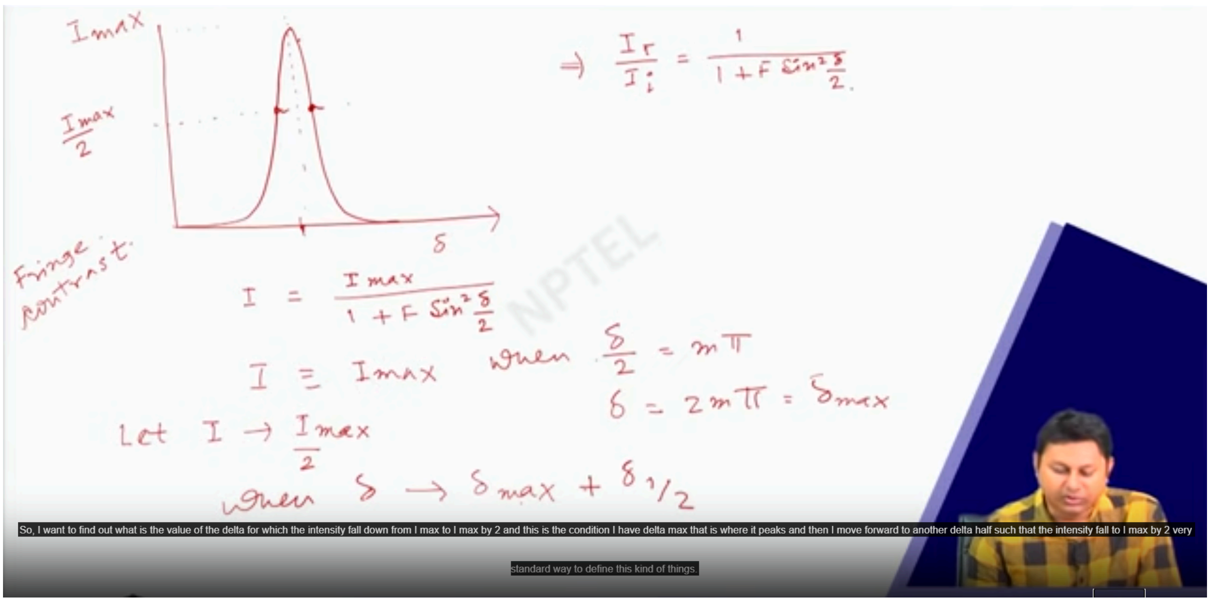

So, F is again 4R divided by 1 minus R square. So, to increase F what we need to increase R needs to be increased. So, that means, the reflectivity of the system needs to be increased a lot to increase the sharpness. That means, this is the Fabry-Perot structure let me draw once again and this surface is a highly polished surface that makes the reflectivity much higher. So, that light is going to reflect this surface. So, this is the way things are happening here and the finesse. If the finesse of the system is large that is f is high which means the reflectivity is also high and if we have a Fabry-Perot interferometer with high reflectivity then the fringe pattern that is produced should be of high quality. So, I can draw a relative fringe to show how these things are happening. So, if I plot say I divided by I max as a function of delta, that is the phase difference experienced by. So, in one case for very sharp fringes, we have fringe patterns like this. So suppose this is 2m pi the value of the delta, this value will be 2m plus 1 pi, this value will be 2m plus 2 pi, this value will be 2m plus 3 pi etcetera. For in the same plot if I plot a fringe pattern with low sharpness it will be something like this. So, this is another way to say that suppose the reflectivity for this fringe pattern is R1 and the reflectivity of this fringe pattern is R2. So, R2 is obviously, much greater than R1 to have this fringe pattern. In one case the fringe pattern is very sharp because the full-width half maxima or this width is small compared to the width we have in other cases and that is happening here because the sharpness of the fringe is high here in this case compared to others and that is because of the value of the reflectivity, the finesse of this lines which is having the sharp fringes is much higher compared to the other line. And, that is directly related to the reflectivity of the system. Suppose, we have two Fabry-Perot interferometers where R 1 and R 2 are the reflectivity where R 2 is much larger than R 1, then we have this fringe pattern with sharp notches. Well, so we need to understand the coefficient of the finesse another I mean note I like to make in a few books I find that the coefficient of finesse F should not be confused with a second commonly used figure of merit F s and that is defined as curly F like this and it is simply called the finesse. t Now, there is a relationship between the finesse and coefficient of finesse, this is finesse F which is pi by 2 root over of F that is the relation.

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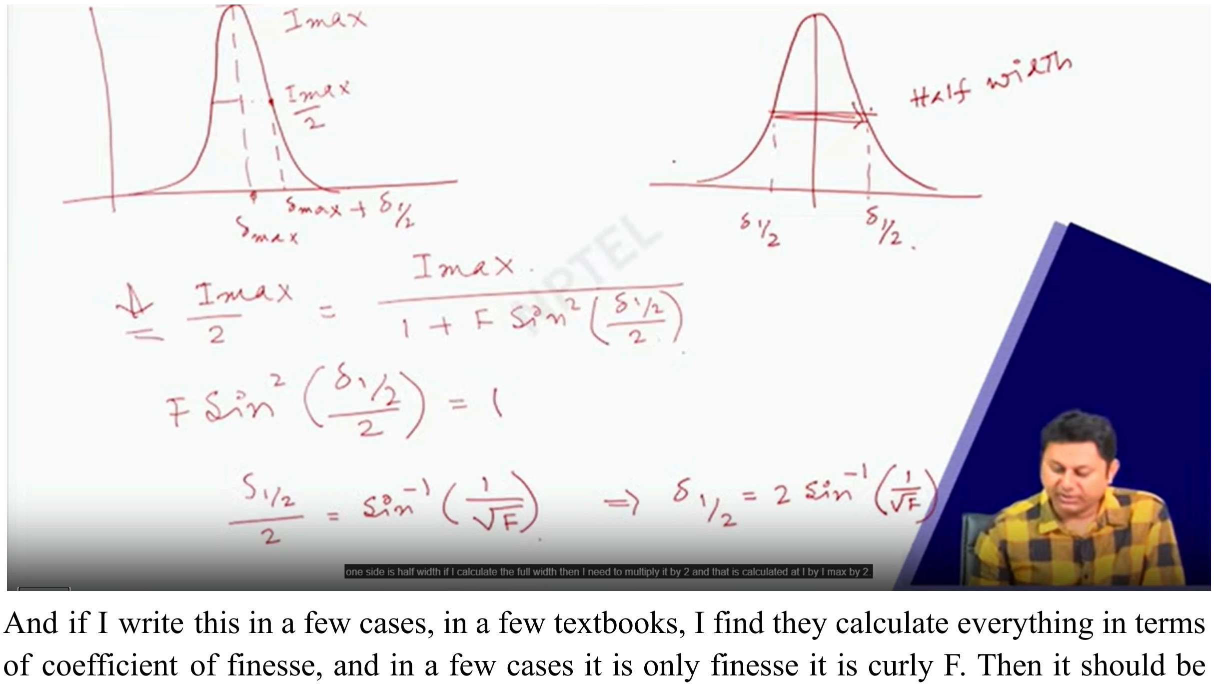

And if I write this in a few cases, in a few textbooks, I find they calculate everything in terms of coefficient of finesse, and in a few cases it is only finesse it is curly F. Then it should be

4R divided by 1 minus R whole square, whole to the power half or in other word it should be pi root over of R whole divided 1 minus R. So, with that also one can write the general expression which we derived that IT divided by Ii which is T is 1 divided by 1 plus F sin square delta by 2. So, T is essentially 1 divided by 1 plus 4 pi just replacing F to this new coefficient finesse F square and then sin square delta by 2. The same thing, just we rename it. Another thing, whatever the pattern we had let me draw it once again the sharp lines. So, this is 2 pi m, this is 2 pi m plus 1, I plot the intensity here and then we have m plus 2 then 2 pi 2m plus 3 pi, and so on. So, another quantity is used here which is called the free spectral range or FSR which is the phase separation between adjacent transmitted peaks. So, in that case by definition, delta FSR is a difference between so, delta, this is delta m plus 1, minus delta m which is nothing, but 2 pi. So, the free spectral range is by definition the phase separation between the adjacent transmitted or the transmittance peak that is also used. So, the half width corresponding to this finesse can also be calculated in this way the half width at half maxima that we already calculated half width at a half maxima. So, we have essentially been talking about this one. So, this is the structure, this is the half maxima and this is the half-width at half maxima. So, this is sin square delta half by 2, and that value we find as 1 by F and we know what is the relationship between F and curly f. So, this is essentially a pi square divided by 4 curly f squares. So, that is the equation 1 we have, in a similar way we can see that sin square delta half by 2 is essentially sin square m pi plus, delta by 2 because I am moving from this point to this point. So, that is the delta half and when we move the delta, this is 2 pi. So, this point is a peak point, it is 2 m pi and this is delta by 2. So, delta is essentially 2 m pi, plus delta half and when we calculate delta divided by 2 because it is sin delta divided by 2, this quantity will be m pi plus delta half by 2. This quantity I am putting here in place of delta half by 2, this is the same thing one can have. So, for a small delta when this is small then, simply sin square delta half by 2 will be delta half by 2 square of that. So, another expression we have is 2. So, from this 1 and 2 from equation 1 and 2.

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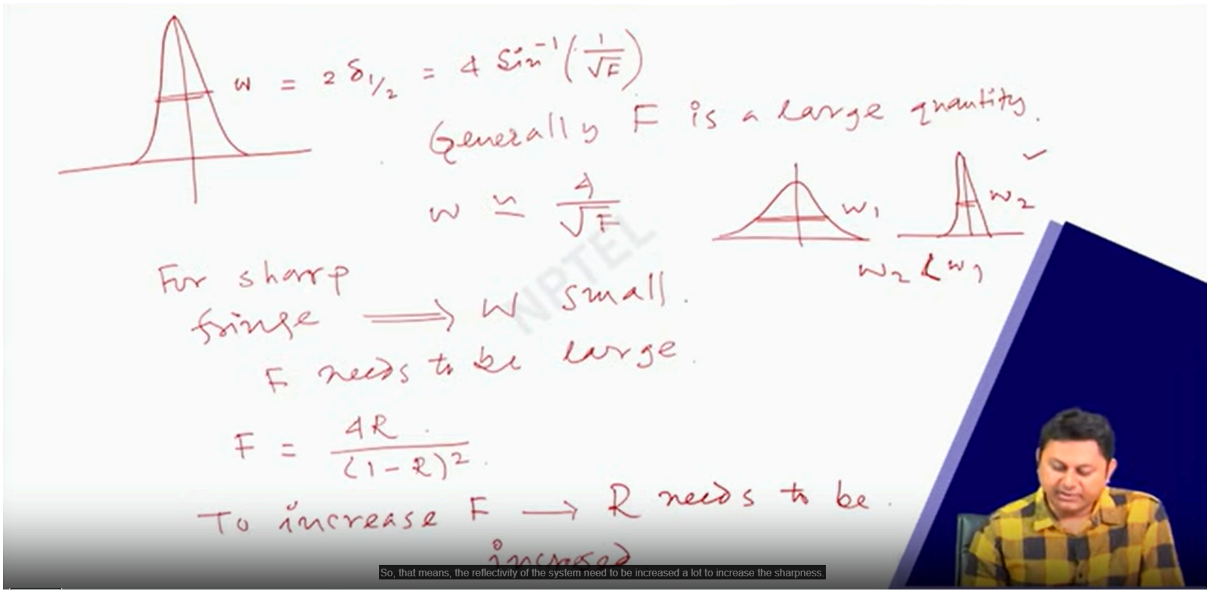

We simply have delta half by 2 is equal to pi by 2 finesse or in other words delta half is equal to pi by this finesse. So, whatever we have is equal to 2 pi divided by 2 delta half, I can write in this way or 2 pi. We calculate that it is a delta FSR or free spectral range, FSR divided by full width at half maximum because 2 of delta half is nothing, but full width at half maxima. So, that means, if I have another peak here then this separation is delta FSR which is essentially 2 pi and this is 2 of delta half which is full width at half maximum. So, the ratio between these 2 quantities is essentially our finesse. So, finesse is high means the separation between these 2 transmission spectra is high and also they are very sharp. So, in this way, one can measure the finesse of these Fabry-Perot interferometer structures. I do not have much time to discuss a few more things related to the Fabry-Perot interferometer. In the next class however, I would like to discuss more about the Fabry-Perot interferometer application-oriented thing and we will see how one can find out the wavelength and the resolution or the resolving power of the system can be measured. If two wavelengths are there how by using the Fabry-Perot interferometer we can resolve these two closely spaced wavelengths and that is called the resolving power of the system. So, what we are going to do or calculate in the next class? So, with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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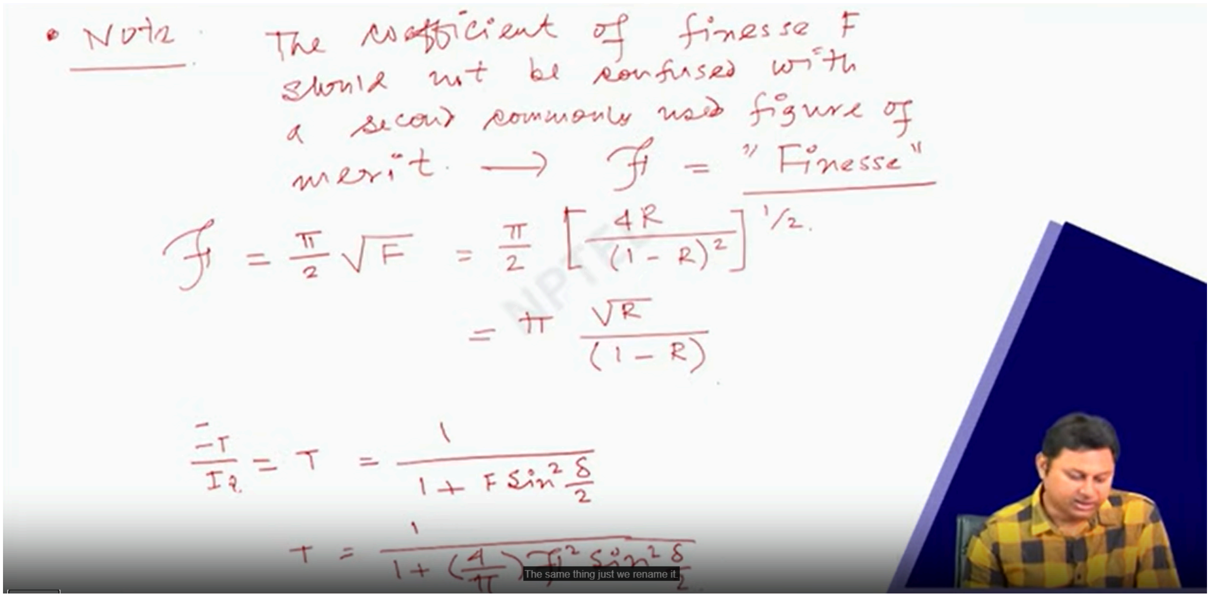

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