WAVE OPTICS

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Lecture - 27: Fabry-Perot Interferometer

Hello, student, welcome to the wave optics course. Today we have lecture number 27 and we are going to discuss the Fabry-Perot Interferometer. So, today we have lecture number 27. So, if you remember in the last class we discussed if we have a film here like this having refractive index nf and if a ray is allowed to fall on this system. Then it will be transmitted and then again going in this direction from this surface we are going to have a reflection from this surface also in principle we are going to get a reflection and the entire structure will be something like this is called multiple beam interference because this beams will interfere in with each other and there is a possibility you will going to get the pattern out of that. So, if this is the incident ray I and this is the reflected ray and this is the transmitted one say IT then we manage to get a relation and that relation was this intensity of the reflected ray is the intensity of the incident ray times and expression, which is 2r square 1 minus cos delta divided by 1 plus r to the power 4, minus 2r square cos delta, where r is the reflectivity of the system and delta is the phase difference that one needs to calculate. Similarly, one can have the transmitted intensity in terms of the incident intensity and one can have an expression like this, 1 minus r square, 1 plus r to the power 4 minus 2 r square of cos of delta. So, these expressions we calculated earlier and then we mentioned what is the maximum condition of this maxima and what is the condition of minima, I am writing this once again. So, the delta here as I mentioned is the phase difference which is k multiplied by the path difference delta, where the big delta was 2 nf, which is the refractive index of the film and the thickness of this film t and then we have cos of theta t where theta t is a transmitted angle this.

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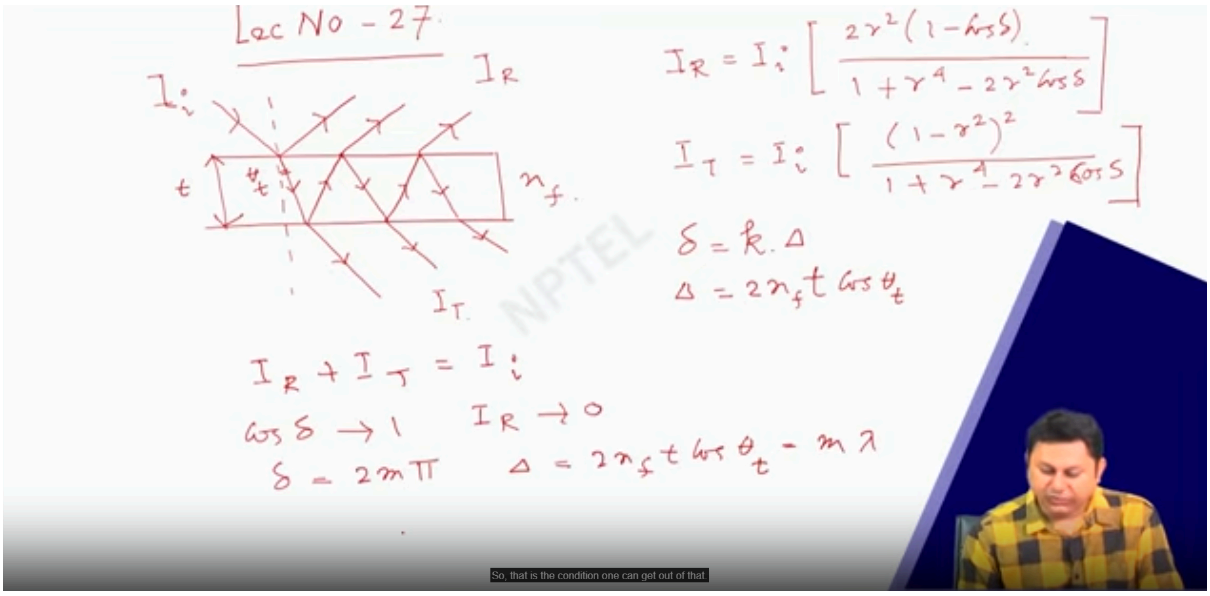

So, based on that one can calculate the delta and you can see that the small delta depends on the big delta and the big delta depends on the angle of incidence or the angle of transmittance here with theta t. By modulating this one can change the phase between these and one can get the constructive or destructive interference pattern for the reflected or transmitted ray. So, if I do that before that one can also find that IR plus IT should be the total intensity incident intensity Ii. Now from this expression also one can see that when cos delta tends to 1, then IR tends to 0, it vanishes, that is the reflected intensity will no longer be there. That means, there is a condition for the extractive interference for the reflected rays. So, that condition arises when the delta satisfies 2m pi or small delta which is 2 ineffective cos theta t that is equal to m lambda. So, that is the condition one can get out of that. Well, one can also find when IR is maximum and that is when cos delta is equal to minus 1 or delta is m plus half of 2 pi or in other words it is 2m plus 1 into pi for that particular delta one can have cos delta minus 1 then IR becomes maximum. The path difference accordingly is 2n effective multiplied by thickness cos of theta t that is m plus half lambda, under that condition what we have is IR is equal to 4 r square divided by 1 plus r square and the whole square of that Ii from the previous expression. Similarly, the transmitted intensity will be 1 minus r square, 1 plus r square, and the square of this and Ii this one can be found from the general expression that we wrote in the previous. Here by just putting cos delta equal to minus 1 one can get these two expressions, whatever is written here from these two general expressions. If I put the condition that cos delta is minus 1, 1 can come to this expression, okay, here 1 square is missing I need to put. Okay so, that was the thing we discussed in the last class. Now, we go forward and try to understand the working principle of the Fabry-Perot interferometer, which is a very important interferometer. So, today we will discuss the Fabry-Perot Interferometer. So, the setup first we discuss this is the same setup which we already discussed in the last class, but in a more realistic manner. So, what we have is a source plane and we have these two fine polished blocks, this side is polished with a very high-resolution right lambda by 50. So that is the flatness order of these sides and I will write that then we have a system of lenses and here somebody is going to see the pattern.

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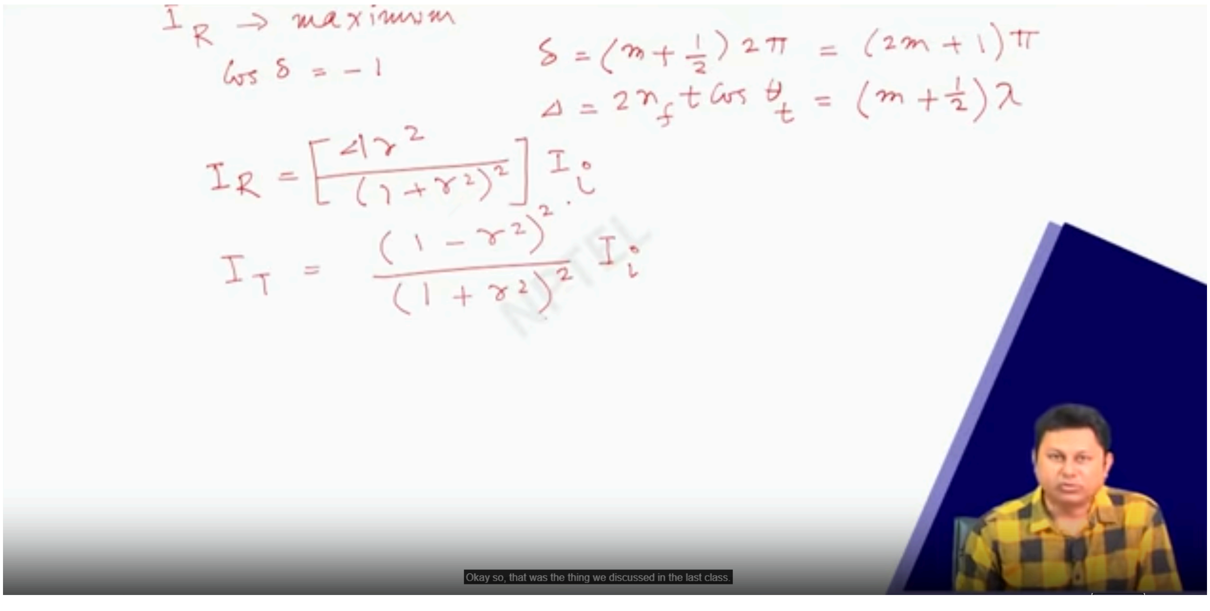

So let me draw how this works. Here suppose I have a source point, here in the source plane and light is coming here with an angle and then goes like this and then emits and then experiences multiple reflections here like multiple-beam interference and then this way again from here and then merge to some point with the lens system. So this is the incident ray and this is the way the light is reflected back from these two systems which we call the cavity. This angle is theta t then this angle will be essentially theta t and these 3 rays will merge by a lens system I call L and here all these rays will interfere, is the transmitted ray will going to interfere and as a result what we see here is some pattern, for example, a fringe pattern will be observed here, maybe it is a maximum, maybe it is a minimum depending on the phase relationship we can find a fringe pattern. My fringe is not looking very symmetric but it should be a symmetric fringe pattern. So this is the thick glass or quartz whose surface flatness as I mentioned is even better than lambda by 50 lambda is the wavelength of the light that is used. So, very highly polished surfaces are there. Now, this is the P point where all the transmitted rate is superimposed and now this point will be a maxima or minima depending on the path difference between the successive parallel beams. Whatever the parallel beams we have here, the successive parallel beams have a path difference with each other and that path difference is basically determined. Whether this P point where all these rays are interfering has a maximum or a minimum. Now, we know that the delta, which is a path difference, can be calculated with this which is 2d. If I consider this air and this separation to be d, then the path difference is clearly written by this expression. And if I consider the in-between space is air between these 2 blocks if it is air then ineffective is nearly equal to 1. So, for brightness at p one simply has 2d cos theta t is equal to m lambda, for transmitted rays that is the condition one can simply have. So, for a fixed d you can see that for a fixed d the condition is satisfied at a certain angle of theta this theta t. So, this condition will be fulfilled by all the points on the circle through IT mean if I have this source as a circular or a extended circular source then it should be satisfied by all the points circular points going through this point S and we will going to have a fringe pattern here which looks a circular fringe pattern, if the source is circular in nature . So let me now describe what happened to the parallel plate?

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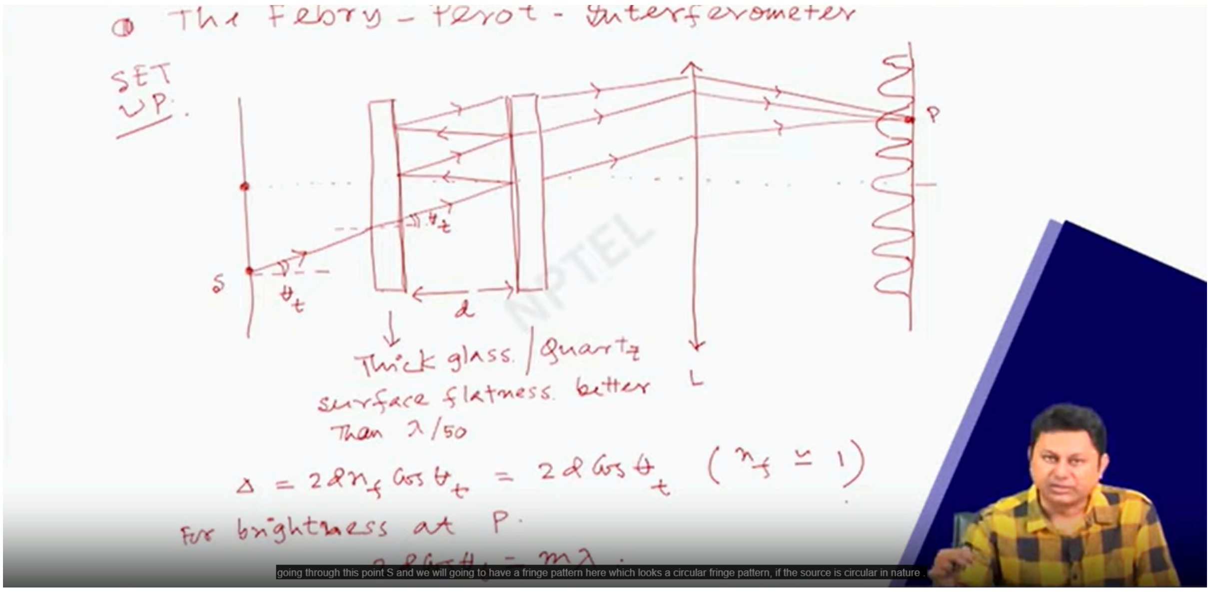

As we already discussed in the last class, the expression is already there. For the parallel plate, we have transmitted ray in this form, this is a general form derived from 1 plus to the power 4 minus 2 r square cos of delta which is the expression we already had. Now, from Stoke's relation. We can have r equal to r prime. That is a Stokes relation we also derived and r square is 1 minus dt prime or big R. That is, a reflectivity is defined by r square or r prime square; this is the reflectivity, where the transmissivity is defined by tt prime. So, that is the condition we have and we also derived this earlier. Now, adopting that condition we can write IT as Ii 1 minus r square we had. So, that will be replaced by a transmissivity square divided by 1 minus r square of that okay this is let me write clearly then. So, that was plus 2 r square. So, 1 minus cos theta was the original expression and I write the original expression in terms of big R and big T. There is a smaller and smaller t exploiting the Stokes relation, I am trying to write it in big R and big T. So, this will be T square this will be 1 minus R square of that then plus 2 of R 1 minus cos delta we write 2 of sin square delta by 2, by writing that I can have an expression like 1 plus, 1 by 1 plus, I defined a parameter called F sin square delta by 2 then multiplied by t square divided by 1 minus r square of that. So, that means, I take 1 minus r square common and then we have 4R divided by 1 minus R whole square. So, my F that we define should be simply 4R divided by 1 minus R square of that is called the coefficient of finesse. It basically determines the sharpness of the fringe. So, the fringe that will generate it determines this F quantity determines how sharp the fringe will be. Whatever fringe we have, it basically determines whether or not this fringe will be sharp. So, that expression we rewrite. So, for the Fabry-Perot interferometer, what we have is transmitted intensity is defined as T square divided by 1 minus R whole square of that multiplied by the factor 1 divided by 1 plus, finesse sin square, delta by 2, where delta is a path difference and Ii, F is a coefficient of finesse, which is determined that where F is 4R divided by 1 minus R square. This is the coefficient of finesse that determines how sharp the fringe will be, we will show how to have this. Now, if somebody wants to have a maxima, somebody should get the transmitted ray IT should be maxima for under maxima condition.

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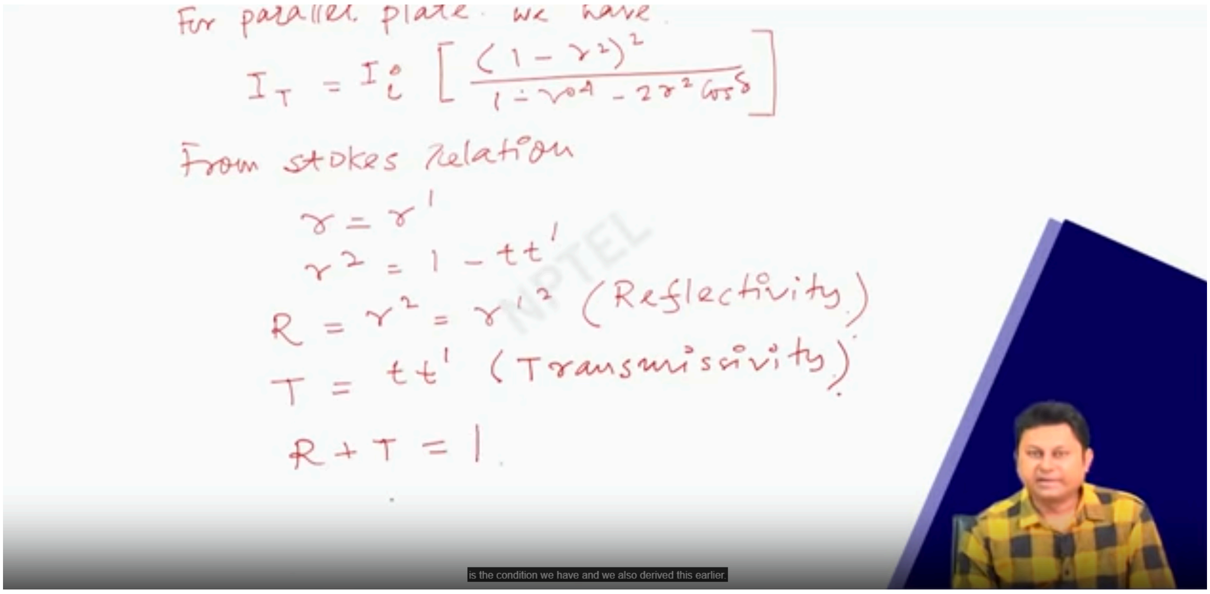

And then this will be if I put this maxima condition that means, it happens when the delta is equal to 2 m pi m being an integer that is in all cases sin square delta by 2 will be sin square m pi and this quantity will vanish. So, the denominator will have only 1 and in that case, IT max will be simply T square divided by 1 minus R square Ii . In a similar way if I want to find out IT minimum one can get this value sin square delta by 2 should be 1, that is the maximum value one can get for sin and then it should be T square divided by 1 minus R square of that and then I have the factor 1 plus F and Ii, as I mentioned in that case delta should have 2 m plus 1 pi that is the value of the delta one can have. Now, what is 1 plus F? This is 1 plus 4R divided by 1 plus R then the whole square of this. And that quantity is if I calculate this is 1 plus R whole square, this is not 1 plus, it is 1 minus R whole square. Let me reach this expression here and the expression simply becomes 1 plus R whole square divided by 1 minus R whole square, this is 1 plus F. Now, once we have the expression of 1 plus F in this way then readily we have IT max as T square whole divided by 1 plus R whole square Ii and IT min is T square divided by 1 minus R whole square this quantity. Now, from here one can calculate the visibility, and the visibility of the fringes is determined by I max minus I min whole divided by I max plus I min. In this case, if I put all this together in this case what we get is simply 2R 1 plus R square. If I put all these values I suggest the students to please check it by yourself just put these two values and then find out whether you will get these things or not. So, here also one can find the T max that is the maximum transmissibility is this maximum transmitted intensity divided by I, and that one can get as T square divided by 1 minus R square. Similarly, T min, the minimum value of transmissivity is transmitted intensity minimum divided by I that value will be t square divided by 1 minus R square. Now, T max minus, T min whole divided by T min, that is an interesting quantity one can get and if they put all this value if somebody put all this value it should be 1 divided by 1 minus R square of that and then minus of 1 divided by 1 plus R square whole divided by 1 divided by 1 plus R square that is equal to 4R divided by 1 minus R, whole square which is nothing, but the quantity F we defined as finesse.

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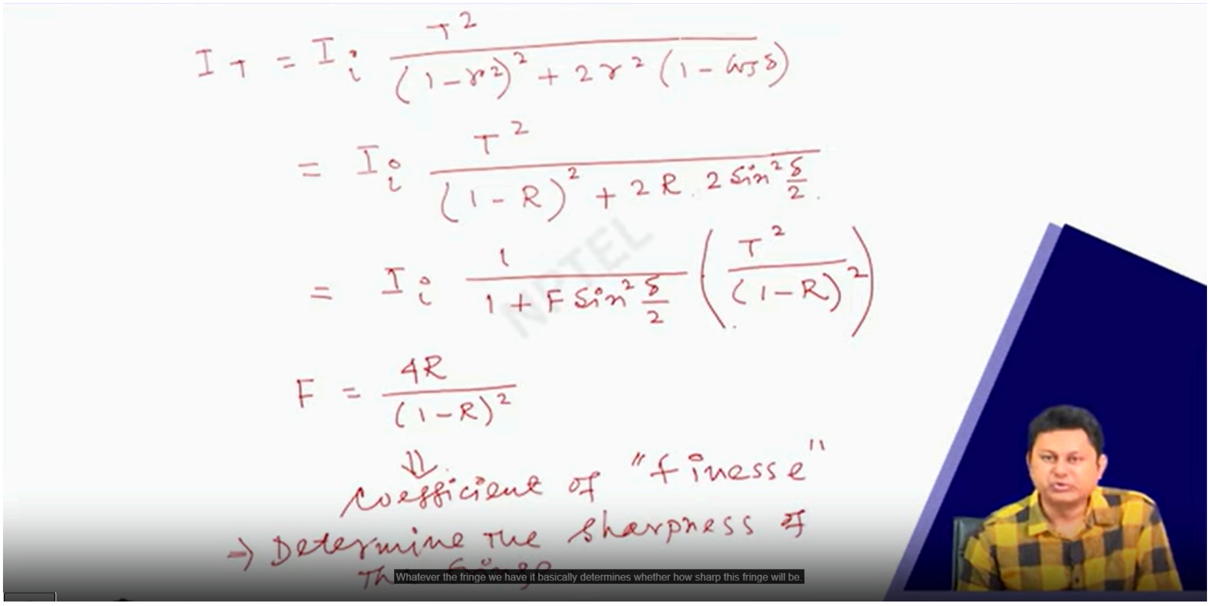

So, in this way today I do not have much time. So, in this way, we find that the Fabry-Perot interferometer is a system and in this system works on the principle of multiple-beam interference. And, we also already calculate when multiple beam interference is there, what is the condition, what is the amplitude or what is the intensity at the transmitted ray for the transmitted ray and what is the intensity for the reflected ray. And, this transmitted intensity and reflected intensity depend on the path difference. So, by adjusting the path difference for different launching angles, what do we find in some cases? We have maximum transmissibility in some cases, we have minimum transmissibility, and this maximum and minimum transmissibility in the output results in a pattern if the source is circular. We have circular fringes like we have in Newton's rings and what we get out of that is a fringe pattern. And that fringe pattern we also calculate that the fringe pattern depends on something called the finesse factor F and we determine the mathematical form of this finesse factor F. In the next lecture, we will discuss more about the sharpness of the fringes and the utility of this Fabry-Perot interferometer. So, with that note, I would like to conclude today's class. Let us meet in the next class and try to understand more about the Fabry-Perot Interferometer and its working principle. Thank you very much for your attention. See you in the next class.

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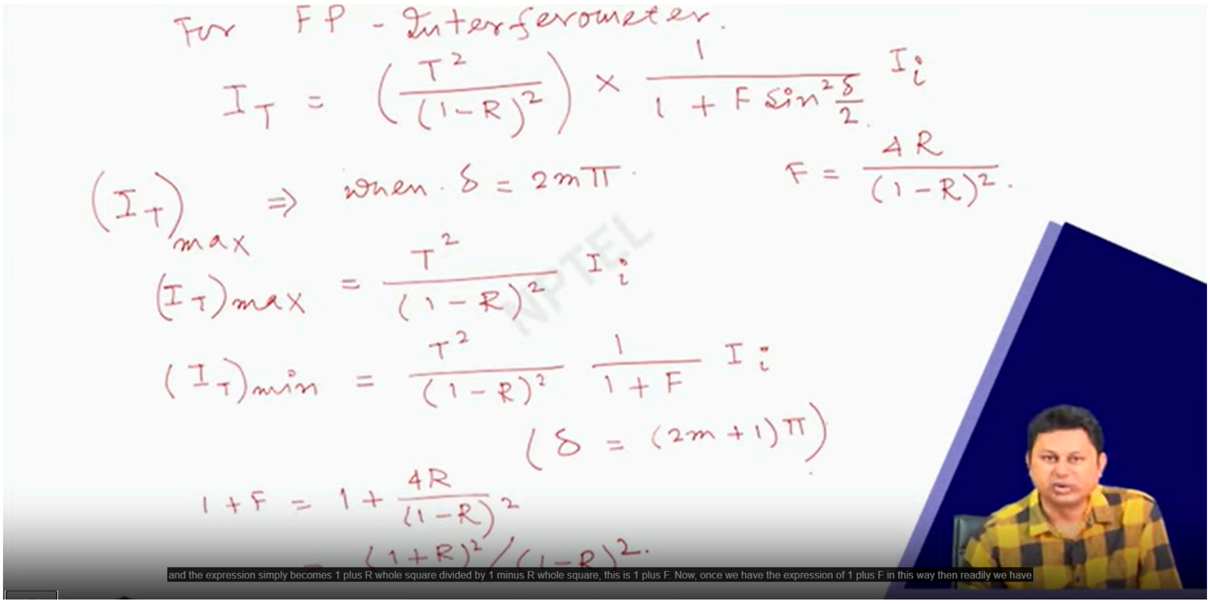

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