WAVE OPTICS

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Lecture - 26: Multiple beam interference

Hello, student, welcome to our course wave optics. Today we have lecture number 26 and today we are going to discuss multiple beam interference. So we have lecture number 26 today. So far we have discussed the Michelson interferometer. So, before that, I should write about that optical interferometer. There are two optical interferometers that we are going to discuss here in this course. One is the Michelson interferometer that we have discussed in the last couple of classes, where we have this source and light is coming, then we have a beam splitter, then there is a reflection it goes to another mirror and then it comes back here, it hits another mirror then, these two light will reflect and they merge together and we can see the beam structure, the interference structure here. So, that is the outline of the setup of the Michelson interferometer. Now, the next interferometer was the Fabry-Perot Interferometer, or FPI for short. In this Fabry-Perot interferometer, the structure is something we have a cavity like this made up of 2 blocks, and then light is allowed to fall here, then it goes back and experiences successive reflection like this. And every time it reflects we can have a transmitted light here in this direction as well. So now this transmitted light if I now put a lens here and capture these two lights to some point P. So, what happened? There will be a pattern that will generate this point because of these multiple reflections there will be a path difference and we can find out what the transmissibility of this system also generates a pattern here. So that is the outline of the Fabry-Perot interferometer setup, very very basic outline of the Fabry-Perot interferometer. But in order to understand how this thing works, we need to understand first today's topic, which is the multiple beam interference for these parallel plates. If we have a parallel plate how this multiple beam interference happens and all this stuff? So with this, we will let me go over our topic before going to the Fabry-Perot interferometer directly. We first try to understand what is multiple beam interference for parallel plates. Now let me draw. This is an interesting kind of drawing. So suppose we have a plate like this and light is falling over this plate making some angle theta i and then get transmitted here in this plane and then again it transmitted and then from here it again reflect back and here we have already a reflected light going this direction, here we have a transmitted light going this direction, here again, we have this transmitted light parallel to this going this direction again it hits.

So let me draw it in a bigger way and then go here if you flick and then translate it again. Okay so this is the structure we had, now this is the way the light ray will go. Let me draw the light rays also suppose the thickness of this plate is t and the light that is coming has an amplitude of E0. This is refractive index n the refractive index of this plate maybe I can write it as nf this film and the refractive index outside is again n, I can also put a lens kind of structure here to capture all the light that is coming from this portion and this will concentrate to point here like this. So here this is concentrating. So I can find out what pattern interference is generated due to these light rays because they have different phase

relationships because they are traveling different paths. So this is a lens system and we concentrate the light here. So this is overall the structure. So E0 is the input light and suppose E1 is my reflected light. If I know the reflectivity of this thing and if it is r then this is equal to r E naught then. Now the light is transmitted here and this angle says it is theta t, if it is theta t and if transmittance is represented by t then the amplitude of this transmitted light is t E naught. Now it is reflected back from this point and this reflection is happening from the lower boundary and it is something. So, here in this case if the reflectivity r which is from n to nf then from nf to n the reflectivity should be something different, and suppose the reflectivity here is say r prime. So, then the reflected light will be r prime multiplied by t E0 because for this light tE0 is the input to the light that is transmitted. Here again, it will be a different transmission because this transmission allows this. So by the way, here if I write down the transmission it should be ttE0 is input, and tt prime E0 will be here also the reflected light, from this point will be transmitted out and if this transmitted out this light if I write E2 then the value here is simply tt prime r prime E naught because originally r prime tE naught was the amplitude and then it is transmitted.
So I write t prime then again it is reflected from this point and we know that the reflection is r prime. So it should be tE naught r prime square, the transmitted light here that is coming simply tt prime r naught prime, sorry r prime square E naught. Then again it is reflected and this reflection I have t is E E0 r prime square. So the reflected light from this point will be simply t r prime cube E naught and if it is coming out like E3 then this value will be tt prime r prime cube E naught. I strongly suggest the students to please check it carefully and do that by your own hand then things will be clear for you how it is happening. Then here in a similar way once you do for one or two rays then the rest of the rays will come automatically. Then it is T r prime to the power 4. It is not erasing, okay what happened? So R prime to the power 4 then E0 and that is coming out is E4 and this value is tt prime, r prime to the power 5 E naught because this reflected light will be tr prime to the power 5 E naught reflection from this. So, this is the complete picture of how the light will reflect and what should be the value of this transmitted light and the reflected light in terms of its transmitted, transmission, transmittance, and reflectance.
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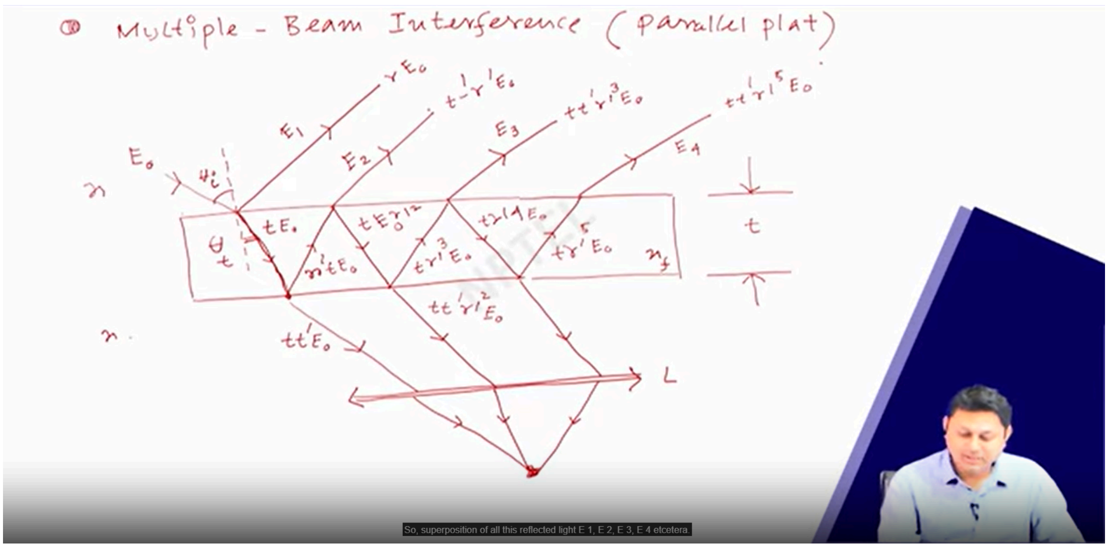

So, once we have this then, next we find out the superposition of the reflected light first. So, our goal is to find out the superposition of all the reflected light. Okay, these lines are now ready. So, the superposition of all this reflected light E 1, E 2, E 3, E 4 etcetera. So, before that, we need to find out what the path differences and so, so what we want to do is superposition of the reflected lights. So, what do we do? So, first, we need to find out what is the phase difference we know that the phase difference is equal to k in a lambda, where lambda is equal to path difference, and in this case, we know it is 2 nf multiplied by t into cos of theta t, where t is the width the separation of this width, of this block and theta is the transmittance angle, this angle is theta t and this angle is theta i, incident angle and theta t is a transmittance. So, once this is known, we know what is the path difference and then we can calculate the electric field. So, E is say E 10 amplitude then e to the power of i omega t E 2 will be E 20 then e to the power of i omega t minus delta, the phase difference that it's, it incurred then E3 will be E30 e to the power of i omega t minus 2 delta and so on. Now, if I try to find out what is my E 10 because we can also represent in the earlier figure it is r E naught and then e to the power of i omega t. What is this? This is tt prime, r prime, E naught, e to the power of i omega t minus delta. So let us go back and I am talking about these outputs, similarly, E3 will be simply tt prime, r prime, Q E naught into e to the power of i omega t minus delta. So, in a similar way I can find out what En is. That is En0 e to the power of i omega t, minus n minus 1 delta, that should be the nth ray that is coming out and the amplitude this is equivalent to the amplitude from this trend. I can find out it should be tt prime and then r prime to the power 2n minus 3, then there should be an E0, then these things will be multiplied by e to the power of i omega t minus n minus 1 delta, it is like that. So, please check that when n is equal to 1, then this value ah this value. So, when this is 2n minus 3. So, when n is equal to 2, then it should be 1. When n is equal to, so n should go from 2 to, it should not take the value of 1 because in this case there will be an issue. So, n can take from 2, 3, 4 and so on. Well, after that we need to find out the superposition. So ER, which is the superposition of all reflected rays, will be simply sum over EN and that value as I mentioned, so plus E1 that value is n 2 to infinity and plus e 1 because n is from 2 to ah infinity.

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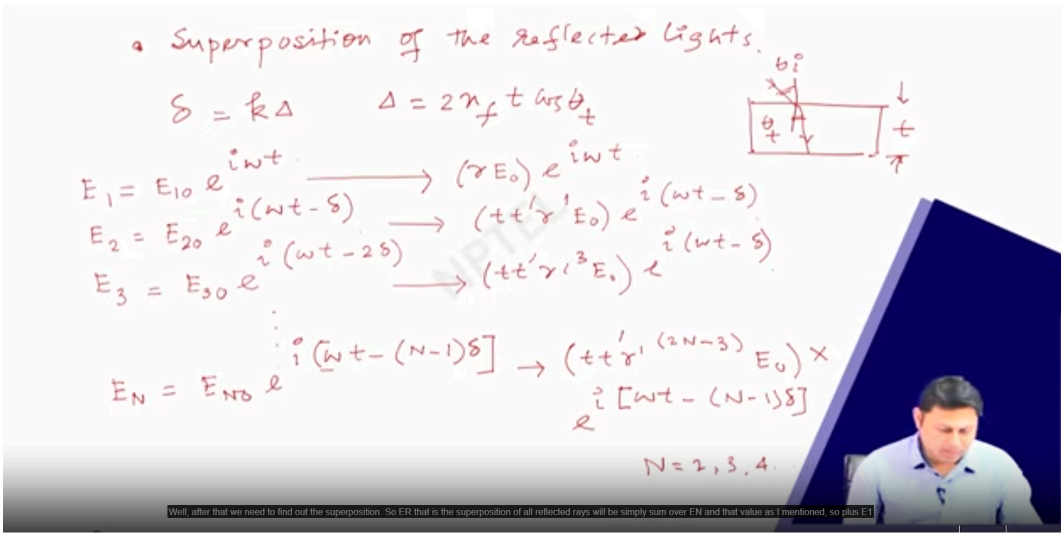

So, let me put it. So, the first value is r E naught e to the power of i omega t plus the sum that we have, which is n equal to 2 to infinity and then we have t t prime then E naught r prime to the power of 2n minus 3 and then e to the power of i omega t minus n minus 1 delta n minus 1 delta, that is the summation that we need to do. So, it looks very cumbersome, but ah, in fact, it is not that. So, let me write E r is equal to let us take E naught, e to the power of i omega t common throughout. Then the first term that I have will be r contribution of E 1 then plus here also we can see that tt prime, I can common and then e to the power I can take minus of i delta common. Why do I take the minus of i delta? Because in that case, we will have this e to the power these things have in common. Let me write then I can get n equal to 2 to infinity and then I have r prime, I take 2 commons then it should be n of minus 2 and then it should be e to the power of minus of i. Then okay here in order to make n minus 2, I need to take r prime outside only it should be 2 n minus 4, and this r that is why I take it outside. So, it should be e to the power of i then it should be n minus 2 of delta because I have already taken e to the power minus i delta outside. So, if I put it inside it should be n minus 1 delta ok. So, I have a structure here r to the r prime to the power 2 n minus 2 e to the power of minus 1 n minus 2 delta. So, n minus 2 is both the cases that is the power. So, I can calculate that. So, suppose this is x and I have n minus 2 power n ranging from 2 to infinity then this sum will be simply 1 plus x plus x square so on and this is essentially 1 minus x where my x is equivalent to r prime square and then e to the power of ah e to the power of minus of i delta, that is my r prime. So, x to the power n minus 1 will be this quantity whatever we have. So, my ER simply will be E naught e to the power of i omega t that was already there then we have 1 r plus tt prime then we have r prime then e to the power of minus i delta, and then that sum term which is 1 minus r prime square e to the power of minus i delta that we calculated, this is over bracket. Well, now I use the Stokes relation to make this equation to be simple. The Stokes relation says that t t prime is equal to 1 minus r square which is one relation and r is equal to minus of r prime which is another relation, that we already derived in an earlier class.

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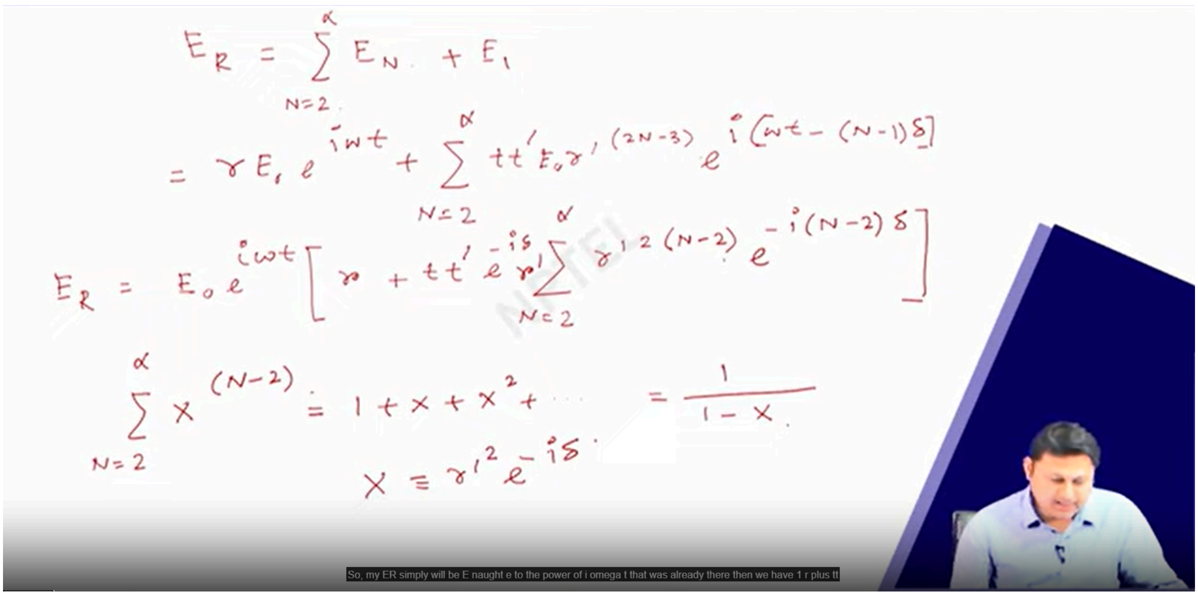

Now exploiting this relation I can have ER equal to E naught, e to the power of i omega t and then r minus 1 minus r square r e to the power of minus i delta whole divided by, just replace this R equal to minus R and take this quantity out then in a denominator. We have 1 minus r squared e to the power of minus pi Delta, few simplifications can be made further. So this is E naught e to the power of i omega t and if I multiply then this one term r multiplied by r that will cancel out. So it should be simply r into 1 minus, e to the power of minus i delta bracket close whole divided by 1 minus r square, e to the power of minus i delta so, okay. So, this E from exploiting this E we quickly calculate what is the intensity this is essentially what we want to figure out. So, I is proportional to the mod of ER square and ER square is essentially E naught square r square, and then whatever we have 1 minus e to the power of minus i delta. So, that is basically ER multiplied by ER star because they are complex quantities and then it is 1 minus e to the power of i delta whole divided by 1 minus r square, e to the power of i delta, 1 minus r square of e to the power of i delta, just multiply by the complex conjugate. So we know that 2 of cos of delta is equivalent to e to the power of i delta plus e to the power of minus i delta. Exploiting that expression we can simplify this a bit ER square is equal to E naught square, r square 2 of 1 minus cos of delta if just multiply I am going to get this, divided by the lower part if I calculate, I will get 1 plus r to the power 4 and then minus of 2 r square cos of delta. So, IR is essentially some input intensity multiplied by 2 of i0 is Ii so 2 of R square cos delta whole divided by 1 plus r to the power 4 minus 2 of r square cos delta. Seems to be a very lengthy calculation, but you need to do this calculation to understand what the outcome is. So, once we have IR. So, let me write down the reflected intensity which is 2 r square, 1 minus cos delta whole divided by 1 plus r to the power 4 minus 2 of r square cos delta. Similarly one can also find the transmitted light because if you remember the light is falling here and then it goes like this. These are the transmitted reflected light and these are the transmitted light. So this is IR sum over all this and this is IT transmitted light and this is the intensity of the incident light. So that was the overall structure? So IT, if somebody wants to calculate it should be 1 minus r square, whole square, whole divided by 1 plus r to the power 4 minus 2 r square cos delta.

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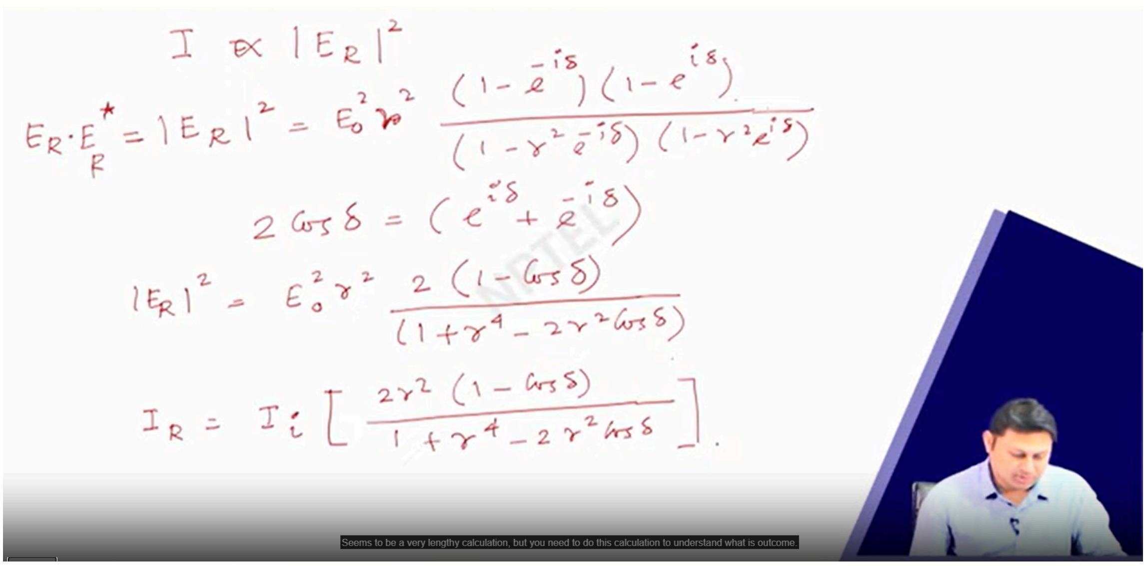

For conservation of energy, you can see that IR plus IT has to be I, that relation needs to be there, that is the conservation of energy. And if IR tends to 0 when this is 0 if we look at the equation when cos delta is nearly equal to 1. That means, when the phase difference is 2 m pi or the path difference 2 nft, this is the refractive index of the film multiplied by t cos theta, t is of the order of m lambda, with this we can get that. So from this condition, one can get that I transmission basically goes to maxima or in that case I transmission is nearly equal to the I incident. So, today what do we calculate? So, I do not have much time, but okay let me finish it with a few more lines. So, when IR is maximum, suppose I want to maximize the reflected ray then similarly cos delta will have minus 1 value instead of plus 1 then delta, the phase difference will be m plus half into 2 pi or simply 2m plus 1 pi, that is the phase difference for which we have delta. The path difference will be 2 nft cos of theta t which will be equivalent to n of m plus half lambda. So, then what happens if I put this condition? So, IR will be simply 4 r square whole divided by 1 plus r square square of that I and IT will be 1 minus r square 1 plus r square Ii. So, the transmitted and reflected intensity I somehow managed to write in terms of the reflectivity of the system. So, we will use it when we understand the Fabry-Perot interferometer later. So, today I do not have much time to discuss because I have discussed everything. There was a lengthy calculation. I request the student to please go through the calculations one by one. I am in a bit of a hurry because I need to complete this within time. But it is a very straightforward calculation, with nothing much in it, it is a very very straightforward, algebraic calculation but at the end of the day what we find is the intensity distribution for the transmitted and reflected ray in terms of reflectivity. So, exploiting this expression of the intensity distribution, in the next class we will try to understand how the Fabry-Perot interferometer works. With that note, I would like to conclude today's class. Thank you very much for your attention and see you in the next class.
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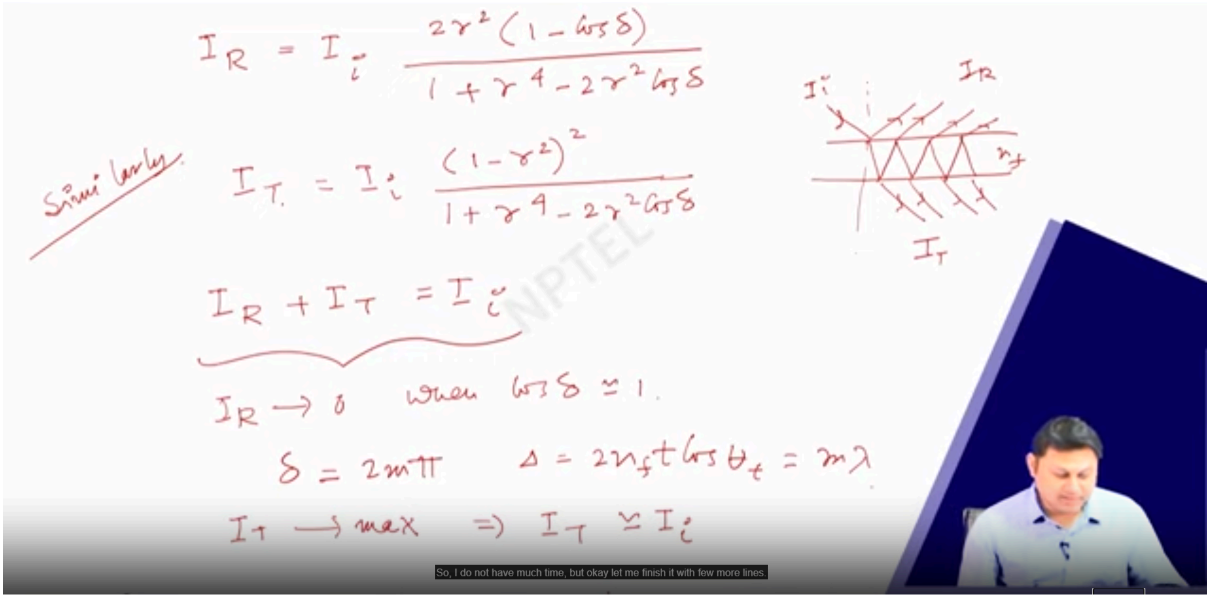

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