

WAVE OPTICS
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Lecture - 24: Optical Interferometers

Hello, students, welcome to our wave optics course. So today we have lecture number 24 and in this lecture, we will be going to investigate optical interferometers. So let me start, so today we have lecture number 24. Before going to study the optical interferometer and discuss what an optical interferometer is, let us first try to do a few things. So the first thing that I will do today is something called Stokes Relation. So what is the Stokes relation? This is essentially the relationship between the transmissivity and the reflectivity or the transmission coefficient and the reflection coefficient. When the light is falling in a system with a boundary then part of the light is reflected from the surface and a part of the light is transmitted, which is the very standard structure we have in ray optics. So if E_i is an electric field, the amplitude of the incident ray E_r is reflected and E_t is transmitted electric field respectively and if the refractive index of this medium is n_1 and this is n_2 , then the reflection coefficient can be defined as r , which is the ratio of the reflected right versus the incident right field. Essentially E_r is divided by E_i , similarly the transmission coefficient t is E_t divided by E_i . So these transmission and reflection coefficients are calculated when the ray is coming from medium 1 to medium 2 and it is experiencing a reflection here and transmission here at the boundary. Okay so once we have this, so that structure in terms of reflectivity and transmissivity I can draw. So in the first case say I have this ray falling here then it is reflected and one portion is transmitted the incident ray is E_i This is effective index n_1 , effective index n_2 . This is E_r as per our notation. But if I want to write in terms of reflectivity or the reflection coefficient, it should be rE_i .

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• STOKES RELATIONS.

Reflection coefficient
 $r = \frac{E_r}{E_i}$

Transmission coefficient
 $t = \frac{E_t}{E_i}$

(RAY REVERSIBILITY)

Well once we know that the ray is moving here this side and then going back then one can also construct a figure with the transmissivity in terms of transmissivity and reflectivity from the fact that now the ray is going from medium 2, to medium 1, it should be something like this.

In a similar way this is a transmitted field but in terms of transmission coefficient, it should be tE_i . Okay, we write the first case, simply we write in terms of the electric field of reflected

and transmitted electric field in terms of the incident electric field E_i with the coefficients r and t , which are obviously dimensionless coefficients, and that is called the reflection and transmission coefficients. So in terms of reflection and transmission coefficient, I draw figure 1 once again. In case 2, however, I will do slightly different things because we know that the ray is reversible. That means if the ray is following this path going from medium 1 to medium 2 if I follow the same thing starting from medium 2, it will follow the same path. So let me draw it once again. This is the ray that was incident and then this is the ray that is reflected and this is the ray that is transmitted but here my arrow will be in the opposite direction because the ray is now moving from the refractive index region 1 to the refractive index region n_2 to n_1 . So this is my incident light and now this light is transmitted in this way. So that means tE_i launched and in this region I get rE_i and in this region, I get the field E_i and it is happening because of the reversibility of the ray. So it is the property of ray reversibility. Well once we know that the ray is moving here this side and then going back then one can also construct a figure with the transmissivity in terms of transmissivity and reflectivity from the fact that now the ray is going from medium 2 to medium 1, it should be something like this, the original figure was this, where this is t of E_i , then it moves. Okay, so in the previous drawing, I made a slight mistake because in the ray reversibility, the ray will be opposite. So, this ray will be opposite, this ray will be opposite, but this ray, I kept this ray the same, which is incorrect. So, let me erase this. Hopefully, this it will be in the opposite direction like this, now then this is in the opposite direction and this is in this direction, this value is $r E_i$ and this value, I mean originally this value was E but now you should, so this is n_2 and this is n_1 . Now note that if this is the incident beam, assuming this is the incident beam that happened, it will reflect, this beam will go to reflect here from this surface, assuming that the rays I really launch a ray in this side and this ray is here. So this ray is falling here it is moving in this direction, so there will be a reflection of this ray. So I will get here the contribution of the two rays. One is from the transmission of tE_i and another is a reflection of rE_i . And here also on this site, I get two contributions. One is a reflection due to this ray from this surface and another is the transmission of this ray that is coming from this side. So I should get two contributions here. So let us put two arrows. So one contribution is the reflection. And if I write the reflection coefficient in this case r' , then r' multiplied by the input here, which is tE_i , that is one contribution in terms of amplitude, another contribution when the ray rE_i is moving in this direction so this is a transmission transmitted wave so if the transmission coefficient is t' , then it should be t' . No, in this case, it is moving from 1 to 2. So the transmission coefficient should not be written t' . It should be simply t because t is the original transmission coefficient we have. So the original transmission coefficient was t and it should be multiplied by rE_i this. What about this one? t t_i is the incident ray and now it is moving in the in from n_2 to n_1 . So this is the transmission and for that transmission, I should write t' transmission coefficient multiplied by tE_i and another coefficient is the reflection and this reflection coefficient should be simply r and then it should be r multiplied by rE_i . So we have an equation in our hands now because I can write E_i . So only originally it is E_i so E_i is equal to from here I can write $r^2 + t t' E_i$ which is one equation, that is coming from this expression and another is this one. Now here originally there was no light so obviously I should get zero contribution because originally there was zero contribution, there was no light and zero should be now

equal to r prime plus r multiplied by t of E_i is the value. So from these two equations, we simply find that tt' prime plus r square is equal to zero, not zero sorry, this should be one and from this expression, we can simply write r is equal to minus of r prime. So that also we can write as a phase change r into e to the power of r is equal to, so that r is equal to r prime, e to the power of π . So the strokes I get are two very important expressions. Let me now write down that tt' prime is equal to $1 - r$ square which is one equation or tt' prime plus r square is equal to r which is one equation and another equation is r equal to r prime with a minus sign and this is my second equation. These two relations are called stokes relations, we're going to use this in our future class but this is the basics of how one can figure out these two expressions. In the book, you can see that these are written directly but we need to understand how one can get such equations. Well after having the knowledge of this equation which we will be utilising maybe in a couple of classes later. Now we will jump into our original topic which is the optical interferometer. So let me give a very basic description of what the meaning of orbit is. Rather than what is the meaning of optical interferometer? So what is an optical interferometer? So an instrument that is designed to exploit the interference of light and fringe patterns results from optical path difference. So basically optical interferometers are instruments that are designed in such a way as to modulate the path difference between two rays. You can play with the interference pattern that is generated due to the light interference and the fringe pattern you can do that. So in this course, optical interferometer, what do we do? We will study two interferometers, one is the very important Michelson Interferometer and another is called the Fabry-Perot Interferometer. So these two interferometers we are going to discuss and both of these interferometers work under this amplitude division. These are Amplitude division Interferometers. Okay so let us start with the Michelson Interferometer. I don't know if today we're going to have the time to discuss everything but at least the setup I can discuss. So let us start with the Michelson Interferometer, in the Michelson interferometer, the structure and the setup are like this.

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$$E_i = (r^2 + t t') E_i$$

$$0 = (r' + r) t E_i$$

$$t t' + r^2 = 1$$

$$r = -r' \rightarrow r = r' e^{i\pi}$$

$$t t' = 1 - r^2$$

$$r = -r'$$

written directly but we need to understand that how one can get such equations. Well after having the knowledge of this equation which we will going to utilize maybe in a couple of classes later. Now we will jump into our original topic that is optical interferometer.

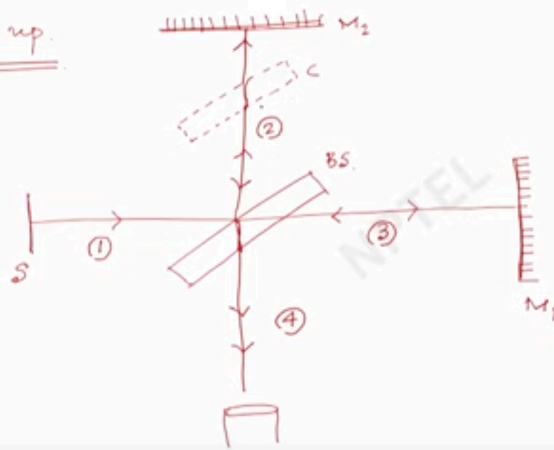
So here we have a source and then we have a beam splitter having 45 degree angle and from this upper surface the ray will be reflected and a ray will be transmitted here as well this

direction and then essentially it hits a mirror that is placed here, this is a mirror, this is mirror one, the reflected ray will also hit a mirror but here we need to put a compensator. Why are we going to put a compensator here? We are going to discuss it. The ray will pass through this compensator and it hits another mirror placed perpendicular to mirror 1. So this is M2, the second mirror and it passes through here hits mirror 2, it goes here and then again comes back. This ray will come back because of the reflection. From here also the ray will come back and essentially it hits here and again one ray the reflected ray we transmitted and this ray from this mirror which is reflected, will be reflected from this point and these two rays will come along this direction and here we have our system to catch these two lights. So let me write it down. So this is our source S, so this is ray 1 that is coming, it hits here then this is the ray 2, that is going this direction towards mirror 2, the transmitted ray is ray 3, that is transmitted and then it hits mirror 2, then again it returns back because it is reflecting and this two reflected thing will combine here and goes along this direction which is ray 4. So few things mirror 1 and mirror 2 they are perpendicular to each other. Second, why did I put in the compensator ray 2? Okay before that I need to write one thing, so ray 2 and 3 are almost equal amplitude. Then here I put c, this is beam splitter bs, so third C is a compensator and 4, S is an extended source. Now before concluding this class let us quickly try to understand why this compensator is there. If you look carefully that ray 3 is passing, the width of the beam splitter three times, this one time, it goes here, then the second time again it comes back, and the third time it goes in this direction. So three times it passes, these beam splitter widths. On the other hand, the ray one when it hits the ray two is just reflecting. So it is not passing three times. So in order to make the path difference the same we put another compensator, another beam splitter, and another block here with the same material of the beam splitter and allowed the ray 2 to pass it. So, when the ray 2 is passing here you can see that it now travels this direction once and then hits back and then comes once. So, 2 times it goes through the compensator and again finally when it is moving here it goes this direction. So, again it moves in this region. So, 3 times it is passing. So, that means, both the ray is passing the width of the beam splitter 3 times. That means, there should not be any additional path difference between these 2 rays, unless we put some path difference by just shifting the m1 and m2, by changing the m1 and m2 then there will be a path difference that we will discuss maybe in the next class. But apart from that there is no additional path difference for this ray 2 and ray 3. However, as I mentioned ray 3 originally passed the beam splitter 3 times but ray 2 is not. In order to compensate for that we put a compensator. So I don't have much time today. So our time is almost complete. So in today's class, we understood first, what is called the Stokes law or the Stokes equation, and second, we discussed the optical interferometer. Michelson interferometer we just put the outline of what is the setup of this experiment or this particular problem. In the next class, we will try to understand how using this Michelson interferometer, one can generate the path difference by shifting these two mirrors. And what is the consequence of that? We're going to get a fringe pattern here. So there will be a shift in the fringe pattern. Based on the shift, how one can calculate a few things like the wavelength of the light, etcetera. So with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.

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1. Michelson Interferometer.

The set up.



1. M_1 & $M_2 \perp$
2. Ray 2 & 3 are almost of equal amplitude.
3. C = Compensator
4. S = Extended Source.

