

WAVE OPTICS
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Lecture - 23: Newton's Ring (Cont.)

Hello, student, welcome to the wave optics course. Today we have lecture number 23, where we continue the concept of Newton's ring, the formation of Newton's ring, and a few more aspects of this topic. So, let me write, today we have lecture number 23. So we already introduced how Newton's rings are formed and how from that spectrum and whatever fringe pattern we have. From that fringe pattern how one can figure out the wavelength of the unknown light? So today we are going to study a few more aspects of Newton's rings. So first let me draw once again the basic structure of Newton's rings, where we have this kind of a glass plate over which. This is the contact point of the glass and then this a plano-convex surface and then this is O is the center of the curvature that it has and the light falls here and then reflected back from this surface and another light falls here and reflected back from this surface and these two lights are essentially interfering with themselves to form the pattern. In order to form a pattern, they should maintain a phase relationship among them. So suppose this separation from here to this is say T of m. So that is the path difference, from which we can calculate the path difference and we calculated that, for maxima, we have the condition $2n t_m$, where n is the refractive index of this region. Normally it is air and it is 1 but let us put this value as n, then that value is $2m$ minus λ by 2, this is for maximum. Similarly for minima, one can find out that this path difference should be simply $2m$ of λ by 2. So in the first case, it is an odd multiple of λ by 2 for maxima, and for minima, we have the even multiple of λ by 2, so here the m value can go from 1, 2, 3 etcetera, and here the m value takes 0, 1, 2.

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Lec No = 23.

Diagram 1: A plano-convex lens of radius of curvature R is placed on a flat glass plate. The thickness of the air film at a distance r from the center of contact is labeled t_m . The center of curvature is labeled O.

Diagram 2: A circular fringe pattern of Newton's rings, showing a central dark spot surrounded by concentric rings.

Diagram 3: A side view of the lens and glass plate showing the air film thickness t_m at a distance r from the center.

At the contact point $t_m \rightarrow 0$
 one can get a spot dark spot at centre.

for maxima
 $2nt_m = (2m-1)\frac{\lambda}{2}$ ($m = 1, 2, 3, \dots$)

for minima
 $2nt_m = 2m \cdot \frac{\lambda}{2}$ ($m = 0, 1, 2, \dots$)

for maxima
 $r_m = \sqrt{2Rt_m}$

m or in other way if I calculate m and R is known then you can calculate t. So that is the formula for m. Now utilizing this formula we can figure out few more interesting things. So let us do that, so already we find that for

So at this point, you may note that at the contact point, the value of T_m is zero. So from these

two equations, one can easily figure out that this condition can happen for minima, so that means at the contact point since t_m is equal to zero, one can get a dark spot at the center. So, if I draw a structure which are concentric circles, this is the fringe pattern that one can see in this experiment but the central spot will always be dark in this setup because of this condition. So after having you know these conditions of maxima, minima we can play with this and try to find out a few more interesting aspects. Also, one important thing we calculated was the, so let me draw it here only, so the fringe is formed based on the structure and suppose some maxima is here at the point r_m , r is measured from this, we calculated that r_m for maximum, r_m was root over of 2 of R t of m that we figure out also in the last class, that means if we know the path difference, if we know the thickness and if we know the R then we can calculate the value of r_m or in another way if I calculate r_m and R is known then you can calculate t_n . So that is the formula for r_m . Now utilizing this formula we can figure out a few more interesting things. So let us do that, so already we find that for maxima one can have, so what we get here, is $2n t_m$, $2m$ minus 1 , λ by 2 . So that I should write here. So for maxima, we get $2n t_m$ that is equal to $2m$ minus 1 λ by 2 , here m goes to $1, 2, 3$ etcetera. In a few cases, it is also written as $2m$ plus 1 , λ by 2 , but these are the same in that case m can also take the value 0 . Well from here, we can find out that t_m for maxima should be $2m$ minus 1 λ by $4n$. So that is the value of t_m , so if I incorporate also we had for bright fringe r_m is equal to root over of 2 of R of t_m , now t_m is this quantity. So for the bright fringe, the radius is this, that is the value I can simplify that R $2m$ minus 1 and then it is λ divided by $2n$, where m is an integer and takes the value $1, 2$ etcetera. So that means if suppose we have a bright fringe here Newton's ring, I'm calculating the value of r_m of that bright fringe. So if I now calculate the diameter of this bright ring, the diameter of the bright ring is simply D_m is equal to 2 of r_m , 2 multiplied by the radius that is the diameter of the bright ring. So, then we can write that D_m square is simply equal to $2m$ minus 1 into R λ all divided by $2n$ into 4 or diameter should be 2 into $2m$ minus 1 λ R by n , that is the diameter of the bright ring.

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For maxima.

$$2n t_m = (2m-1) \frac{\lambda}{2} \quad (m=1, 2, 3, \dots)$$

$$t_m = \frac{(2m-1) \lambda}{4n}$$

For bright fringe.

$$r_m = \sqrt{2R (2m-1) \frac{\lambda}{4n}}$$

$$= \sqrt{R (2m-1) \frac{\lambda}{2n}} \quad (m=1, 2, \dots)$$

Diameter of the "bright ring" $D_m = 2r_m$

$$D_m^2 = (2m-1) \frac{R\lambda}{2n} \times 4 = 2(2m-1) \frac{R\lambda}{n}$$

For a dark ring.

$$D_m = \frac{4mR\lambda}{n}$$

So, what we are doing here, is trying to find out what should be the difference between the m th and say m minus 1 st order dark ring because if I manage to get that value then we can

find out how the diameter of the ring throughout the length is varying and we will see that this value is a function of m . So that means over the distance of the diameter of the ring, the width of the diameter of the ring will be affected by this. It is not the same for all the rings. So if I calculate this for a bright ring then for a dark ring we can simply have D_m equal to, just putting $2m - 1$, instead of putting $2m - 1$ multiplied by λ by 2, you should put $2m$ multiplied by λ by 2 and if I do we'll go to get a term like $4m\lambda$, then R divided by n that should be the value of the diameter for an m th order dark ring. Now, m th order dark ring so suppose this is a dark ring, the central ring, another dark ring can also be possible here, say this is this dark ring is say m th or $m + 1$ th dark ring and this is say m th dark ring, so $m + 1$ th dark ring and m th dark ring I know the general formula of the diameter of m th dark ring. So I can write that D_m square is equal to 4 of m of λR divided by n . So, the difference of the m th and $m + 1$ st order dark ring in the diameter can be calculated simply by just putting m to $m + 1$. So, what I am trying to do is make a difference in the diameters of $m + 1$ st and m th dark rings, you can do that for bright rings, also m th dark rings. So, that difference if I write ΔD_m that should be $D_{m+1} - D_m$. Now, from here we can write D_{m+1} square is equal to $4\lambda R$ divided by n into $m + 1$. So if I simply put this result here then it should be $2\sqrt{\lambda R/n}$ and then $\sqrt{m + 1} - \sqrt{m}$. Now this value is independent of m . So that means if I increase the value of m then the separation will be smaller and smaller. So one should note that for a large m , the value of ΔD_m is small, which means the rings gradually become narrower as their radial increases. So that means if I look at Newton's ring we can see that the separation between these two rings is consecutive, and dark rings will gradually reduce, which means over a large distance what happened, the rings are gradually becoming narrower and that is an interesting feature of Newton's ring. So if I see the ring then there are many ring structures available in this superposition of two lights in different systems. We can also see there is a Michelson interference problem where also we have a circular fringe kind of pattern.

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$D_m^2 = \frac{4m\lambda R}{n}$ $D_{m+1}^2 = \frac{4\lambda R}{n} (m+1)$

• Diff. in the diameters of $(m+1)^{\text{th}}$ & m^{th} dark ring is

$$\Delta D_m = D_{m+1} - D_m = 2\sqrt{\frac{\lambda R}{n}} (\sqrt{m+1} - \sqrt{m})$$

Note For large m , ΔD_m is small that means the rings gradually become narrower as the radius increases.

Here also find a circular fringe but the identity of this kind of fringe is this quality if ΔD_m

is small then the rings gradually become narrower. So that is one aspect I wanted to discuss and then next that is important is how somebody can measure the fringe of the width of these rings because these rings are not like this line, it should have some width. If I draw one ring very close, my drawing is not perfect, but try to draw. So this is one ring. So now this is one like a fringe. So I want to find out, what the fringe width of this system is so that one can also figure out exactly the way we figure out for the case how the width will go, and how the separation between these two fringes is changing. So here we are determining fringe width, so it is simply with calculation, we already have a bright ring, so let us start with the bright ring in this case. So for a bright ring, we have D_m^2 is equal to $2 \times 2m - 1$ λR divided by n . What happened to the next ring? m should be replaced by $m + 1$ that's all. So we should write at $2 \lambda R$ divided by n and then $2m + 1$ minus 1 . So once we know the value of D_m^2 and D_{m+1}^2 of that then we can calculate this, so their difference, D_{m+1}^2 minus D_m^2 is simply this minus this. And it is easy to just check, it is $2 \lambda R$ divided by n . If I take this term common, then the rest of the term is simply $2m + 1$, $2m - 1$, $2m + 1$ minus, $2m - 1$. So, you should have simply 2 terms here and that leads to 4 of the λR divided by n . Now I write as $D_{m+1} - D_m$ and $D_{m+1} + D_m$ which will be equal to 4 of λR whole divided by n . Now we checked in the previous calculation that when m is large then the separation between $m + 1$ and $m - 1$ is reduced. So this value I can write as this is equivalent to 2 of D_m . If that is the case, then we simply have $D_{m+1} - D_m$, that value is equal to 4 of λR divided by 2 of D_m which is equal to 2 of λR divided by D_m straightforward calculation I am doing. I am showing that by utilizing the expressions I can find out many aspects or different results related to the structure of the fringe pattern. So the fringe width, interesting quantity fringe width β which one can define as $D_{m+1} - D_m$ divided by 2 because we are dealing with diameter that's why I need to put it divided by 2 is simply λR whole divided by $n D_m$. Again you can see that β is inversely proportional to the value of D_m .

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o Fringe width calculation.

\Rightarrow For bright ring $D_m^2 = 2(2m-1) \frac{\lambda R}{n}$

$D_{m+1}^2 = \frac{2\lambda R}{n} [2(m+1) - 1]$

$D_{m+1}^2 - D_m^2 = \frac{2\lambda R}{n} - = \frac{4\lambda R}{n}$


$(D_{m+1} - D_m) (D_{m+1} + D_m) = \frac{4\lambda R}{n}$

$\approx 2D_m$

$D_{m+1} - D_m = \frac{4\lambda R}{2D_m} = \frac{2\lambda R}{D_m}$

fringe width $\beta = \frac{D_{m+1} - D_m}{2} = \frac{\lambda R}{n D_m}$

That means if I go to the higher order fringe then the beta value will reduce compared to the lower value of radius.



That means if I go to the higher order fringe then the beta value will reduce compared to the

lower value of radius. Finally, I like to show another thing that one can determine, The next day we can discuss this briefly, utilizing this Newton swing structure one can also determine the refractive index of a liquid. So this is the determination of the refractive index of a liquid. So the structure is simple, in one case we have the conventional setup, where this plano-convex lens is placed on a glass block and this is where this quantity is air having refractive index n_a . In other cases, what happened? That I have the structure, this plan convex lens is here but instead of the air we have some liquid placed here, I am setting this region too, so this is now liquid with some refractive index say n_l . Now this is case one and this is case two. So for air, we can calculate this quantity, which we already calculated actually. So, I am not going to go into detail that $D_{m+p}^2 - D_m^2$ this is for air that we calculate and this value is $4p$ where p is the order λR divided by n_a exactly the similar thing we can do for the liquid, where we can calculate this quantity that $D_{m+p}^2 - D_m^2$ for liquid, under the placement of the liquid I calculate this experimentally. Now the left-hand side I can calculate experimentally. So, if I now divide this equation we simply have $D_{m+p}^2 - D_m^2$ for liquid, which we calculate experimentally divided by $D_{m+p}^2 - D_m^2$ for air, this is liquid also can be calculated experimentally both the thing we calculate experimentally which simply comes out n_l divided by n_a . For the air, normally n_a is very close to 1. Then utilizing this expression, one can simply find out what is the value of the refractive index of the liquid by just calculating the left-hand side using the experiment setup. So on that note, I would like to conclude here, I don't have much time to discuss more about this. But roughly we cover Newton's rings problem, which is an extensive problem. In many cases it is used, I believe the students who are taking this course in their lab also may have these experiments. That's why I wanted to go through these things in detail and I show all the aspects of the formation of Newton's rings and the related calculation. In the next class what I will do is I will calculate What happens when we use some sort of interferometer, optical interferometer and what is optical interferometer. It is a system that is such that changing the cavity, means changing the length of two mirrors or two plates. I change the path difference essentially, I am changing the path difference, and by changing the path difference what we can do? We find the change in the interference pattern that is produced and from that also we can calculate a few things like web lens etcetera. So that we're going to cover in the next class. So with that note, I would like to conclude here, thank you very much for your attention and see you in the next class.

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a Determination of the RI of a liquid.

①

air n_a

For air $(D_{m+p}^2 - D_m^2)_{air} = \frac{4p\lambda R}{n_a}$

②

Liquid n_l

For the liq. $(D_{m+p}^2 - D_m^2)_{liq} = \frac{4p\lambda R}{n_l}$

$$\frac{(D_{m+p}^2 - D_m^2)_{air}}{(D_{m+p}^2 - D_m^2)_{liq}} = \frac{n_l}{n_a}$$

In the next class what I will do is I will go to calculate