

WAVE OPTICS
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Lecture - 22: Newton's Ring (Cont.)

Hello, students, welcome to our wave optics course. So today we have lecture number 22. We are going to study Newton's rings, which we already introduced in the last class. So today we will study how one can find out the condition for maxima, and minima of these rings. So today we have lecture number 22. So today our topic is Newton's ring. Well in the last class, we have already discussed how Newton's rings are formed, the formation of Newton's ring, and the corresponding experimental setup. So today we will extend this idea, to find out what should be the path difference and what is the condition for getting maxima and minima in terms of those rings. So let me draw once again the geometry. So this was the slab and over that, we have a wedge-like structure like this and so this is R , the radius of curvature of this curve and if I join that then, suppose the light is falling here and then gets reflected from the lower surface and get interfered and we have the interference. So this is ray 1 and this is ray 2, these two will interfere. Now this length is a path difference, I write as t of m and if this is the origin from here to here. This point is called r_m , so the point is to find out r_m this radius because that radius is essentially the radius of the rings that we want to form in terms of this path difference with the geometry that we have here. So let us try to do that, let this be R and this is t_m . So if I put O here and this point is say A and this point that is touching here is B and this point is D . So my OD is equal to R , which is equal to OB but OB is equal to OA plus AB , my OA then is equal to OB minus AB . From this picture, it is easy to show that OB is nothing but R and AB is nothing but t_m . So OA , this quantity is simply R minus t_m . Now from this simple geometry, we can write that r square is equal to r minus t_m square minus r in AD square.

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Lec No = 22
 Newton's Ring

$OD = R = OB$ $AD = r_m$
 $OB = OA + AB$
 $OA = OB - AB$
 $= R - t_m$
 $R^2 = (R - t_m)^2 + r_m^2$
 $R \gg t_m$
 $R^2 = R^2 - 2Rt_m + t_m^2 + r_m^2$
 $r_m = \sqrt{2Rt_m}$
 Path diff. at the point r_m
 $A = 2nt_m$

$AD^2 + AO^2 = OD^2$

that is, this point according to our figure, that is delta is equal to $2nt_m$, where n is the refractive index of this region, if it is pure air then it is simply 1

So, AD is equal to r_m and I am just using the Pythagoras theorem which is AD square plus

OB square is equal to OD square, where my AD is rm , OB is, no , this is AO square, AD square, plus AO square. Can I make a mistake? Here we have AD square, plus AO square, this is AO square. So AO square is equal to R minus T in the whole square plus rm square. Okay now usually this r radius of this curve is very much greater than this quantity tm . So we can neglect a few terms related to tm square. So in that case we have R square is equal to R square minus 2 of R of tm and I am neglecting the tm square because tm is very very small compared to r , then it is simply plus rm square. So r square, r square will cancel out and I simply have rm as root over 2 of R of tm . So if I want to find out what is the path difference at point tm , then the path difference will be for normal incidences, in this case, all are normal incidences the path difference at point rm that is, this point according to our figure, that is Δ is equal to $2n Tm$, where n is the refractive index of this region, if it is pure air then it is simply 1 . So that we know rm and this is the path difference we know. Now as I mentioned at this point, the two rays will interfere. So let me draw here, how these two waves are going to interfere. So this is the structure we have, this wedge-like thing, this is the slab we had, and the ray will fall here first and go here and another ray will be reflected from here. So these two rays essentially interfere with one and two and this path difference, the distance is tm and that is the structure. So ray 1 and 2 will interfere, so the total phase difference then because there is a reflection from this surface if it is n and if it is n_g , so n_g is greater than n . So there is a reflection from this surface. So the total phase difference will be k naught Δ plus π , due to this reflection and that is 2π by λ , path difference is $2n tm$ plus π . Now for maxima, if this interference generates a maxima there is a circle with maxima intensity for maxima What do we have? We have 4π divided by λ then $n tm$ plus π that is equal to 2 of $m\pi$ integer, 2π integer multiple of 2π or $4n$ into tm divided by λ , this first term is equal to $2m$ minus 1 , π , π will cancel out or in other word it is. So tm is $2m$ minus 1 multiplied by λ divided by $4n$. So that is the condition we have for maxima, for minima similarly, we can have the condition. So From this knowledge let us check how one can use Newton's ring to you know to form the to find the wavelength of the light that is falling over here. (Refer slide time: 15:03)

Ray 1 & 2 will interfere

Total Total phase diff.

$$k_0 \Delta + \pi = \frac{2\pi}{\lambda} \cdot 2n t_m + \pi$$

For maxima

$$\frac{4\pi}{\lambda} n t_m + \pi = 2m\pi$$

$$\frac{4\pi t_m}{\lambda} = (2m - 1)$$

$$t_m = \frac{(2m - 1)\lambda}{4n}$$

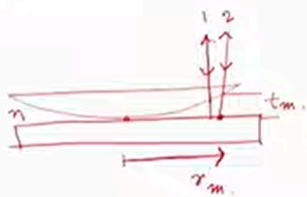
So, from that you can find out and the minimum I am not going to do that it is some sort of exercise that you please do

So how do you do the wavelength measurement that I like to show here? So that is the

expression we have, this t_m where we have the maxima is this, now for dark ring in the similar well not going to do this problem because that is the, okay so for dark ring what happened, that this condition the path difference which will be the phase for maximum, phase difference which is $2m\pi$ it will be just the integer multiple of for destructive condition it is just $2m - 1$ $2m - 1$ into π by 2. So, from that you can find out and the minimum I am not going to do is some sort of exercise that you please do. So, the next thing that I do is this: how to measure how to measure the wavelength. Ok so for, so this is the structure. These two rays will interfere and form a bright ring or dark ring. So we already had, I mean we are already had, the expression that if this is my r_n , that is the radius of, suppose it is forming a dark ring then this dark ring radius is r_n and r_m is equal to root over of $2R$ and this thickness t_m . Now if I multiply this with 2 then it is $2r_m$ is equal to 2 of root over of $2r$ t_m , where this is my t_m path difference. So, for air if this is n we know the path difference is 2 into n into t_m for these two wells, now for air n is equal to 1 and the path difference Δ will be equal to this path difference, Δ will be simply equal to 2 of t_m . Now for the dark ring, we are talking about the dark ring because it is easier to calculate that this path difference plus this change of phase which leads to another path difference Δ_r is the total path difference. If it is λ by 2 integer multiple of then we have destructive interference dark rings. So, Δ_r is a path difference due to the phase difference of π . Now, in this condition, there is a, I mean, it is, it is a multiple of λ by, so let me erase this. Let me first find out what is the path difference of Δ_r . So, Δ_r is a path difference due to the π phase difference of reflection. So, Δ_r is equal to λ by 2. So, for this factor, we have a condition like this that it has to be the odd multiple of λ by 2. So, then Δ the path difference for it is for minima then the Δ will be simply $2m + 1$ λ by 2 minus Δ_r , which is equal to $2m$ multiplied by λ by 2 or m of λ that is the path difference we have for destructive interference. So, the Δ here I already calculate it is $2t_m$ and that is equal to m of λ . So, if I put it back here, this $2t_m$ value, then will be 2 of root over R multiplied by m λ .

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1. How to measure the wavelength



$$r_m = \sqrt{2Rt_m}$$


$$2r_m = 2\sqrt{2Rt_m} = 2\sqrt{Rm\lambda} = D_m$$

Path diff.
 $2nt_m$
 for air $n=1$
 $\Delta = 2t_m$

$\Delta + \Delta_r = (2m+1)\frac{\lambda}{2}$ (For minima)
 $\Delta_r =$ Path diff. due to the π phase diff. of reflection
 $\Delta_r = \frac{\lambda}{2}$

$\Delta = (2m+1)\frac{\lambda}{2} - \Delta_r = 2m\frac{\lambda}{2} = m\lambda$
 $2t_m = m\lambda$

that we figure out



And that is again the diameter of the dark ring, D_m is the diameter of the dark ring, so D_m is

$2\sqrt{r m \lambda}$, that we figure out. Now if I write it here suppose we have a dark ring, say this is the dark ring that we find and this is the m th order dark ring whose diameter D_m is as I calculate. Let me see what the calculation d_m is $2\sqrt{R m \lambda}$. So $2\sqrt{R m \lambda}$ and this wavelength λ , where r is the radius of curvature of this curved surface that we had to find the Newton's rings this is m th order. There are other rings also because this structure is a concentric structure. Suppose these are the different rings and I'm just talking about this dark ring that is the MS. Suppose it is the m th dark ring so I can have D_m square here as $4 R m \lambda$. So I can measure the m th order of the dark ring and if D_m is measured then I can find out that this value will be $4 R m \lambda$ square. Similarly, I can also measure some other order, suppose D_{m+p} plus m th order dark ring, and for that, this value will be $4 R (m+p) \lambda$. If I subtract this D_{m+p} minus D_m square, we can measure experimentally this left-hand side and then we play with the numbers then we have $4 p \lambda R$. So, λ then one can find out easily with the expression that $D_{m+p}^2 - D_m^2$ whole divided by $4 p R$. Now, we measure experimentally, what we measure experimentally D_m and D_{m+p} can be measured, so through experiment we can measure D_{m+p} and D_m , where the value of the p is also known I know which order I am measuring, and then exploiting this expression by knowing the value of R . So R if somebody provided then all the right hand side is known. So I am doing an experiment without knowing what the λ is. Then Newton's ring experiment. So in Newton's ring experiment, this is the structure. So light is falling here, like this experimental setup I mentioned also in the last class: how light will come from here, reflect, and make a ring-like structure like this. Then by using the microscope, we can find out what is the value of the diameter of these rings, and knowing all this value in the right-hand side with the value of R I can find out what is the value of λ . So this one is an important experiment through which you can measure the value of the unknown wavelength. I mean it is possible that one can measure what is the value of this region, this region means the refractive index of this region. Suppose I have this experiment by putting some kind of oil here instead of air. If I put some kind of oil here whose refractive index says it is in oil, then the refractive index of the oil can also be measured, also be calculated using this experiment. So today I don't have that much time and also in the next class maybe I'll do more calculations to show how these rings are formed and how the width of the ring is changed with the order of Newton's rings. If I increase the order how the width will change and also like to show the technique through which an unknown liquid or any unknown substance is placed here in between these wedge-like structures. Then how somebody can calculate the refractive index of this exploiting this very interesting Newton's rings system? So with that note, I would like to conclude, so today that what we learned is how one can find out the unknown wavelength λ by exploiting Newton's rings experiment and all the mathematics for Newton's rings experiment, where by knowing the value of r , r means the radius of curvature of this convex glass substrates, so one can find out the radius of this ring as well. So, with this note I would like to conclude, so, in the next class I will do more on Newton's rings where we will show how the refractive index of the unknown well can be measured with the same technique. So, I hope you will enjoy that treatment as well. Thank you very much for your attention and see you in the next class.

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m^{th} order dark ring.

$$D_m = 2\sqrt{Rm\lambda}$$

$$D_m^2 = 4Rm\lambda$$

$$D_{m+p}^2 = 4R(m+p)\lambda$$

$$D_{m+p}^2 - D_m^2 = 4p\lambda R$$

$$\lambda = \frac{D_{m+p}^2 - D_m^2}{4pR}$$



Experimentally.

D_{m+p} & D_m
↓
Known.

$R \rightarrow$ Known.

then these refractive index of the oil can also be measured, also be calculated using this experiment. So today I don't have that much of time and also like to in the next class maybe I'll do more calculation to show that how these rings are formed and how the width of

the ring is changed with the order of

