WAVE OPTICS

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Lecture - 19: Interference by division of amplitude

Welcome, student to our wave optics course. So in the last class, we develop the concept of interference and then introduce how the double slit will work then also we introduce that, there are a few other techniques through which you will going to get the interference pattern namely Lloyd's mirror experiment and also the Fresnel's biprism where using these two instruments you can manage to get the interference pattern. However, there are other techniques also. So, in today's class in lecture number 19, we are going to discuss the interference by division of amplitude. So, let me start with that. So, today we have lecture number 19 and this is lecture number 19 what we are going to study is interference by division of amplitude. So far in the case of you know Young's doubles list experiment what we had that we had a system like this. We have two sources here or two holes A and B and then from one source this is the wavefront that was there and then from A to B two separate wave fronts were generated like this and throughout the process what happened was that they interfered at some point on the screen say p. So here we have the division of the wavefront because wavefronts are divided so we have a division of wavefronts in today's class. However, we try to understand what happened instead of wavefront division. We have amplitude division and how it happens let me give you a very simple example here suppose you have a single film and if light falls on that then one part will be refracted and one part will be reflected from this point this is ray number 1, which is reflected and this ray which is refracted will again reflect from this point and get refracted like this, this is ray 2.

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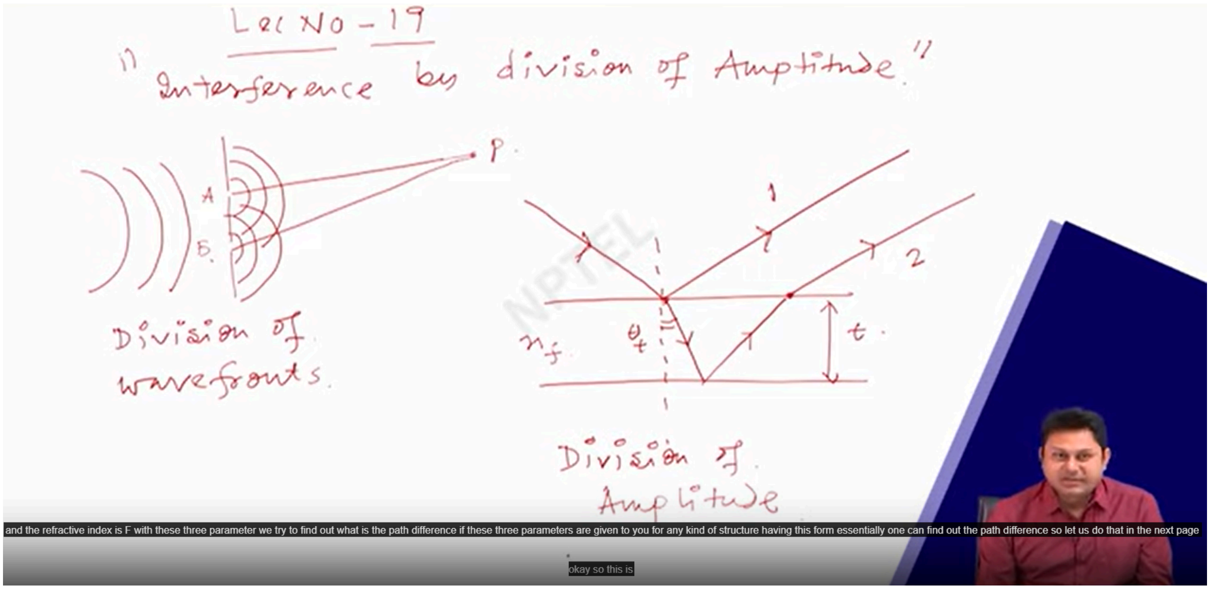

So, I have ray 1 and ray 2 and essentially they can if I put a screen here they can interfere with themselves. Now this ray 1 and ray 2 the interference that is happening due to ray 1 and ray 2 is due to the division of amplitude because here the amplitude gets changed in this one ray and two rays should not have the same amplitude.So that is the basic structure one can have when trying to understand how by division of amplitude interference takes place. So this is due to the division of amplitude. From this figure, it is also clear that the ray that is travelling ray 1 directly from this point and the ray that is travelling from this point which is going through the film. So these two rays should not have the same path because there is already a path difference that is encountered by this ray 2 during this process. So if somebody calculates the path difference and then calculates what is the path difference then like in the previous case the path difference will determine whether these two interferences will get a maxima or minima at some point. So we are going to check today that how what should be the calculation, what should be the path difference in terms of these parameters. Suppose this has a refracted angle phi t the thickness here is t and the refractive index is F with these three parameters we try to find out what is the path difference if these three parameters are given to you for any kind of structure having this form essentially one can find out the path difference. So let us do that on the next page. Okay, so this is single-film interference. Due to single film, so single film interference. Okay so in order to find out the path difference I need to draw it once again. So let me draw exactly the same figure. I'm going to draw this as the ray that is falling here then is refracted and it is bounced back here and then again goes to this direction on the other hand I have one ray that is going this direction after reflection from this surface. So I have this, now let us consider this angle to be an incident angle to be theta i and this angle to be theta T transmitted angle refractive index here says no and the refractive index of the film is nf and also the thickness that I mentioned is T. Now I need to find out the path difference. So let me first write down, this is A, This point is A. Suppose this is B, heating in point C. And if I draw a perpendicular here. Then this is D. And this angle is easy to show that this angle should be theta i. Note that this theta i angle and this angle whatever the angle say x is pi by 2 because this is 90 degrees and this angle and this angle theta i is also pi by 2. So from that it is easy to show that this angle should be theta i, the incident angle. Well if I draw a perpendicular here Why I am doing that I will explain. There are several ways through which you can find out the path difference. I am just showing one of them. And if you draw a perpendicular here from this point. So this is say point G. This point is E. And this point is F. So the optical path difference let me calculate here based on this figure so light is falling here reflecting back so the optical path difference delta is equal to The extra part that is travelling, this is ray 1 and this is ray 2. The extra part travelling by ray 2 that we need to calculate, is this AB plus BC. So, nf is multiplied by AB plus BC and then this part AD is the additional path that is travelling by ray 1. So I need to subtract so minus no multiplied by AD So, that is the path difference between ray A, ray 1 and ray 2. So, I can split this in this way for simplicity of the calculation. So, let me write down the delta as equal to then nf AB, I define AE plus EB plus BF plus Fc. I just split AB and BC in this way. There is one part here AE plus EB and BF plus Fc then minus n naught AD. So that is my path difference. Now I need to put some kind of geometry to find out what these values are in terms of nf theta t. So let me write here since of theta t, by the way, let me find out what is this angle. This angle is theta t, this is theta t minus 90 degrees. So this angle has to be theta t. Similarly, this angle is also theta t. So let me write it clearly. So angle AGE is equal to angle FGC is theta t very easily one can check that. So sine theta t in that sense should be AE divided by AG or AE is equal to AG of sine theta 2. Now, by symmetry, we can write AG is equal to AC divided by 2. So, essentially I have A e is equal to A c divided by 2 then sin of theta t okay. Now, we also have AD which is here to here and this angle is theta i is equal to AC into sine theta i. Now, from Snell's law, we have n0 sine theta i is equal to nf sine theta t just using Snell's law. So, if I calculate n0 ad then let me write down if you calculate this thing n0 AD, that is the last part of this path difference. This is 2 of AB, I am writing 2 of nf AE why is that, let me check because, let me do that in this way so 2 of AE from here from this equation I get this is equal to AD sine of theta t divided by the sine of theta i, AD sine of theta i AD divided by the sine of theta i is AC and this is. So here I have 2 of AE is equal to AC of sine of theta t. So that AC I write from this equation, I write AD divided by the sine of theta i which is 2 of AE. So this quantity I can write AD from Snell's law. I just use this: this is n0 divided by nf Now, also we have cos theta t is equal to EB, cos theta t means this, this is EB by GB because this angle is theta t. So, cos theta t is EB divided by GB. Now, this GB is nothing but the thickness. So, our aim is to find out everything in terms of thickness t, theta t and the refractive index nf. So, this quantity will be simply eb divided by thickness t, so that we know, now let me go back to the equation that we had a delta equal to nf AE plus EB plus BF plus Fc and then n0 AD n0 AD. I already calculate this is nf ah into AE and this AE means this to this. Now AE these two of AE can be written as nf multiplied by AE plus FC because AE and FC are same. So this I can write is nf multiplied by AE 2 of AE I write AE plus fc. Now, all the components I believe I figured out I do not just need to put it here. So, if I do that then my delta is equal to nf then Eb plus BF which is equal to 2 of nf EB. So, EB plus BF means this is EB and BF EB and BF are the same so 2 of EB is EB plus BF in a similar way I can write EB plus BF is equal to 2 of 2 of this EB now that quantity is equal to 2 of nf of t of cos theta t because you remember that, cos theta t here it was EB by t that I calculated here. So, cos theta t was EB divided by thickness. So, EB I can calculate in this way.

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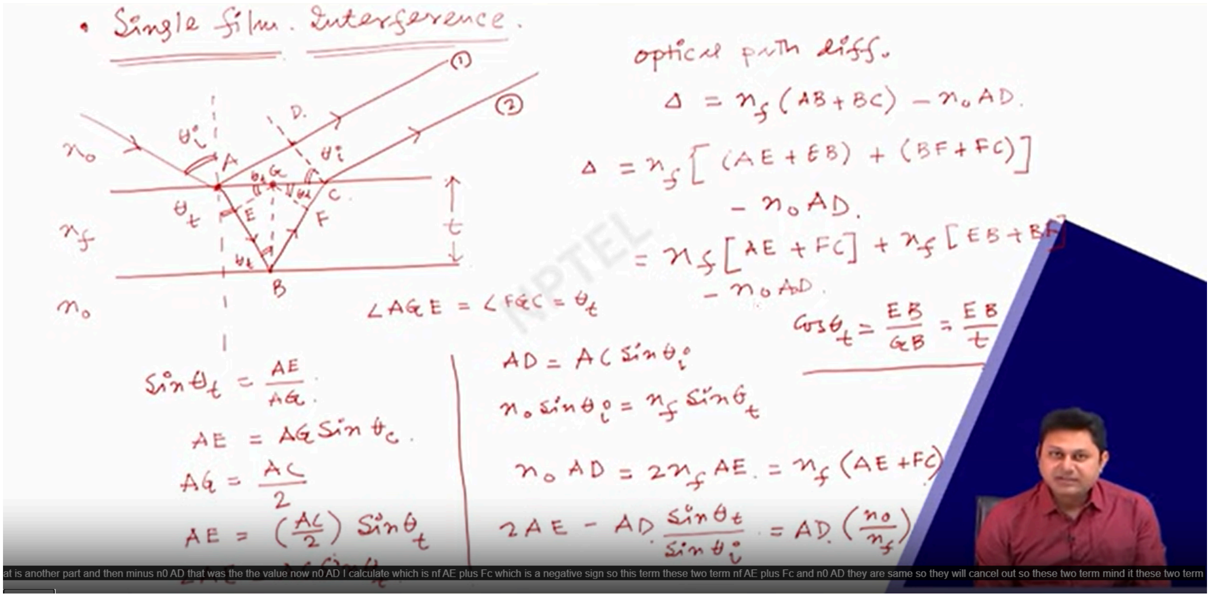

So the path difference finally We can have this quantity because the rest of the quantity is canceling out. So, here I see that this quantity n0 AD, n0 AD was NF AE plus FC and nf AE plus FC was already there. FC is equal to FC and AE this quantity is here. So I can write it as so let me write it once again here. So I can write nf multiplied by AE this and FC from here plus Fc is one part then plus nf EB plus BF is another part and then minus n0 AD that was the value now n0 AD I calculate which is nf AE plus Fc which is a negative sign. So these two terms nf AE plus Fc and n0 AD are the same. So they will cancel out so these two terms mind it these two term will cancel out. So, these and these are the same, the rest of the term is nf EB plus BF, and essentially we calculate in the next plate nf EB plus BF which is equal to nf 2 of nf t theta. So finally the path difference, let me draw once again. If this is my structure single film ray is falling here, then if this thickness is t this angle is theta t and the refractive index of the film is nf then the path difference between ray 1 and ray 2 delta is simply equal to 2 of nf then thickness t and cos of theta t. So that is the path difference we have and note that when the ray falls normally then theta t value is 0. So we can have a normal incidence of light, theta t is 0. So the path difference will be simply 2 of nt into t, and the optical path difference will be simply this quantity but there are a couple of things that we need to know here also and that is the reflection, due to the reflection the additional phase shift occurs and this additional phase shift is of the amount of pi. So an additional phase shift is arising from the reflection which means if this is n1 and this is n2, we have a pi phase shift and this pi phase shift occurs when it is reflected from a denser medium to a rarer medium. So, in this case, n2 will be greater than n1, and we will get a pi phase shift here that we need to consider every time. So what is the condition for constructive and extractive interference? Let us quickly now check that the phase difference obviously leads to a path difference, this phase difference pi. So for the phase difference pi due to reflection leads to a path difference I write the delta r of lambda by 2. This path difference arises solely due to the fact that light is reflecting from the n1 medium to the n2 medium and this n2 medium is higher than the n1 medium the refractive index n2 is higher than n and in this case pi phase difference is there. So, if there is a pi phase difference there should be a path difference associated with the path difference that we need to consider and that is lambda by 2.

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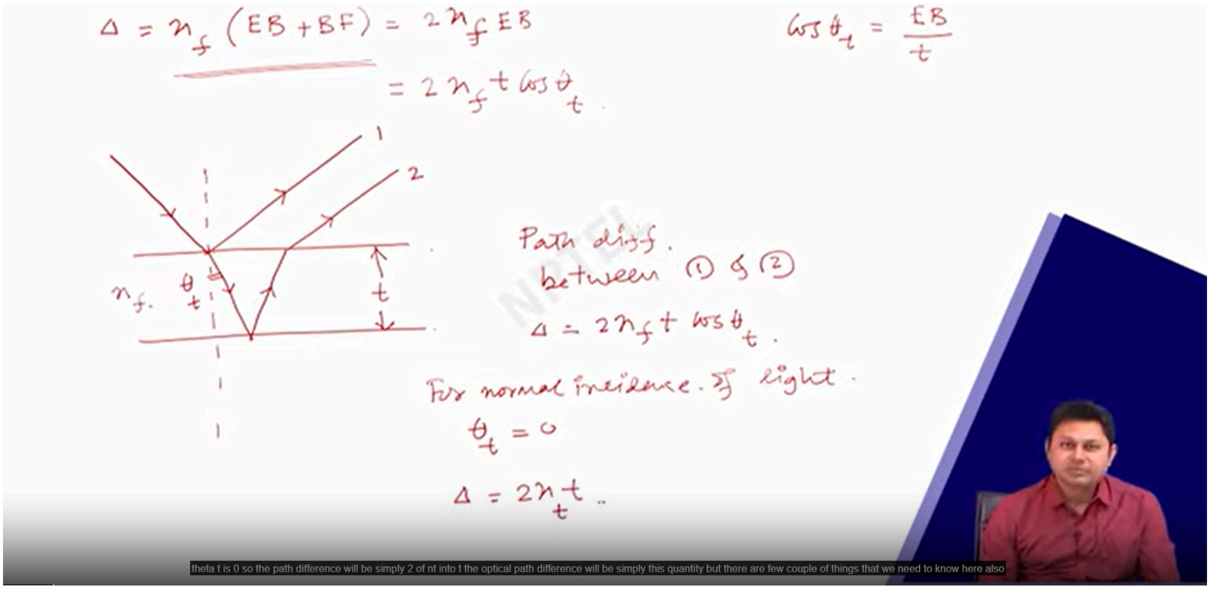

So the total path difference will be defined by the path difference due to the ray that is passing through this film that we calculated 2 nT, T cos theta that we calculated cos theta T, but you need to add an additional path difference with that because of this reflection issue. So, that should be the total path difference. Now, for constructive interference. Now for constructive interference, we can have the path difference delta plus delta r to be the integer multiple of lambda, and for destructive interference, we have the path difference delta plus delta r equal to m plus half lambda. So, with that note today I would like to conclude. So, what we have learned today is that. In the previous class, we understand that when two waves are interacting through this double slit experiment then there is a division of the wavefront, and because of that interference is happening. There are other conditions also to have the interference and the conditions are that these two beams should have coherence in nature. However one can have this interference by using this division of amplitude method. In the division of amplitude what happens if the ray is falling over a surface a part of the ray can be transmitted and it can again reflect to the second boundary and then come back. Also, another ray that is transferring from the surface of this interface is there and these two are going to interfere. This interference is happening because of the division of amplitude. Here the amplitude is divided and we will find that because of that there is interference that is happening and also the condition that we need to take care of here when there is a reflection. Because of that, a pi phase difference occurs every time and this occurs when the refractive index n2 is higher than n1. So n2 is higher than n1. So that condition gives us an additional path difference lambda by 2. So during the calculation, we need to take account of these facts. So with that note, I would like to conclude here. So, thank you very much for your attention, and see you in the next class, where we are going to learn more about the interference due to division of amplitude. Thank you very much. See you in the next class.

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