WAVE OPTICS

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Lecture - 18: Young’s double slit experiment (Cont.)

Hello, student in the wave optics course. So today we will have lecture number 18 and we will continue with the Young's double slit experiment that we started in the last class. So let me first describe once again the concept of the double slit experiment. So we have lecture number 18. So in Young's double slit experiment, we saw that if we have two point sources S1 and S2 this separation is d then the light is coming from S1 and S2 we are going to interfere with some point P. So this is Z direction and this is Y direction everything say, is happening in the YZ plane. Now because of the different path that is propagating by the light s1p and s2p, there is a path difference and that path difference leads to a phase difference and we got an interference pattern and that interference pattern looks like a cos square function, like this. This is the direction of theta and we find that the intensity which is a function of theta can be represented by the formula for I dot and then cos square pi A, then sine theta divided by lambda. Now for small theta, this expression changes a lot and we get an expression in terms of other parameter cos square pi a divided by lambda and then y by d, where, y I measure, from here to here, this is my y and this S to S1 was a this is d, a,y, and lambda. All the parameters are now known to me and then we find out the fringe width that is the separation between these two fringe delta Y was lambda multiplied by D divided by A.

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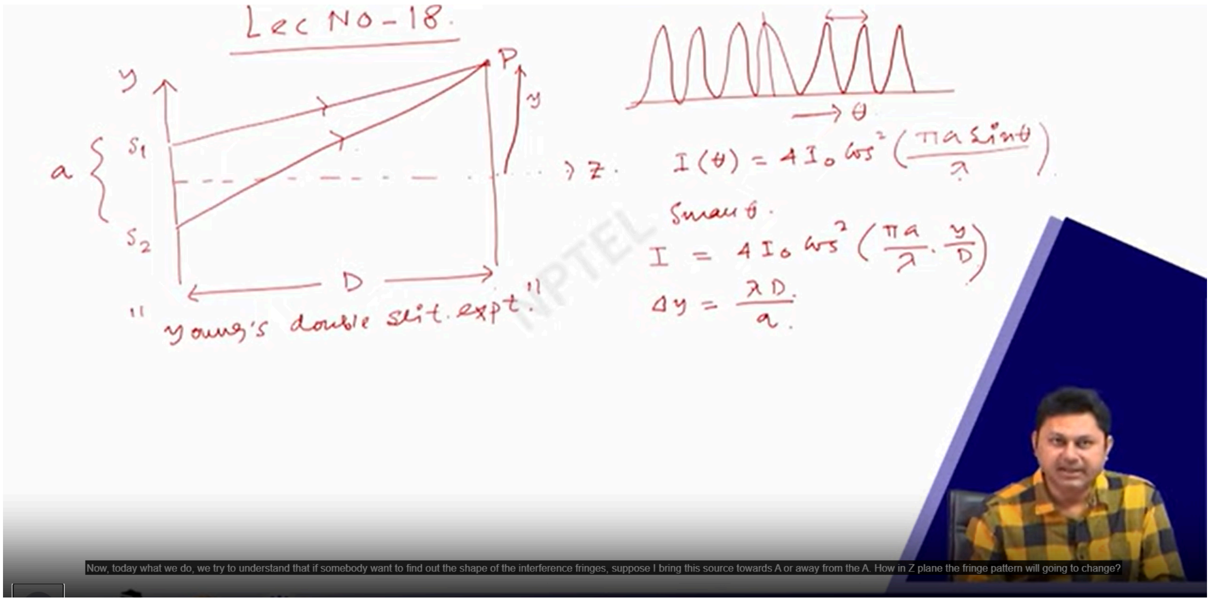

So, if the value of A lambda and D is given, then one can find out what is the fringe width for this system, this Young's double slit. So this is Young's double slit. Now, what we do, we try to understand that if somebody wants to find out the shape of the interference fringes, suppose I bring this source towards A or away from the A. How in the Z plane will the fringe pattern be going to change? That we want to study. So, let me draw it once again. So, our goal today is to find out the shape of the interference fringe, and how the shape of the interference fringe can determine what we are going to see. So let me draw, so we have two sources and this is the point P and the coordinate of the point P, say, Z Y . So this is the axis y direction and this is z, so the coordinate of the p, says, z y. So if I now shift this dotted line towards this z-axis, I mean over this z-axis right and left-hand side then what happens, is that the fringe pattern will change and the shape of the fringe will change accordingly. So we will going to find out what is the I mean equation that describes the shape of the fringe, that is what is the locus of this point P. So, this is the origin of this point I say this is my 0 point, that is x y equal to 0 and z equal to 0, and this value as usual is A. So this value is say o which is the origin. So the location of S1 in terms of coordinate is 0 a by 2 and the location of the coordinate, the coordinate of this point S2 is 0 minus a by 2. Now if I calculate simply S1p with all the known values. So, S1p square of that is simply z square plus y minus A by 2 square. Similarly S2p square is z squared plus y plus a by 2 square, these two. Now, the path difference if I calculate that comes out to be a delta equal to s to p minus S1p. So, from here I can have an equation in terms of the delta between S1p and S2p and that is delta plus S1p and the square of that is essentially the S2p square of this. So we have delta square then S1p square. I can find it from here because this value is known as plus z squared plus y minus a by 2 square and there is a 2ab term.

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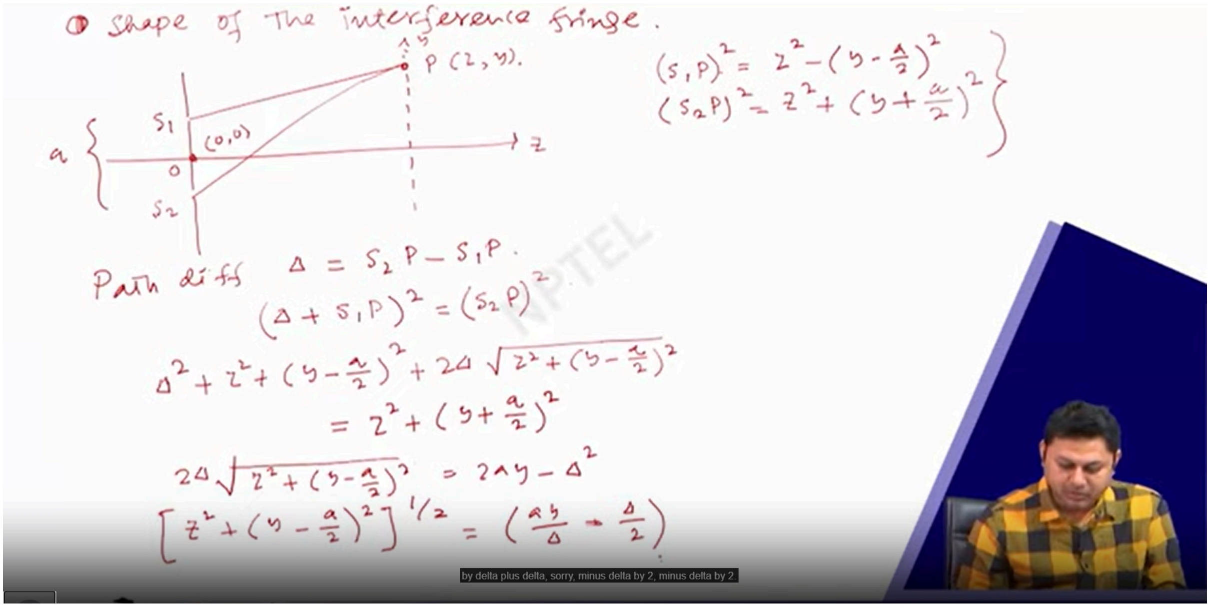

So, I should write plus 2a and b will be the root over of that thing and that is the root over of z square plus y minus a by 2 square. That thing should be equivalent to z square plus y plus a by 2 square of x. Now there are several terms that should be cancelled out. For example here z square z square will cancel out y square y square will cancel out, a by 2 square term will be there so that will be cancel out. So removing all this term we are going to get 2 delta then this term root over of z square plus y minus a by 2 whole square is equal to 2 of Ay that term will remain here and minus of delta square. We can divide this here so z square plus y minus a by 2 whole square whole to the power half is equal to ay by delta plus delta, sorry, minus delta by 2, minus delta by 2. So, if I square both sides if I square both sides, then we are going to get z square, plus y, minus a by 2 squares of which is equal to y a divided by delta, minus delta by 2 squares. So here we have z square, plus y square, minus y a, plus a square by 4, that is equal to y square, a square by delta square, minus ya 2ab term becomes y a and then plus delta square divided by 4. So y term will cancel out so, essentially we have z square and then minus y square if I take common then it should be a square by delta square minus 1, and on the other side, we have the delta terms I am keeping all the z and y term one side and it should be 1 by 4 and then delta square minus a square. So, I can rearrange these things and I can have an expression like y square divided by delta square by 4 minus z square divided by a square minus delta square whole divided by 4 is equal to 1. Just dividing everything and rearranging we can simply get this. So, this is an equation in the yz plane how the point p if I shift the point p how the point p will going to behave or what is the locus of this point p when a is fixed and delta is the corresponding part difference? So this is essentially the equation of the hyperbola, in the yz plane. So, for a fixed path difference delta, If I now want to see how this plot will be, I'll need to go to next. So, let me write down the expression once again.

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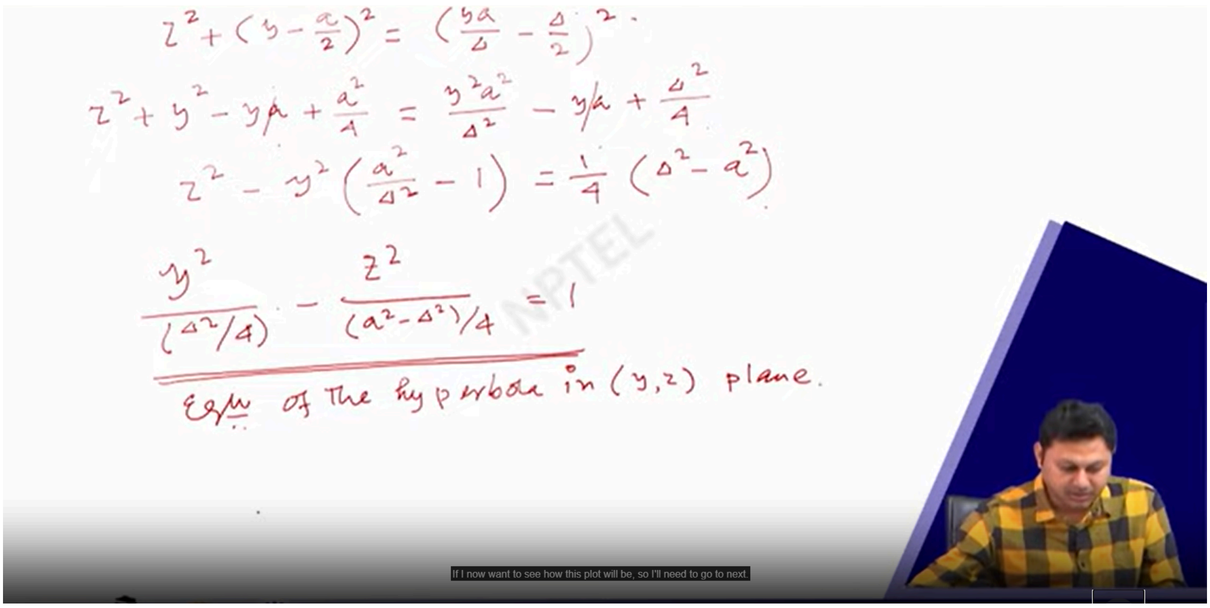

This is y square divided by delta square by 4 minus z square divided by a square minus delta square divided by 4 is 1. So, if for a fixed delta that is path difference we shift the point P in such a way the path difference is fixed. So, for a fixed delta that is the path difference one can have so this is my Z, and the points S1 and S2 are here if I draw this I can have point P, which is over this line, so this line will be something like this. This is hyperbola so, it goes like this, then it goes like this. So, we know that when an expression is given by this, then the eccentricity of the system is given by this 1 plus q square divided by P square. So, for this hyperbola, this increase is given by this. In this case, if I compare this equation to this equation, then my E comes out to be the root of the delta square by 4 plus a square, minus the delta square, divided by 4 into just replacing these things and simplifying into 1 divided by delta by 2. So that means it is 4 delta square so I can simplify it further. It is simply divided by the delta. So generally the order of delta is 10 to the power of minus 8 centimeters, which is the path difference we are talking about and the order of a is of the order of 10 to the power of minus 2 centimeters, which is the separation between these sources. So, if I put this value, we can see that E is very high. And when the eccentricity is very high it almost behaves like a straight line for large distances and we get in the figure also in this way so that means if I move over this, you know my screen, this is in general, there is this screen is present here and if p is over the screen and if I get a maximum here, If I move this screen, the points that are here, this straight line over which all the P points, then this maximum will be there only and that maximum corresponds to the same path difference that is generating. So, we are going to get an expression of hyperbola for the shape of these fringes. So that is the way one can calculate by simply doing whatever the geometry they have. Okay, after that we quickly discuss two more systems. A couple of systems where one can also find out the interference pattern and this interference pattern are generated due to the virtual sources.

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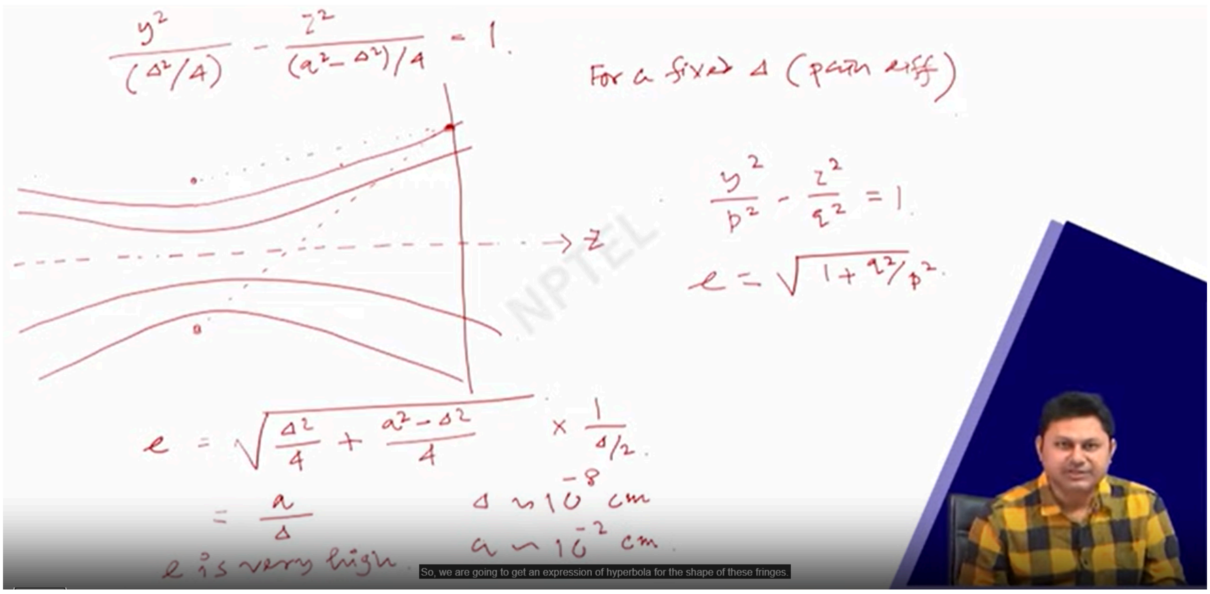

Here in two-slit experiments, we have two sources which are real sources but it is possible that even without having any real source due to the virtual source we can generate the interference pattern. Two cases we are going to discuss quickly so first, let me write down. So this is interference with virtual sources. Interference with virtual source, so the example one is called the Lloyd's mirror. So, what is the Lloyd mirror and what is the setup? So you need to understand this setup and then the rest of the things you understand quickly. So the setup is I have a source here and a finite mirror somewhere here, so one light can directly fall here over this mirror and it reflects. So this is supposed to be my screen where I want to find out. Okay, my drawing is bad so I need to change it, and if I extend this here we can have my virtual source which is s prime. Now from here to here, the length is a, so it reaches here and one ray one can go directly here. So this is ray one and one ray is going from here and bounce back and another ray from this side is also going to here at this point and it is also coming out from this virtual point. So that is the setup, so this is ray 1 that is coming here and this is ray 2 coming here. One is going directly and another is going like this. So you can see that there is a region where the direct ray and the ray that is coming from this virtual source will interfere. So if this length this is it is exactly like a double slit experiment if I understand properly then this is one source S1 and this is S2. Instead of S1, I write S which is a real source and this is a virtual source. The interference is happening because of the interference due to the light that is coming from the real source and it is reflected light that is coming from the virtual source. So the interference pattern should form somewhere here like this. So, this is the region where one can expect the interference pattern, the fringes having bright and dark patches. So, the fringe width, if somebody wants to calculate it, is equivalent to, you know, the double slit X kind of experiment.

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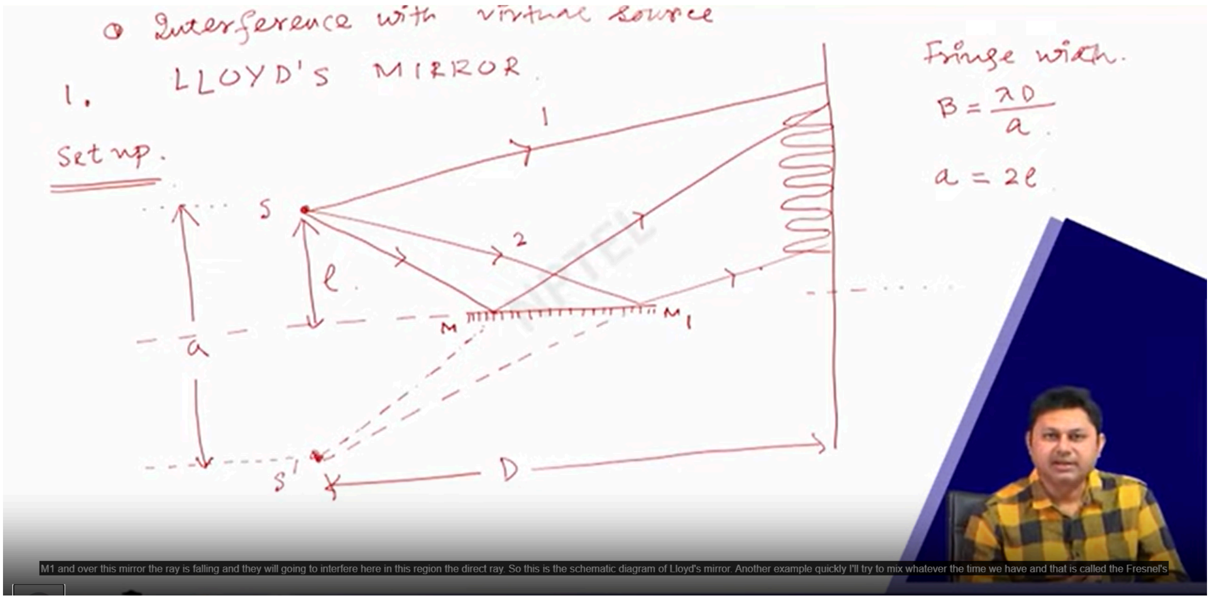

The fringe width one can calculate and the value will be like beta is equal to lambda D divided by a. So, the a is basically the separation between these virtual sources. So, if you know what is the value of S, this length you know, So, this is L then A is nothing but 2 of L the perpendicular distance from the mirror 2. So, this is the mirror, say M M1, and over this mirror, the ray is falling and they are going to interfere here in this region with the direct ray. So this is the schematic diagram of Lloyd's mirror. Another example is quickly I'll try to mix whatever time we have and that is called the Fresnel's Biprism. In Fresnel's biprism, we have a plane here where we have a source sitting here. This is the real source. But we have a prism with a very small angle. This is biprism. Now what happens is the light from the S will come like this. It will go here and then fill the part of the light but it goes like this. We have another light that will reflect like this and suppose we have a screen here so one light also will come here and then come this direction, another light will also go there move here, and diffract. So when I extend these two, they will form a virtual source here at this point. Similarly, at this point, this is S1 which is a virtual source, and S2 which is a virtual source and because of the light coming from s1 and s2, there will be interference patterns one can observe. So interference patterns will be observed in this region in this way. This is the interference pattern one can observe here and this will be along y direction if I go to maxima so this is mth order maxima. Well, if I have this angle alpha then we know and this is the minimum deviation delta m so we can have a relation that delta m will be alpha into the refractive index of the prism minus 1, that we know from the geometry and if this is a by 2 and this separation from the prism to this virtual source is d and this separation if it is d then this delta m for virtual source I mean, separation can be approximated as delta m is nearly equal to a by 2 divided by small d for a small angle for small delta m. Now from here, I can find out a is equal to 2 of d delta m or 2 of d alpha n minus 1 because delta m is n minus 1. So this is my value, which is a separation between these two sources s1 and s2. Once you know this, then the maximum point can be simply calculated as m lambda and d here will be simply replaced by d plus small d, the same equation that we are using, that we have used for the double slit problem, divided by a, which is 2d alpha n minus 1. So we have the formula for mth the length for mth order maxima and that is defined by this. So using this biprism one can generate this kind of fringe and from the fringe by calculating the fringe width or the location of the fringe, one can also calculate the lambda once the d, small d, d value, and angle of prism are known. The refractive index is also unknown. So unknown lambda can be figured out. So this is such an experiment that is done in the laboratory, the optics laboratory. Where using this technique basically the fringe width is calculated and also from the fringe width one can calculate the unknown wavelength as well this is an interesting kind of experiment that you can you may encounter in your course and the structure of this experiment is schematically shown here, which is nothing but like a double slit experiment. But here instead of having one source, we have two sources but they are situated in a virtual position S1 and S2. So, I will not discuss much because we do not have much time today. So, today what we did is we try to understand that if the light is interacting in a double slit experiment, then the fringes are formed, and once the fringes are formed how the shape of the fringe will want to change. So we find that it is like a hyperbola, the pattern will be like a hyperbola, also we understand that if we put some proper arrangement then there is a possibility that I can split this wavefront and these two split wave fronts can interfere with each other and can form a fringe, two examples are given. One is Lloyd's mirror experiment, where using a mirror, you figure out you put a source and then this source will give you a reflection, and as if the reflected uh the reflected rays coming from this virtual source that is the virtual image that is formed by the mirror. And we're going to get an interference pattern. Similarly, we can split the wavefront here by using a biprism. The figure is there already in front of you and where the light is split through this prism and two virtual sources are generated from these two virtual sources the light is coming and they are interfering with each other and forming the interference pattern. And from this interference pattern one can calculate many things. One of the important things is web length. So, that is for today. So, we will not discuss this phenomenon. In the next class, we are going to discuss how instead of splitting the wavefront I just split the amplitude. Then these two split amplitudes can also interfere with each other and can form the interference pattern. Several nice examples are there like Newton's ring and the Michelson interferometer etc. So that we will discuss it in future classes. So thank you for your attention and see you in the next class for more. Thank you.

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