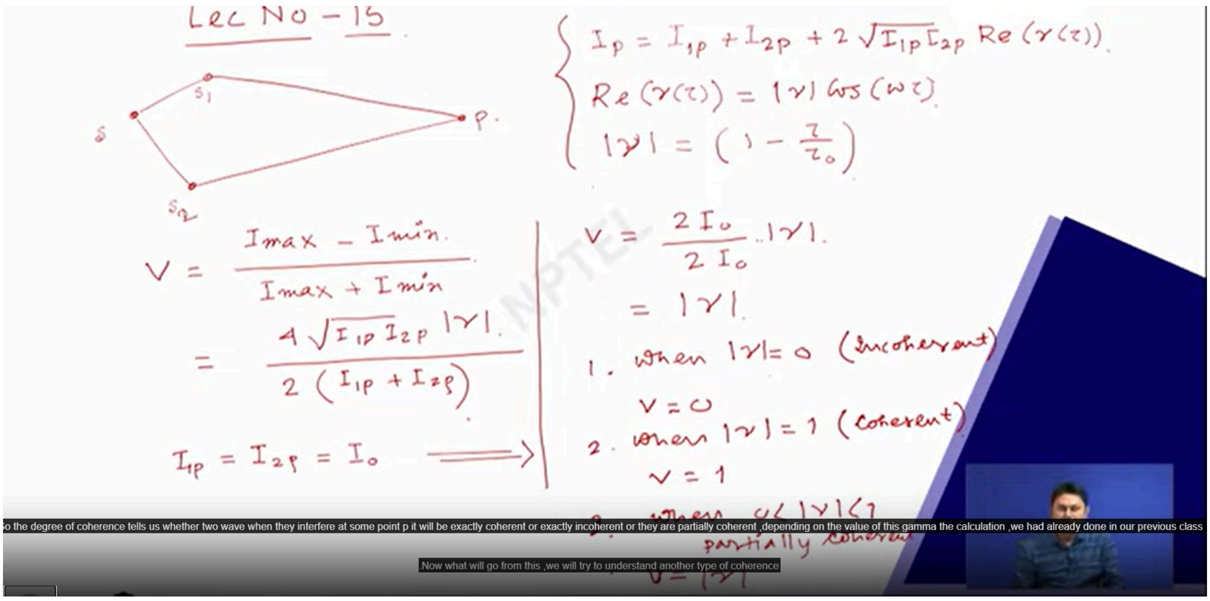
WAVE OPTICS
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur
Lecture - 15: Concept of Coherence (Cont.)

Hello, student, welcome to the Wave Optics course. So, today we will have our lecture number 15 and we will continue the concept of coherence. For the last couple of weeks, the last couple of classes we have been doing this concept, trying to understand this concept. So let us continue with that. So we have today's lecture number 15 and we're going to study the concept of coherence again. So let me remind you what we have done. So, far if we have one source point s and the waves are emitting from this source point, through another two source points S1 and S2 say, this is S1 and this is S2 and it is reaching some point p . Then the two waves that is coming from the source S1 and S2 will go to superpose at this point p and after doing all the calculation we find that; if we try to write down the intensity at the point P, I can write it the intensity in this way. The superposition of these two waves will produce the intensity at point P in this way and then we have an interesting term called the real part of gamma tau. Where this real part of gamma tau determines the degree of coherence, the light that is coming here, and how they are going to interfere themselves at the point is basically determined by this. So what was the real part of gamma tau? We calculated that in the last class.

(Refer slide time: 8:52)

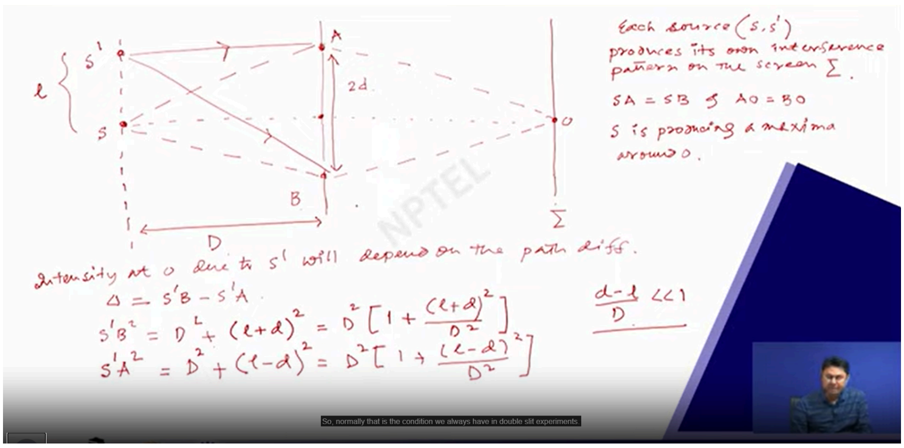


I could write it as some amplitude and since it is a real part I write cos of omega tau that was the value and what was in the amplitude, the amplitude of this gamma contains term 1 minus tau by tau naught but tau naught is the coherence time and this quantity is basically determined by the degree of coherence. We remember that if this is the general form then we can find out something called visibility. We define that visibility as I max by definition, minus I mean divided by I max, plus I min. So the degree of coherence is something to relate with this visibility and we checked the four conditions in the last class, what happens if it is completely coherence, if it is completely incoherence and thirdly in between these two either it is in between complete and this means the gamma value is neither 0 and nor 1 in between that value. So this visibility, by the way, if I write in this form. Then from this expression, I can write it as you know that, this maximum value gives me maximum minus minimum. If I calculate then there is a cost term, so based on this I can write something like; I max minus I mean that means, Ip max minus Ip min, and the maximum and minimum value is determined by this cos omega tau term, if the cos omega tau is plus 1 then, that should be the maximum and minus 1 then, that should be the minimum, if I put this together then what I get, it is 4, in the numerator we have 4 roots over of Ip, 2p and then, we have this mod of gamma term divided by this plus this, the interference term will cancel out and we are going to get 2 of this, that is a general form of v in terms of gamma. Now what happens if Ip,1p for the same source say, this is i naught, if that is the case I can write v is equal to this, one two will cancel out. So it should be two of I naught divided by these two that are not there. So it is two of I naught then gamma mod so, the visibility is essentially determined by the gamma naught when two sources are the same. Now three conditions I can put together so, one when gamma is equal to mode of gamma is equal to zero which means it is completely incoherent, then what we have the visibility is zero incoherent, and the second is when mod of gamma is equal to 1, that is the degree of coherency I'm writing here in terms of gamma that is completely coherent. Then .v turns out to be 1 the visibility now has a maximum value and finally when gamma is in between mod of gamma is neither 0 nor 1 but it is in between then it is called partially coherent. In that case, the v will be equal to the value of the gamma that it takes. So the degree of coherence tells us whether two waves when they interfere at some point p will be exactly coherent or exactly incoherent or they are partially coherent, depending on the value of this gamma calculation, we had already done in our previous class.

Now what will go from this, we will try to understand another type of coherence and that is called spatial coherence. So today we will discuss the concept of spatial coherence. So, what is spatial coherence? So, basically, it is the correlation in phase between two spatially distinct points of the radiation field. So if for example, I have an absolute point source S it is an absolute point source and from this source, the radiation is emitted and it is falling on the screen having two points A and B. So points A and B here whatever we have drawn are spatially coherent. If s is a point source A and B are spatially coherent, if S is a true point source, what is the meaning of that? That means there is no spatial distribution here over this source and if there is no spatial distribution over this source then the correlation of the phase between two spatially distinct points, here there is no two distinct points. It is only one point so since there is only one point automatically these two points A and B have a proper phase relationship. So that's why A and B are called spatially coherent. However, the situation is slightly different. If we have a distribution suppose, the source has a distribution like this, "distribution" means this is an extended source. Suppose, this is an extended source, what is the meaning of an extended source? I can consider many point sources over that, here we have only one point source but an extended source means it will be the collection of many point sources and they are mutually incoherent sources. That means these sources are emitting light but this light doesn't have any phase relationship among them. So extended source and it can be this extended source in general can be treated as a collection of mutually incoherent point sources. So, that means the wave that is emitting from this point and the wave that is emitting from the other point, don't have any phase relationship between them. So this lateral dimension of the source over which the radiation you know remains correlated in a space is essentially the measurement of the spatial coherence. That means among this extended source there is supposed to be another point here and these two points maintain that this is the highest distance, the lateral distance among which the two points on the extended source maintain their phase relationship. So these two are considered to be the spatial coherence point. So, how you measured that, how you quantified that, that is interesting, and what we are going to do next.

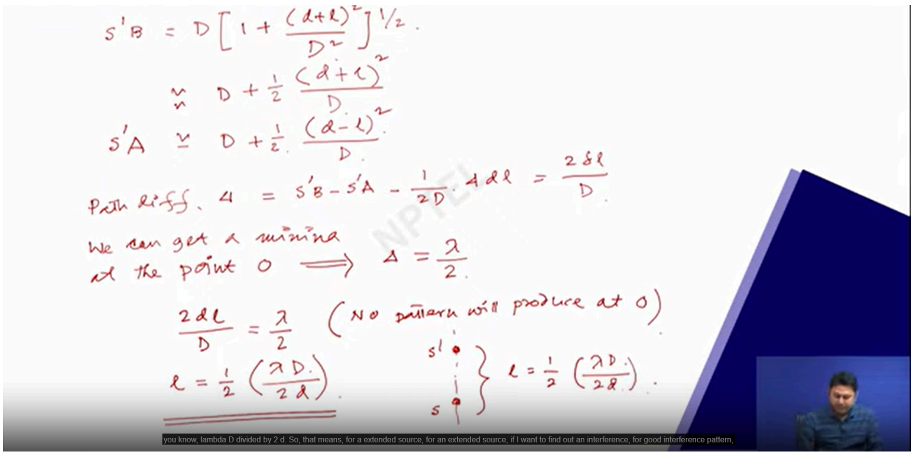
So, suppose we have an extended source and this is the extended source dotted line and I take two points, one is say S, this is the point S and another point is S'. Over the lateral distribution, these S and S primes are located over this extended source. Now, from S, the radiation is coming and we have, say, two points A and B. And it is going like this. I put a dashed line. So here we have A, then we have B, like some sort of double slit kind of experiment, we are doing here and then over A point P, it is reaching or over A point O is reaching this is O, over the screen say, I'll define this like sigma. So what is happening here, S is a point and light is emitting from here hits points A and B . Suppose there is a pinhole and then the light goes to point O superimpose and they're superimposing there. So this is, we assume, this is an extended source and this length, say, D and this is the axis and this length, A to B, we consider this to be 2D. Okay, a similar thing can happen since this is the extended source not only from the s but from s prime also the light can emit and if it does then, it goes like this. Let us draw this by this solid line. So, here are the two rays that are coming from the point s prime and do the same thing from here it goes to over o and they superimpose. So now we need to find out the condition for which s and s prime do not have the minimum length of s and s prime, where they don't have any phase relationship. So suppose each source point s and s1 is producing, you know, its own interference pattern at point o. So let me write down here each source in this case it is s and s prime produces its own interference pattern on the screen this sigma. So now SA, we know that SA is equal to SB and AO is equal to BO, so that means S is producing A maxima around this point O. So the point source S is producing a maxima around O. Now if S is producing some maxima, then we need to check what happened for the other wave. That is what happened for the other source S1. Will the S1 produce a maxima or minima? However, it depends on the path difference between S1B and S1A. So, the intensity at the O depends on the intensity at O due to S prime will depend on the path difference and here the path difference means if I write delta, that path difference is s prime B minus, s prime A. Now we can put some geometry here. So I can have s prime B square using the Pythagoras theorem, you can readily find s prime B whole square is D square plus l, say, let me write this as l, so, this is okay.

(Refer slide time: 25:42)



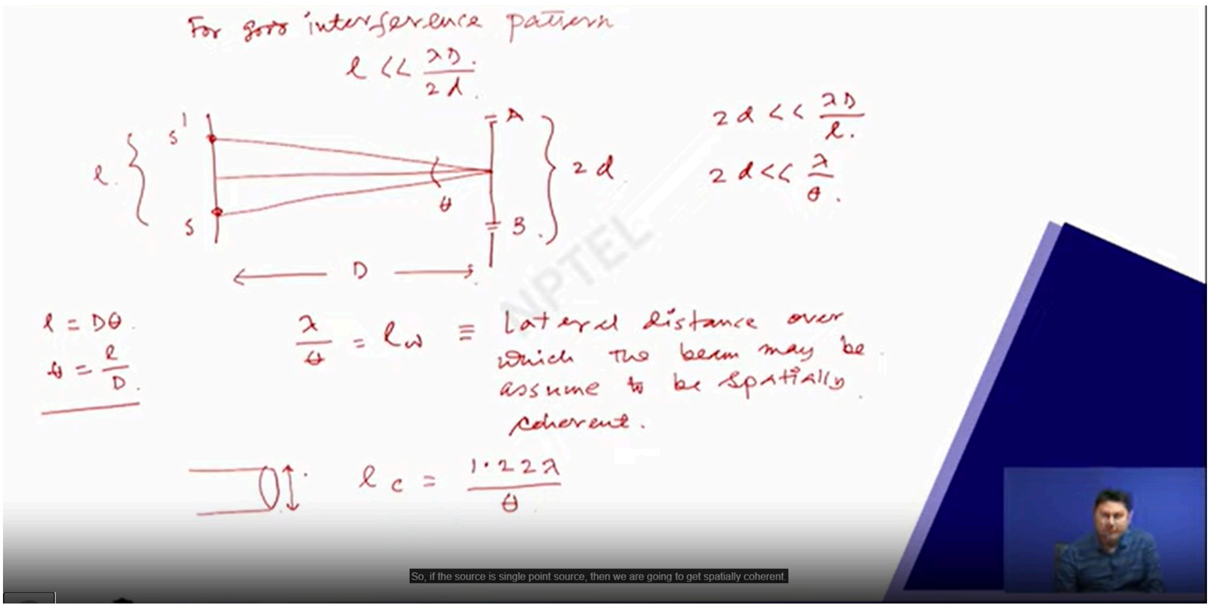
So I need to define this as well. So this is a length separation between these two, I need to put it otherwise, I will not be able to calculate that. So now if I put then it should be D square plus, l plus d, whole square because d is from this to this point and I am calculating this. So this square plus, l plus the length that is here from this so this is d so this will be s prime b square. So I can write this as d square 1 plus, l plus d divided by D square of these. Similarly, a square of that is D square plus, l minus, d square of which I can write D square, then 1 plus, l minus d, a whole square divided by D square. Now this quantity d minus l or l minus d whatever, divided by D, this quantity is very very less than 1. Because of this source, they are very close to each other and these points are also considered to be very close compared to the length between this point and this screen. So, normally that is the condition we always have in double-slit experiments. So, this condition always holds. If that is the case, then we can simplify this quantity, s prime, B and s prime, B will be D into, let us first write in this way, then we put our approximation plus d plus l divided by D square of that whole to the power half. This is essentially D plus half d plus l whole divided by D. Similarly, I can write S1 S prime A is nearly equal to D plus half half d minus l divided by D. So the part difference delta which is equal to s prime B minus, s prime A is essential. This minus this if I do. So, there is a square I missed here. This will be a square. It is essentially 1 divided by 2D and then 4 of d of l or that is 2dl divided by D. Now, we are going to get a So, we can get a minimum at the point O when the per difference delta will be equal to lambda by 2. Okay, the lowest value I take I can also multiply by 2 that is the condition for which we can get the minimum but let us take the lowest one which is lambda by 2.

(Refer slide time: 32:25)



So, in that way, S will produce a maxima there and S prime will produce a minima there. So, no fringes will be there because of that condition that is the condition where. So, no interference pattern will be there because of this. Now, if that is the case that S prime is producing a minimum there, then what we are getting is 2 d of l divided by D which is equal to lambda by 2. So, that is the condition that no interference and no pattern will be produced at O from here we can find out the separation of the source which is half of lambda D whole divided by 2d so we can find a value of l that is the separation between these two source point for which we will going to get no fringes at the point so l is here half s and s prime and l is that value half of you know, lambda D divided by 2 d. So, that means, for a extended source, for an extended source, if I want to find out an interference, for good interference pattern, we require that extended source, the separation between two points for extended source should not be very close to the value lambda D divided by 2L, rather much much smaller than that, because you know, the last case we find that, this is a half term. So for each source, we have another source in the opposite direction, such that this condition is satisfied. So, in general, l should be very very less than lambda D by 2 d to have a good interference.

(Refer slide time: 35:13)



So, if I draw that S and S prime they should. So, here we have the slits A and B, and geometry wise this is my D and this separation is l, see if I calculate what the angular separation one can have from these two points. So this angle, if say, is theta and this is our 2d that is the geometry we have. So 2d you already find that it has to be very very less than lambda D divided by l, same equation, I am writing here so then 2d has to be very very less than If I write in terms of angular separation it should be lambda divided by this theta, "l" because here we can see that l is equal to d into theta, for small theta and theta is equal to l divided by D. So that value I put here, l divided b to theta, so that is the angular separation. So lambda divided by theta, that is the length I write lw we call as lateral distance over which the beam may be assumed to be spatially coherent. So, that is the length for which we can consider that the beam can be considered to be a spatially coherent beam. So you know for a circular source, however, a circular source that is so, lc is around 1.22 lambda divided by theta. This we will discuss later, we will discuss the array function and all these things. So that is the separation one can have, one can consider if we want to find out what happened for a spatially coherent case.

So, if the source is a single-point source, then we are going to get spatially coherent. If it is not, then it is not a spatially coherent thing. So quickly if I draw what happens when a spatially coherent beam and a spatially incoherent beam is propagating? So this is the way one can expect the propagation of spatially and so we have a point source here and the light will propagate like this. In all space points, we have a well-maintained phase relationship, so this is the way the light will propagate when we have a spatially coherent light beam. However, for spatially incoherent light beams this is Z and this is said over X, so this is X and this is Z . The light will propagate, and then suddenly there will be a phase distortion and it will be like this there will be no phase relationship if it is an incoherent kind of wave front moving from an incoherent kind of source. So this is the way the coherent and incoherent beam will propagate in space when we have in terms of spatially coherent and incoherent. So today we're going to understand, so we don't have much time today to discuss more about this. So we're going to discuss in the last couple of classes which is important, how the light, when they interfere with each other then how coherency, the property called coherence is important to find out whether they will produce a very nicely looking interference pattern or not. So, that is a major criterion that if two lights interfere with each other they will produce a very nice looking fringe and this will happen when they have the coherence property in them. So the degree of coherence we calculate for which we can find that when they are absolutely coherent, they will produce a very nice fringe in terms of visibility. The term is one if they are not coherent at all, they will not produce a very nice-looking fringe. In fact, we will not get any kind of fringe pattern out of that. And that's why the visibility is almost zero there. But we can also get in between the condition where we have partial visibility and the degree of coherence determines what is the value. Also, we learned the meaning of spatial coherence. If we have a point source, and it is emitting light so that light when it is, propagates the wavefront they have a well-maintained phase relationship among them but if we have a source that is emitting light but is an extended source the two points that are emitting the light there if they are not especially coherent then the wavefront there, that is moving is not precisely having among the phase found in this propagation they don't have a precise phase relationship among them and we call this incoherent coherence. So incoherent radiation, so in two slit experiments this is very important that the light should be especially coherent to get a fringe. In the next class, we will discuss more in detail that, when two lights will going to interfere then, what should be the pattern and how for you know two slit cases the patterns are formed, etc, with that note I would like to conclude in today's class thank you very much, for your attention and see you in the next class.