WAVE OPTICS

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Lecture - 14: Concept of Coherence (Cont.)

Welcome, student to the wave optics course. So in the last class, we discussed the coherence length and what is the meaning of coherence, etc. Now in today's class which is lecture number 14, we continue with the concept of coherence. So the calculation that we started in the last class that we are going to continue. Ok, so we have today's lecture number 14, and the concept was, we have a source here and then with two different paths this is S and we had S1 and S2. So lights are reaching some arbitrary point p, light is first goes here by S1 and S2. So want to find out what is the intensity at point p and in order to calculate that, so what we find, let me write down that the intensity at point p is intensity, so that the wave that is coming on from the S1 source, S1 intensity for the source S2 and their interference term half of epsilon naught c. Then we had two real parts of the E1 and E2 star, that was the interference term. Then I know what my E1 and E2 are. E1 is related to the source field like this t minus T1 and E2 is related to beta 1 E t minus T2. They are related to the source S, the source will be S with this form.

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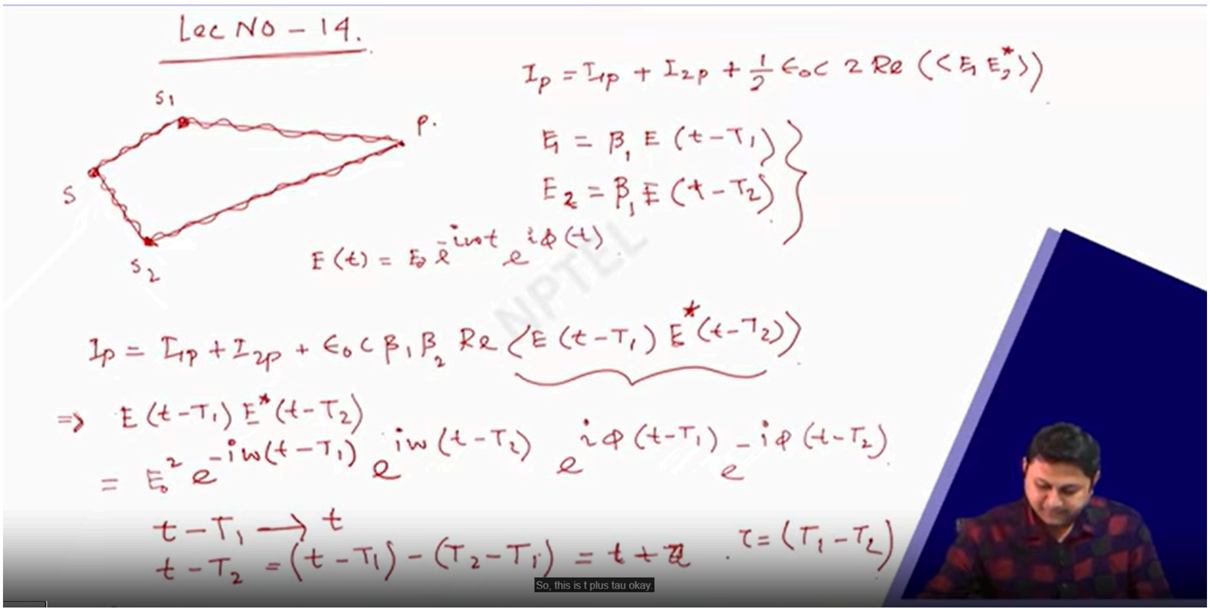

Then we introduce these into E1 and E2 with the form E and also we have the explicit form of E as E equal to E t is equal to E0 e to the power of minus i omega t e to the power of i phi t that we had. Now imposing these we get, let me write here, Ip is equal to I1p plus I2p plus epsilon naught c. Then beta 1 beta 2 real part of E t minus T1 and E t minus T2 average of that, we will get a star here. Okay, then we try to find out what this term is. So E t minus T1 and E star t minus T2. That quantity one can have as E naught square e to the power of minus of i omega t minus T1, e to the power of i omega t minus T2, e to the power of i phi t minus T1, and e to the power of minus i phi t minus T2. This is the way we can write this quantity E t minus T1 star, E star t minus T2. Now, if I write t minus T1 rescale to some t then t minus T2 I can write as this is equal to t minus T1, minus T2 minus T1. I just rescale that t minus T1 is t. If I rescale t minus T1 as t, t minus T2 in terms of t minus T1, I write this. So, this is essentially t plus tau, delta t plus tau, or delta t. Let me write it as tau, where t minus T plus tau, where tau is equal to T1 minus T2. So, erase this completely. So, this is t plus tau okay. So, with this new notation, I can have a new rescaling factor. I can have IP, the irradiance at point P is equal to I1P plus I2p, plus epsilon naught c, beta 1, beta 2 the scale factor, due to amplitude reduction and then real part of E t nu rescaling and then E star t plus tau. This is a very important factor in understanding coherence, this is called the correlation function. The correlation function that correlates the electric field at a point at some time t, to multiply it by the complex conjugate of the same electric field at some later time t plus tau. This is given by a factor gamma. So, gamma tau is a function of tau is the average time average of this quantity, this physical quantity E t and E star t plus tau so that factor is called the correlation factor or correlation function. I can normalize this correlation function as gamma. So gamma tau, I can define gamma as half of C epsilon naught, beta 1, beta 2 whole divided by root over of I1p I2p.

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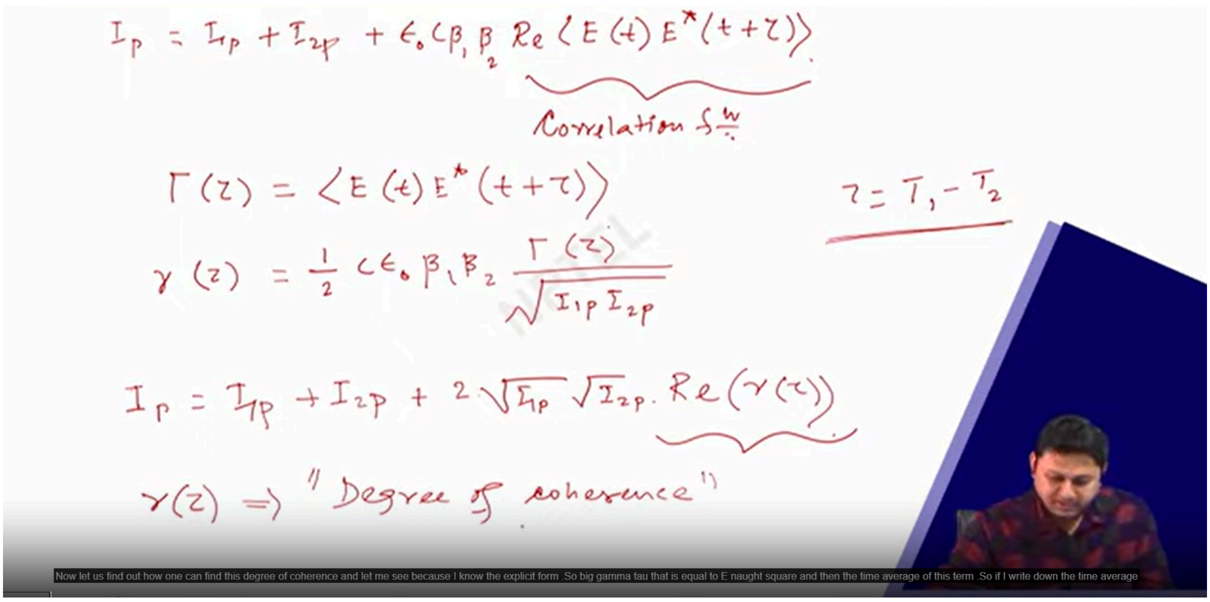

So once I write small gamma in terms of big gamma then this Ip whatever, the Ip I wrote becomes a very compact form and this is I1 p plus I2p, plus 2 of root over of I1p root over of I2p multiplied by the real part of gamma, which is a function of tau. Okay, so essentially this is the term that is the heart of the interference. So this term is the heart of the interference term and it is a function of tau. Therefore, it is a location of p as well because tau is the time difference between tau was T1 minus T2, that is the time difference between the time of flight of two waves reaching point p. So gamma is a function of tau and it depends on the location of p as well. So this gamma tau is the term essentially called degree of coherence. Now let us find out how one can find this degree of coherence and let me see because I know the explicit form. So big gamma tau that is equal to E naught square and then the time average of this term. So if I write down the time average it is e to the power of minus of i omega t into e to the power of i omega t , plus tau, and then the phase term e to the power of i phi t multiplied by e to the power of minus i phi t, plus tau. I just simply wrote down the expression that we had. Now, e to the power i omega t and e to the power i omega t here cancel out. So the only thing that we have to the power i omega tau, which does not depend, which is the time difference between the time of flight. So I can take it outside. It should not be a part of the time average. So, it should be E0 square e to the power of i omega tau. In the bracket, we have e to the power of i phi t into e to the power of minus i phi t plus tau, the time average of all these things. Now when we have the time average we know that if I have a function time average of the function ft then by definition it is 1 divided by T0 to T that function dt. This is the way we make a time average. So, here also our goal is to find out what the time average of these things is. So e to the power of i phi t to the power of minus i phi t plus tau time average is 1 divided by time a large amount large time.

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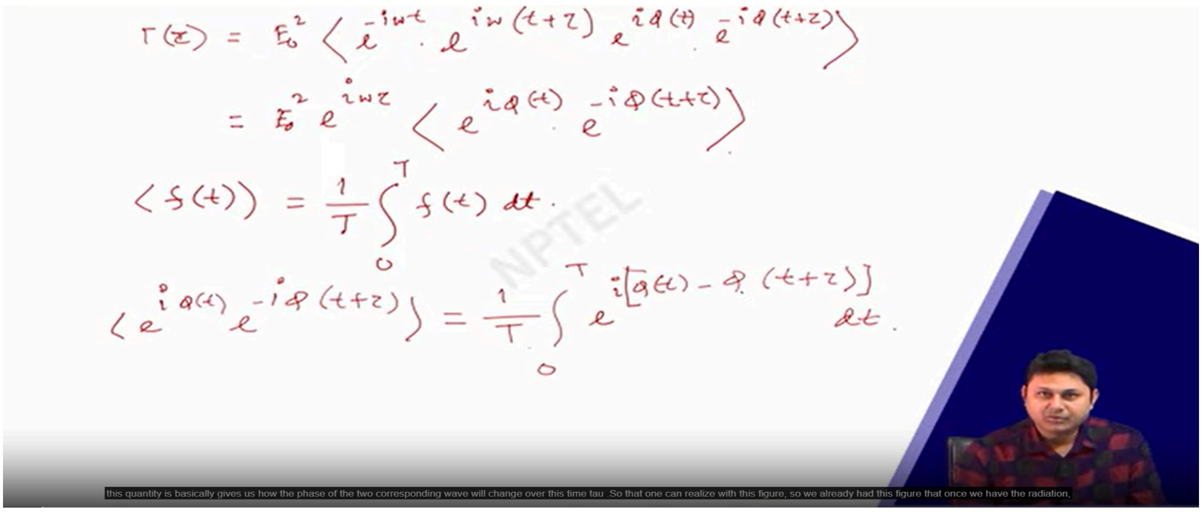

Over which we want to integrate this term so it is e to the power of i phi t minus phi, t plus tau, bracket close, i integrate over time so dt. Now this quantity basically gives us how the phase of the two corresponding waves will change over this time tau. So that one can realize with this figure, so we already had this figure that once we have the radiation, these harmonic waves are emitting but it is emitting without phase relationship with a very long period of time . After some point, it breaks and we have radiation like this; where no phase relationship is there. So let me draw it once again. So there is a sudden phase jump here and so on. So since there is a phase jump from this point if I draw this phase jump in the same plot, then it looks like it goes here and there is a jump here and then it goes here there is a sudden jump here, at this point like a step function then it goes here there is a sudden jump here, then it goes here there is a sudden jump and it goes here there is a sudden jump and this drawing, it suggests that this is for phi t over the time. This is the way the phase changes. Now, the same thing one can have for another time and that is for phi t plus tau and when we have t plus tau, we will get a similar curve but there is a jump here and then there is a jump here, then there is a jump here, then there is a jump here and there is a jump here. So we will have this jump but at different times and this different time is essentially tau. So from here to here we have tau, so over this, I plot time and this is happening for a time period tau 0, that is the coherence time. So at this point, it is tau 0, at this point, it is 2 tau 0, at this point it is 3 tau 0, at this point it is 4 tau 0, and so on because it is happening at the period say, tau 0 . Now when we find out the difference if you carefully note that it is Phi t minus, Phi t plus tau, that was T plus tau. That was the term we needed to calculate and if I plot that again in the same plot.

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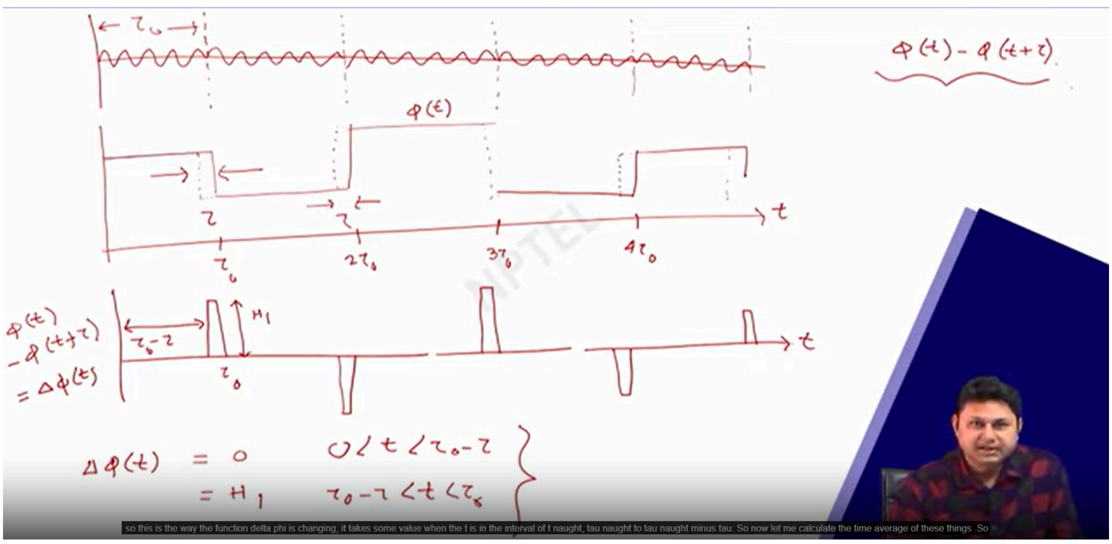

So I am plotting here phi t minus, phi t plus tau, then we get a step function like this; maybe it is on the positive side, the negative side there is random, so it can have certain jumps and we're going to get some blocks like this. So from here to here, it is tau 0 minus tau and this is tau naught, and for this function this height says, h1, and so on. Over time I have this picture. Okay, so once we have these functions, let me call this function the delta phi function of t. So, I can write down that the delta of phi is equal to 0 when t is in between t tau 0, minus t and 0, that is in this region but it has a value H1 when t takes the time this, So this is the way the function delta phi is changing, it takes some value when the t is in the interval of t naught, tau naught to tau naught minus tau. So now let me calculate the time average of these things. So small gamma tau, that was the degree of coherence, which is essentially related to big gamma tau, is proportional to that quantity, 1 by T, this integral, time average, 0 to big T, e to the power of i, delta pi t dt that is the quantity. Now note that for large T we can have many periods. So that is equivalent to n multiplied by tau naught, tau naught is the period. So, the integral that we have is gamma tau with certain constants, it will be the power of i omega tau that was already there multiplied by the integral of this. So, 1 divided by n tau naught and then integral two part 0 to tau naught minus tau, where the function phi is 0. So we will have e to the power i multiplied by 0 dt and other parts which are tau naught minus tau, to tau naught and we have e to the power of i h1 dt . Okay now this term is also there for next so we will get a this is for n equal to 1 but a similar term, because we have h1, h2, h3 etc. So like this here we have another, so this is h2, this is h3, and so on. So I need to integrate overall discontinuity where, during the place, there is a phase jump. So if I start adding all these terms we will get similar terms for n minus 1 successive intervals.

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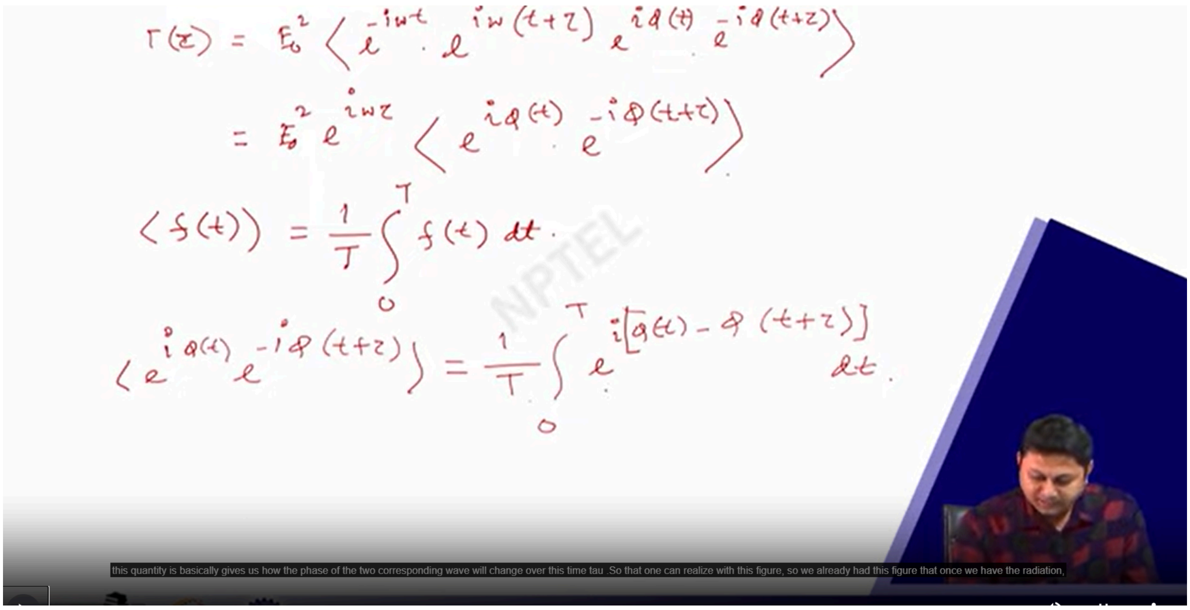

Similarly, successive terms we are having so what value then gamma we are getting from here, gamma tau is e to the power of i omega tau divided by N tau naught, and then I am getting this integral, and when I get tau 0 minus, tau plus this value. So it is if I do the integral, it will be I tau and tau minus this thing. So essentially I get plus tau, e to the power of I h1 that is my first term. Similarly, my second term will be tau naught minus tau and then plus tau e to the power of i H2 and so on. A large number, N is a large number, so I have something like this. So, my gamma, that is the degree of coherence that I wanted to find out will be related to a term like e to the power of i omega tau divided by N of tau naught. And if I accumulate all these terms tau 0 minus tau, tau 0 minus tau all this tau 0 minus tau, I accommodate there will be n number of tau 0 minus tau. So, it should be N multiplied by tau naught minus tau, that will be one term and then there will be a sum over tau. If I common there will be a sum over j equal to 1 to N e to the power of i H j. Okay now H j is a random nature as I show, because of the random nature of this sum, whatever the sum that should vanish. So I should write because of the random nature of H j, the sum should vanish for large N. If it does then what value essentially we get so, is if that is the case then from here, we can write gamma tau is essentially equal to e to the power of i omega tau divided by n of tau naught multiplied by N into tau naught, minus tau. So we're going to cancel out and essentially we have something like 1 minus tau divided by tau naught into e to the power of i omega tau. Now if you go back and check then we need to essentially find out the real value of gamma tau. That was the term that was sitting there in the interference, the intensity calculation. The interference term is related to the real value of that. So, when we put the real value, then this becomes 1 minus tau divided by tau naught into cos of omega tau. So this is basically the magnitude of the degree of coherence.

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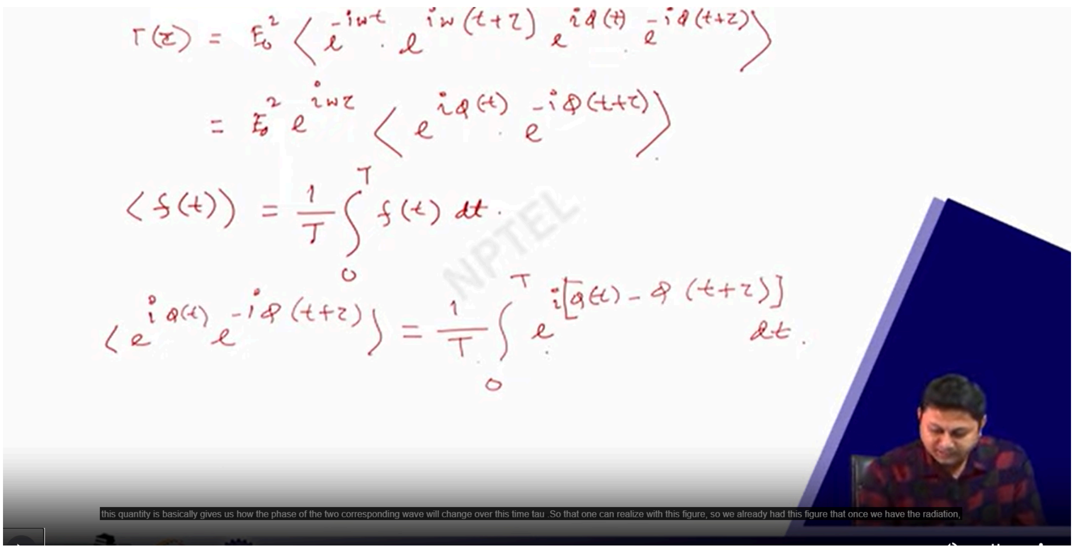

So if I just consider what is the mod value because this is the magnitude and from this equation gamma tau, if I only take the magnitude, the gamma tau is essentially the term 1 minus, tau minus tau naught. Whereas, gamma tau is the mode of gamma e to the power of i omega tau. The real part of this gamma tau is mod of gamma and then cos of omega t. So because of this term actually what we get is, sometimes we get, this is basically the interference term and I can get. So let me go back and check what was the interference term related to this. So let me write down ip was I1p plus, I2p plus, 2 root over of I1p, root over of I2p, and then the important term is the real part of this. So the real part of gamma tau that was the expression we start with is the main expression and this is the interference term. Now we are in a position to find out what this quantity is, the real part of gamma tau is based on whether this value can give 0 based on this value it can give 1 or it can give minus 1. So, I can also define something called visibility. Visibility is when this is, let me write down first. This is I max minus I min divided by I max plus I min. That is the visibility. Now, let us concentrate on this term and try to find out what is happening. So, let me go one by one back and then we are going to finish after that. So, we write in complete incoherence. So, what is the meaning of complete incoherence? When this tau tends to tau naught, that means the value of mod of gamma tends to 0. If that is the case, then we simply have the interference of two waves as intensity due to the single summation of the intensity of these two single sources. I can write it as 2 of I naught if these two sources are equal for equal source.e For equal source, this is 2P. And what is the visibility? If I find out the visibility, it should be 2I naught minus 2I naught divided by 4I naught. So, there will be 0. So, visibility will be 0.

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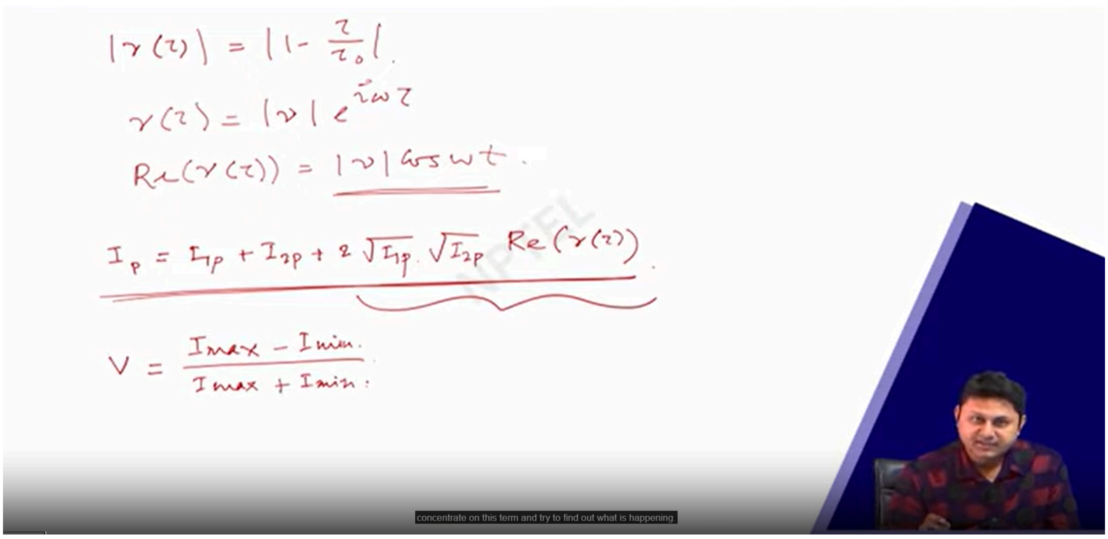

So, when we have complete incoherence,then there will be no visibility at all because these two waves are simply canceling out as canceling, which means there will be no correlation between these two. However, when we have complete coherence then tau is zero and tau tends to zero, that is the condition for complete coherence. If that is the case. The mod of gamma, that is the degree of coherence is 1. And the real. The value of gamma tau is simply mod of gamma, cos of omega tau. So IP, in that case, becomes I1 p, plus I2 p, plus 2 root over of I1 p, I2 p, and cos of omega tau. Now, this cos of omega tau has a maximum and minimum. So, I can write I max as Ip. This is basically the I1p root over of I1p, plus the root over of the I2p whole square of that is the I1 . And if I1 and I2 are the same then this value is 4 I10 for the same source. Well, for I minus in that case, the IP minimum will be root over of I1P minus root over of I2P and the whole square of that. So, that is essentially 0 for the same source. So, I can have visibility in this case 4 I naught divided by 4 I naught which is 1. Very, very strong visibility one can have. And this is the most ideal case when we have two coherent sources interfering. But that is not always the case. The general case that we always get is partial coherence. That is the topic that we start with. So partial coherence is the case we always have, where tau is in between 0 and the coherence length. So that means the mode of gamma which is the degree of coherence is between 0 to 1 . Then Ip will have something like I1p plus, I2p plus, 2 root over of I1p, I2p then the real value of gamma and I don't know what is the value of the gamma definitely. So this value will remain for the same source. It is 2 I naught and then 1 plus root over of, sorry, it's not 1 plus root over of, this is just gamma. 1 plus the real part of gamma for equal beam. And if somebody wants to calculate what is the V visibility, I am not going to do this calculation.

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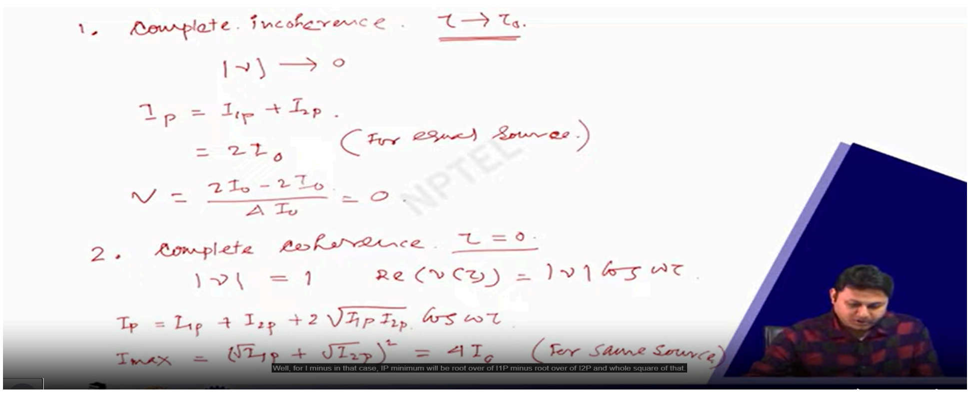

Rather, I ask the student to please check it using the formula I max minus I min all divided by I max plus I min. And that value essentially gives you mod of gamma, the degree of coherency. So that is the visibility one can find out. So today we don't have much time to explain more about coherence than different aspects of coherence. So I'm going to stop here . Hopefully, you understand I did all the detailed mathematics with the physical correspondence to make you understand that, when two lights are coming from the same source but two different paths there is a time lag and this time lag how leads to the concept of degree of coherence and we get the concept of partial coherence. And when we have absolute coherence then the visibility the way we defined is maximum, that means if two waves are interfering, we are going to get a very nice interference pattern in the later part, of course, we are going to see that. However, if there is no coherence, that means the coherence of two beams is not coherent to each other, we are not going to get any visibility in zero. So, we are not going to get any kind of pattern formation. So, that is why interfering two lights will be visible in terms of the production of the pattern, different kinds of fringes will be understood depending on the coherency, the property something called coherence, which means how the source emits continuous harmonic waves and how the source is monochromatic. If it is highly monochromatic, it is expected that it will generate a very lengthy harmonic wave and we will get sufficient coherence length and we're going to get a very strong fringe. So in the next class, we will discuss another aspect of this coherence in terms of wavefront, which is called spatial coherence. In order to get a nice fringe pattern also spatial coherence is important. So, we are going to understand what is the definition of spatial coherence and how this spatial coherence is important in generating different fringes. So with that note, I would like to conclude here, thank you very much for your attention and see you in the next class for more. Thank you.

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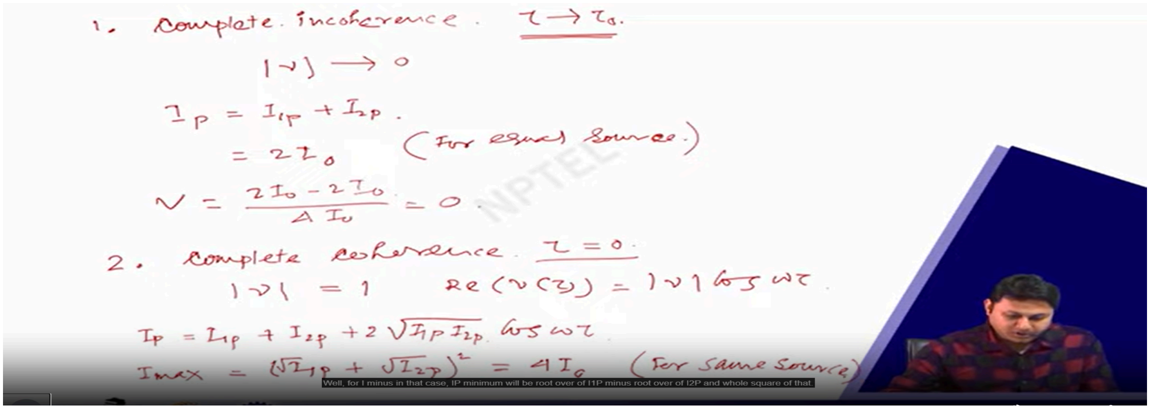
1. Complete incoherence. τ → τ0.
|ν| → 0
Ip = I1p + I2p.
= 2I0 (For equal source.)
ν = (2I0 − 2I0) / 4I0 = 0.
2. Complete coherence. τ = 0.
|ν| = 1 Re(ν(τ)) = |ν| cos ωτ.
Ip = I1p + I2p + 2√(I1p I2p) cos ωτ
Imax = (√I1p + √I2p)^2 = 4I0 (For same source)
NPTEL
Well, for I minus in that case, IP minimum will be root over of I1P minus root over of I2P and whole square of that.