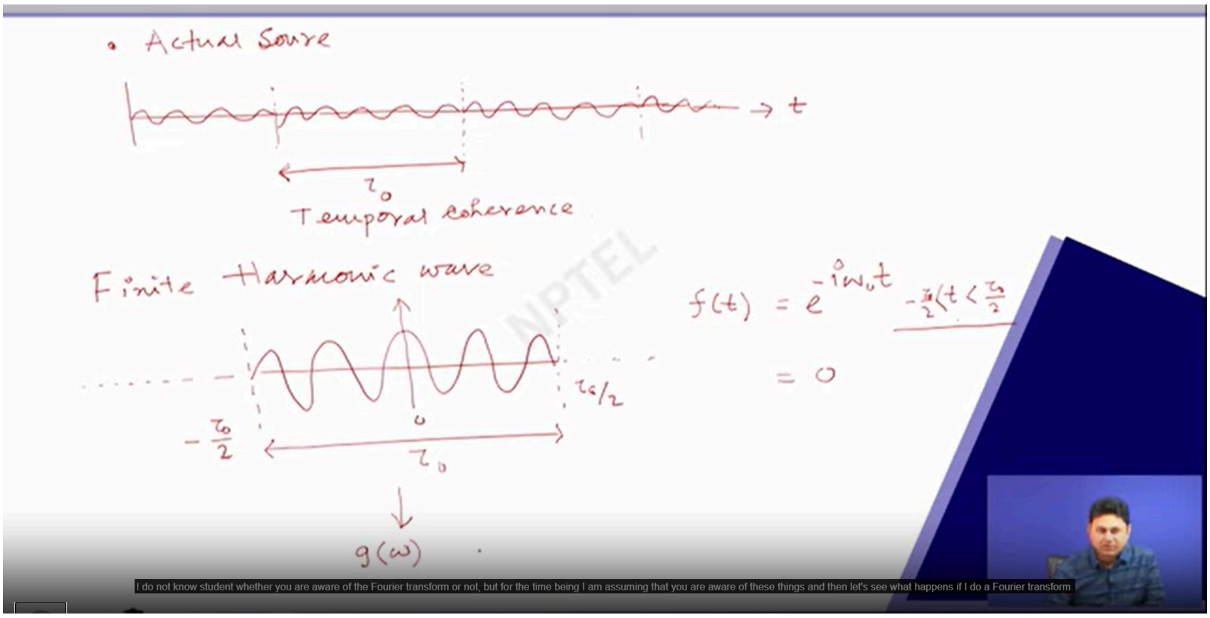
WAVE OPTICS

Prof. Samudra Roy
Department of Physics
Indian Institute of Technology Kharagpur

Lecture - 12: Concept of Coherence

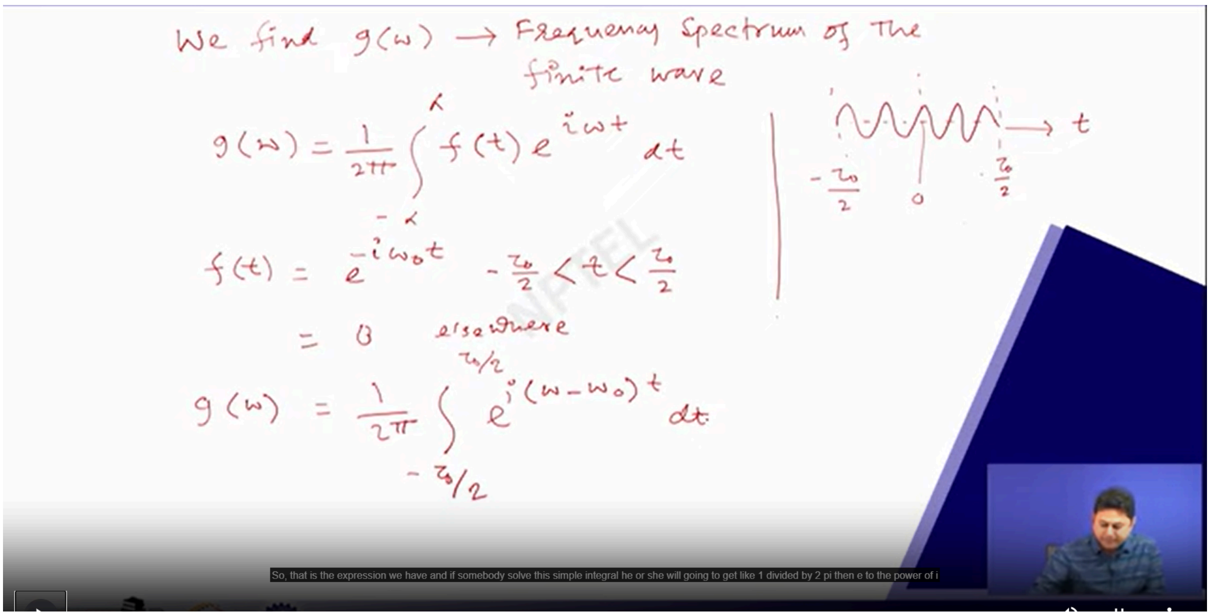
Hello, student to our wave optics course. Today we have lecture number 12 and today a very very important concept we are going to learn is the concept of coherence. Okay so let me start with the basic idea that we have. So today we have lecture number 12 and today our topic is coherence, a very very important topic conceptually, as well as in understanding wise. So when we have a pure monochromatic wave, then that pure monochromatic wave, it is emitting the light. And this is the most ideal case, that is for a pure monochromatic wave, that is a wave having a single frequency, that is the meaning of the monochromatic wave. Then what happens if it emits light? That light wave can be represented as this harmonic wave, the electric field, which is a function of time over time, it is moving like this and for a perfect monochromatic source that is where the frequency is very precise. If I draw the frequency it should be like this and this is the frequency of this particular wave very precisely then we can have an infinite wave, which is moving in a sinusoidal fashion. So for a perfect monochromatic source the displacement, whatever the displacement I draw here remains sinusoidal for t. So I should write the displacement remains sinusoidal for a time in the limit minus infinity to plus infinity. So eventually we have a continuous flow of waves without any jump or without any phase distortion in it over the time period. This is the most ideal case and we called it the pure monochromatic wave. However, in the actual source we don't have. So in actual fact what happened? We don't have a very specific frequency, rather we have a frequency distribution and what happens is that the electron goes to a higher energy state and goes back to a lower energy state and it emits some light. When it goes up and then goes down and emits some sort of light within this time period whatever the light it emits.

(Refer slide time: 14:39)



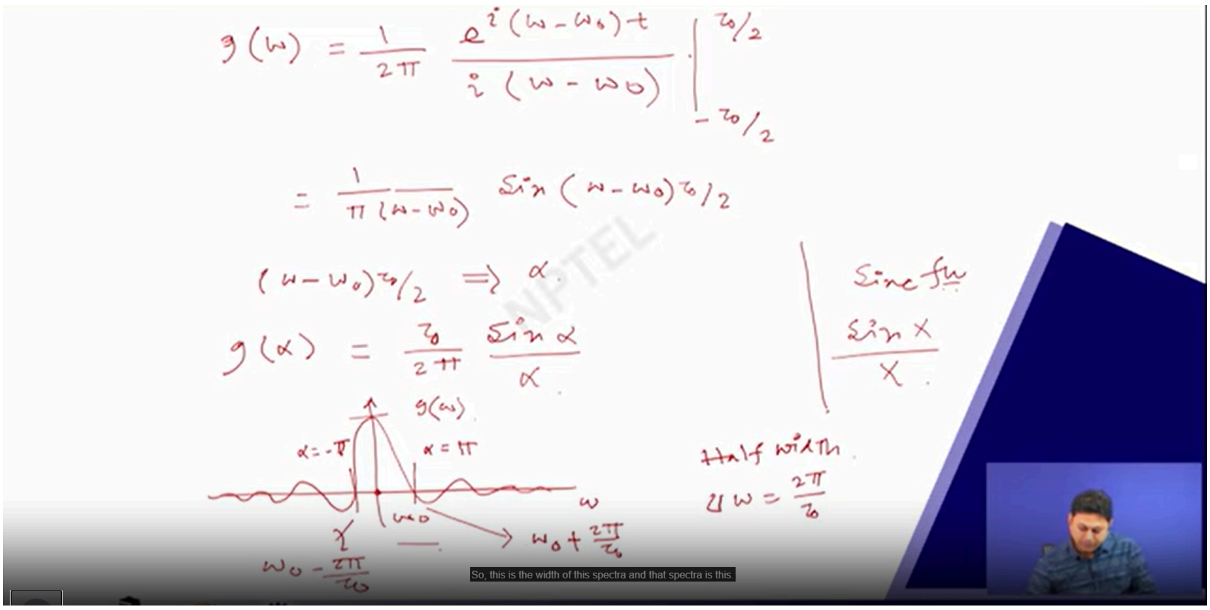
It has a sinusoidal structure in that however there are different energy states in the source and in different phases they emit light. And what happened is that the radiation that is coming out is some sort of discontinuous over this phase. So this kind of discontinuity one can expect when we have the actual source. This is for the actual source. So, if I try to understand this with whatever the figures I draw, if I write in a nice way then for actual source the disturbance that is coming. Let me draw this. So, suppose we have a radiation and then there is a sudden jump of the phase and we have a radiation then there is a sudden jump of the phase, we have a radiation and there is a sudden jump of the phase, we have a radiation it is something like that over t. So from this to this you can find that from here to here in this region there is no sudden change of the phase that is marked by this vertical dotted line here. There is a sudden jump in the face but in between this region where we have a almost perfectly harmonic wave without any phase distortion we don't have any sudden jump and this time where we have a definite phase relationship, we can write this time as tau 0, this span of time rather. So, this finite span of time, where it remains sinusoidal, is called the temporal coherence of the beam this time. This is our temporal coherence. So what is the meaning of temporal coherence in actual source? So the definite phase relationship exists only for a finite span of time in the previous case when we have the purely coherent source or purely monochromatic wave then any point if you go and then you can predict at some point t, in other point t what should be the phase. So there is a definite phase relationship throughout the time. So the coherence time here is in general in principle it is infinite. However, for actual sources what happens the electron can go from lower energy state to higher energy state and it then goes back and some radiation will happen and it goes this will be done in a random fashion. And as a result for a small span of time we have the wave that is coming from this kind of system, for a small period of time we have a definite phase relationship of this radiation. And if I draw, as we have drawn here that we have a span of time where, we have a different phase relationship of this radiation and here we can see these are the jumps one can expect of actual source because the phase relationship is not throughout the time but there is a span of time tau 0

(Refer slide time: 20:42)



and where we have a phase relationship and that time T 0 or tau 0 is called the temporal coherence. Now, these are the finite harmonic waves. If I want to calculate for this finite harmonic wave. So, let me first write it, these are not like the previous case. In actuality, I mean in pure monochromatic waves what happens is that we have a wave-like structure, we have a disturbance that is moving from plus infinity to minus infinity over time. So this is an infinite harmonic wave but here in this case we have discrete waves which have a definite phase relationship and this span of time we have finite harmonic waves. So if I draw this time span. Let me draw it. So it is something like this. S So, this is my finite time from here to here, it is tau naught and if I write this is my 0 position. So, this is tau naught divided by 2 and this is tau naught divided by 2 with a negative sign. Okay, so, this is the distribution we have which is a finite harmonic wave and if I want to write this functional form mathematically then it is a function f of t which is having the value e to the power of minus i omega naught t, that is the expression of a harmonic wave but it is true only for t less than t naught by 2, greater than minus t naught by 2. In this region this is true and rest of the cases it is 0 when rest of the case is 0 means if I go beyond that ,so we don't have anything, so only it is a finite harmonic wave with the time span this like a section of these waves that is coming from the actual sources is a section of that. Okay once we have that finite wave in the time domain it is possible that we can find out, what is the spectrum of this finite wave, what is the meaning of the spectrum. If we have a function in the time domain then in principle we can find out what is the corresponding function in the reciprocal domain or in the frequency domain that is g omega. So, g omega is something that one can find by making a simple Fourier transform of this finite wave. I do not know whether you are aware of the Fourier transform or not, but for the time being I am assuming that you are aware of these things and then let's see what happens if I do a Fourier transform. You may note that in the previous case when I mentioned that if a wave is moving infinitely without any phase distortion that is definite phase relationship is there, then the the corresponding frequency of these, whatever the wave that is coming is a very precise one omega and if somebody want to describe this kind of function, say, suppose this frequency is omega at omega naught that is the frequency of this radiation, then whatever the way we have if I describe this as e 0 e is equal to e 0 e to the power of minus i omega naught t and there is no restriction over t this is from minus infinity to plus infinity. And if I do a Fourier transform to find out what is this corresponding frequency, then one can do the Fourier transform easily and essentially one should get a delta function with this form, that for a particular frequency omega 0 there is a sharp peak and then the rest of the value is 0. That means we have a very precise frequency when we have an infinitely extended monochromatic wave but for an infinitely extended harmonic wave. But for finite harmonic waves if I do the same treatment I try to find out what is my g omega. We need to do a Fourier transform. Now the frequency spectrum one can calculate. So here we find g omega, which is the frequency spectrum of the finite wave. For this finite wave we want to find out g omega, how the frequency is distributed and in order to do that we need to do a Fourier transform. So, g omega is simply 1 divided by 2 pi goes from minus infinity to infinity that is the standard way to write the Fourier component of a given function f t e to the power of i omega t dt, here I should write once again what was our ft. So it was a finite wave and ranging from tau 0 by 2 to minus tau 0 by 2, this is my tau 0 equal to 0, this is the not tau 0 equal to 0 notation is not correct, here this is just the zeroth location and this is basically time t. So, my function was let me write down the function here only that ft is equal to e to the power of i omega naught t with a negative sign that is the way we define for the time range. This and 0 elsewhere in all the other points is 0, except in this region it has a harmonic structure. Now if I put it back to the expression that we had. So, g omega will be equal to 1 by 2 pi. So, the function is 0 for all other points except this limit. So, I will put that limit. It should be minus of tau naught divided by 2 and it is tau naught divided by 2 and we have e to the power of i omega minus omega naught t dt. So, that is the expression we have and if somebody solve this simple integral he or she will going to get like 1 divided by 2 pi then e to the power of i omega minus, omega naught t all divided by i of omega minus, omega naught, that is the integral we have with the limit minus tau naught by 2, to tau naught by 2, that is the limit we have, this is omega naught. So, this value is essentially 1 divided by pi omega minus, omega naught and then e to the power i omega minus, omega naught multiplied by tau by 2 minus, into the power minus, i omega minus, omega naught tau by 2. If I put this limit, it is easy to show that this value is essentially sine of 2 of i of sine and this 2 will cancel out and i will cancel out, so 2 i i will cancel out. So essentially we have sine of omega minus, omega naught tau naught by 2. Now if I write omega minus, omega naught multiplied by tau naught by 2, a new variable, say, alpha, then my g alpha will be something like tau naught divided by 2 pi, 1 tau naught will be there because omega minus, omega naught tau 0 divided by 2 that we took alpha. So, here omega minus, omega naught is there. So, if I write the entire thing in terms of alpha it should be sin alpha whole divided by alpha it is something like this, where alpha is defined by omega minus, omega naught tau 0 divided by 2. Well this expression suggests that this is a sinc function because this function we know is called the sinc function which is defined as sin x divided by x. So here also we have a function like this and if I plot this function I'm going to get a structure something like this. So this is my g omega function and here if I plot in terms of omega then this value is 0. So that is omega 0. So around omega 0 it should have a span, initially if you remember. So I'm going to show them side by side. So let me find out what this value is? This value is essentially at alpha equal to pi and this value is alpha equal to minus of pi this is the coordinate we have.

(Refer slide time: 26:44)



So when alpha equal to pi means this value is at omega 0 plus 2 pi divided by tau naught. If I put alpha equal to pi here, then I can find out what is the value of omega and this value is omega naught minus 2 pi by tau 0. So, we have a width here along which the function have a value and that width if I calculate we call this at half width that value is essentially delta omega is equal to 2 pi divided by tau 0. So, this is the width of this spectra and that spectra is this. So, let me know the tally 1 by 1. So, for an infinite wave this is an infinitely extended wave we have a frequency distribution, this is in the time domain. Suppose we have a function f of t that doesn't have any restriction. So it is having in frequency domain, it is having a very sharp looking function like this and this is omega naught and here the g omega is equal to, mathematically if I write it should be a delta function, delta omega by minus omega naught but here in actual case the wave that is emitting is not infinite, it is finite and if I draw that wave in between minus tau naught, sorry this is plus to minus tau naught by 2 something like this. So in this span it is finite, apart from that in the left hand side on the right hand side it is 0. So here the functional form ft was e 0 exponential of minus of i omega 0 t but there is no restriction over t, here the function is finite. So we have a restriction over t. So my functional form was minus i omega naught t with a restriction over t that it is not for all the t's but only a small span of t which is minus tau naught divided by 2 to plus tau naught divided by 2 and 0 otherwise. As soon as we have a restriction of the function in time domain, we find the g here is having a form like this g of omega here is proportional to a sin kind of function, we have sine of omega minus, omega naught tau naught by 2 whole divided by omega minus, omega naught down naught by 2. This is the functional form or in other way it is sin alpha divided by alpha. But the point is here instead of having a sharp value at omega naught, we have a span here. And that span, if I write half width, so delta omega is a half width and that half width we find the value, it is 2 pi divided by tau naught. Well, now it is interesting to note that if tau naught goes to infinity, what is the meaning of that? If tau naught goes to infinity that means whatever the finite harmonic wave we have is no longer finite, it goes to plus infinity to minus infinity. So, this reaches to the actual case. Then it is delta omega, then this delta omega that is the width of the source, the frequency width of the source will go to 0. So, that is the relationship between case one and case two, for finite cases we have a frequency span. So the wave trend can be represented not by a particular frequency omega naught but some other span is there. In case of finite wave trend and in other cases when we have infinite wave trend then the frequency will be defined very precisely as omega naught and because of that what happened we have a definite phase relationship between the wave whatever the time we have. So in case one has coherence length, that tends to infinity. So this is the purely coherent light. So absolute coherence is there and this is the most ideal case which normally does not happen in physics. However we have more realistic case in case two where we have finite waves, finite wave means the wave trend that is coming here, it is finite over time and we have a finite harmonic waves here and for finite harmonic waves we find that if it is finite and this finite thing can be quantified by this time span tau 0, which we call the coherent time. So if it is a finite coherent time then in the source there is a frequency distribution. So, omega naught is not only omega naught it is omega naught plus pi by 2 pi divided by tau naught and omega naught minus this side also it is minus of 2 pi. So, we have a span over that. So, if I quantify the half width at delta omega. So, that means we have omega naught plus, delta omega, omega naught minus delta omega that is the span. So, 2 of delta omega is the width of this span. So, we have a span over frequency and that is why we have. So, we have discrete radiation. There is no phase relationship between this harmonic wave that is radiated. So, today we learn a very important concept and the concept of coherency and what is the meaning of the concept of coherency that we find. This is a temporal coherence. We find that if we have a source and if this source is emitting light then there is a phase relationship one should expect for a finite span of time after that there will be no phase relationship and we say that the coherency is only for that period of time after that the coherency was destroyed. Today we don't have much time to describe more about these things, maybe in the next class we will going to extend our discussion and we will going to discuss how the coherency will be also considered in as a spatial in the case of spatial distribution of the light we call the spatial coherence. And in this case we are going to have temporal coherence. And we also quantify in the next class to a very specific parameter called the degree of coherence. In degrees of coherence we will try to understand what happens if we have absolute coherence, then two light will interfere and what happens, if we have partial coherence if two light interfere what happens. And if there is no coherence at all, if two lights interfere, then what happens? So, we quantify the degree of coherence in a mathematical way, in the next, maybe in the next class and then things will be much more clear to you that how one can calculate the coherency, how it is related to the visibility of light, if two light interfere to each other and if I want to find out the fringe then the a particular pattern we call the interference pattern, we will go we will going to discuss these things in the next few classes. Then coherency is a very very important property that one should have to find a very distinct fringe and why it is coming, what is the background mathematics, what is the background physics of that we want to discuss in detail. With that note I would like to conclude in today's class, thank you very much for your attention and see you in the next class.

(Refer slide time: 33:43)