WAVE OPTICS

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Lecture - 10: Material Dispersion

Hello, student, so today we have lecture number 10 of our course wave optics where we are going to discuss material dispersion. So, in the last class, this is lecture number 10 and today our topic is material dispersion. So let me write it here, in material dispersion, what we are going to study is how the light, when the light is propagating inside a medium, then based on the wavelength of the medium, what happens? That the different frequency components of the light will propagate at different frequencies. So how it happens, we are going to understand in detail in today's class. So far what we have done, we have two waves of different frequency and we have a superposition of these two waves. When these two waves with different frequencies are superimposed we find a resultant structure having the form like this: they are forming small bits. So these are called bit formation. So these are the resultant wave that is generated and this wave has two components one is called the envelope, which is these amplitude modulations and another is carrier wave. So they are propagating with frequency vg and vp. And we find that when they are propagating with these two frequencies that means when the envelope and the carrier wave are moving then these two different frequencies or the same depends on certain. So, if we find then try to find out the relationship between vg and vp it comes out to be vg is equal to vp into 1 plus lambda by n and then we have dn d lambda. So that is the expression we had. Now the important expression here is this dn d lambda. So if dn d lambda is 0 then what do we have?

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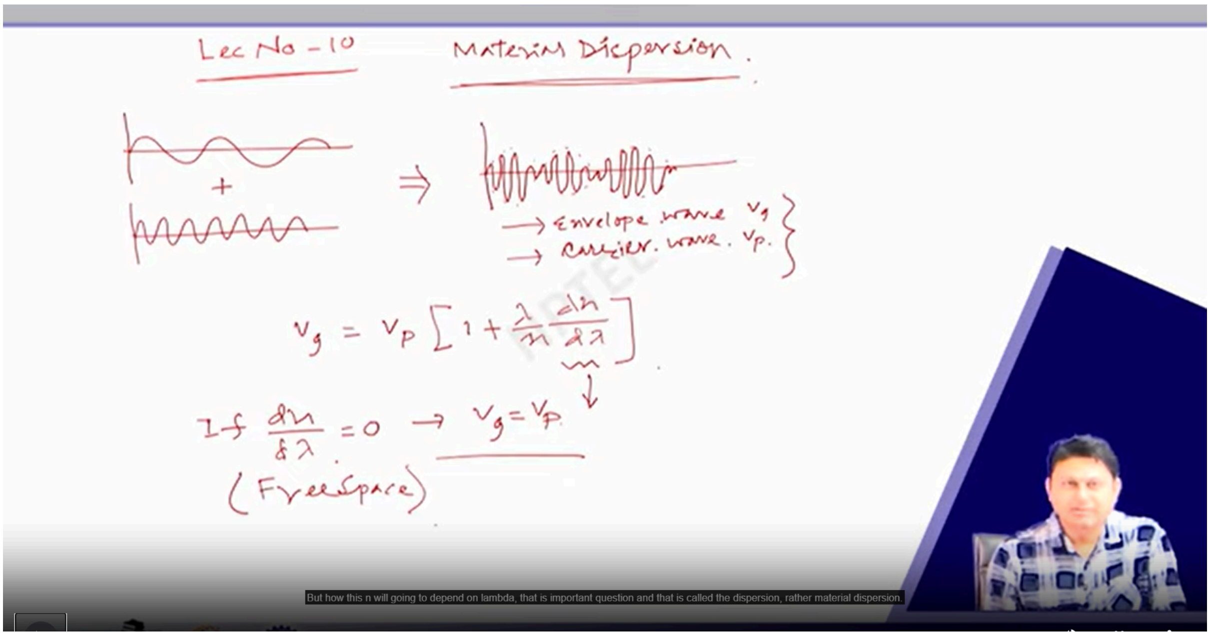

As a result we have vg equal to vp and it happens only in free space but in general if we have a medium through which a light wave is moving having two different frequencies then obviously dn d lambda will no longer be the same, the dn d lambda should not be zero. So it will be different from zero and we will get a condition when Vp and Vg are not the same, they are different. But how this n is going to depend on lambda, that is an important question and that is called the dispersion, rather material dispersion. So, let me now study the meaning of this material dispersion. So, this is essentially the fact that different wavelengths under material dispersion, different wavelengths propagate at different group velocities. So different wavelength components will travel at different group velocities. So what is the reason? Simply the refractive index is a function of wavelength or frequency refractive index is a function of wavelength and frequency and that is the reason we have this n derivative. So n is a function of lambda. Now we are going to find out how the n is a function of lambda, y is a function of lambda, to understand that we need to understand the Lorentz model of dielectric. So what is that? So in the Lorentz model what happens when we launch an electric field varying electric field say this is the varying electric field, I launch the dipole. So because of the presence of the electric field what happens there will be an orientation of the dipole and if the electric field is vibrating so what happens that the dipole will oscillate based on the frequency of this propagating wave that is propagating through the medium. So if I write this wave as E equal to e naught, e to the power i omega t. So, that is the time varying electric field we launch inside the material and in the material we have the dipole and this dipole is going to vibrate we call this dipole oscillation. So, the dipole is going to oscillate. So, this is dipole oscillation, this dipole oscillation can be modelled through this law range model and from that we can extract the value of the refractive index. So the polarization we know how the refractive index comes to the picture that we can understand. So, the polarization which is the dipole moment per unit volume is essentially epsilon naught susceptibility of the material multiplied by the electric field. So, that is the expression of polarization. Now, if I manage to get this susceptibility we can find out the refractive index because the refractive index is equal to root over of 1 plus this susceptibility.

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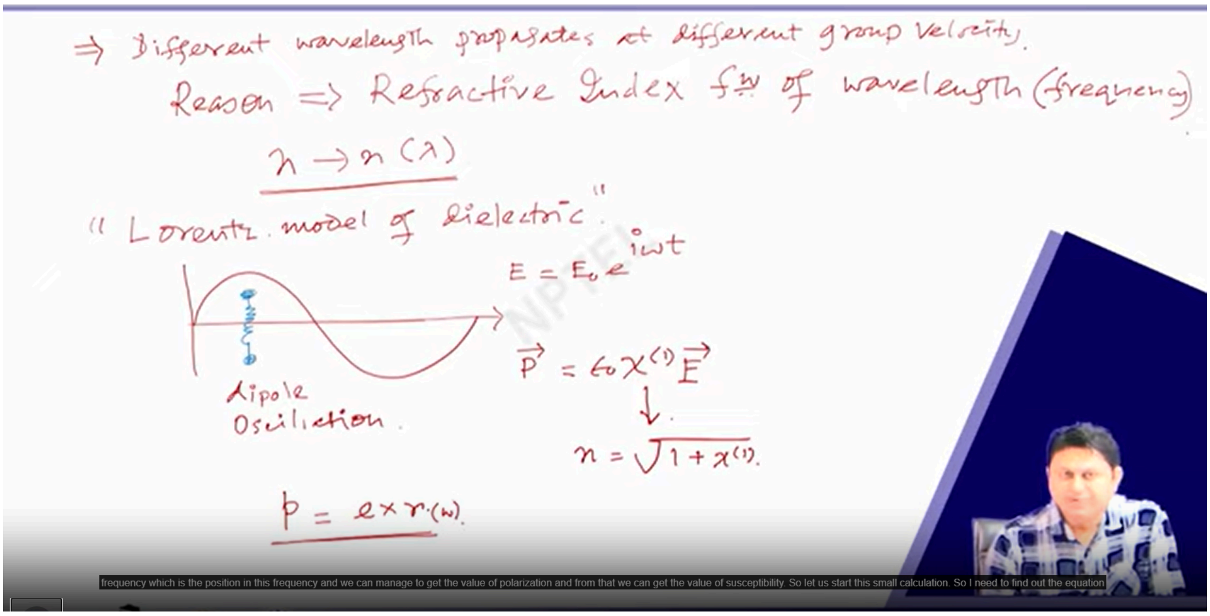

So, if my E is known and from that if I somehow manage to calculate the value of the polarization which is dipole moment per unit volume, then from this expression, we can find out the susceptibility and essentially we can calculate what is the refractive index, how the refractive index varies with respect to lambda. So, the equation of motion for this oscillating dipole, I can say how I find the polarization by the way the dipole moment. So, dipole moment is small p, dipole moment is charge multiplied by the distance r. So, if I manage to get this quantity r from this differential equation, that electric field that is launched and because of that dipole is oscillating. So, obviously r will be a function of omega frequency which is the position in this frequency and we can manage to get the value of polarization and from that we can get the value of susceptibility. So let us start this small calculation. So I need to find out the equation of motion of this oscillatory dipole because the presence of this external electric field that has a time variation is simply m x dot, let us take this problem as one dimension. So, I can see that how the dipole is vibrating in one dimension is equal to f total. So, this is the model we have that is the dipole they are vibrating because of the presence of this electric field which is the time variation. And we try to write the differential equation making this dipole like a spring mass system. What we are doing is trying to write a differential equation to find out the value of its position x. That is my aim. So, what is the value of total force we have in total force? We have three different forces, one is the external force that is coming through the electric field then, we have the damping force we consider as a spring mass system. So we can also consider some sort of damping and another is the restoring force. So these three forces we can have once we deal with this system as a spring mass system. Now, F e which is the external force is simply electric field multiplied by the charge and it is simply E e this is essentially the driving force or external force I write this is driving force. Then we have the damping force fd, the damping force we know is proportional to the velocity. So proportionality constant I write gamma and velocity x dot. So it should be a damping force and then we have the restoring force, simply, Hooke's law, it is kx, which is restoring force. So, once we have these three forces, we can write down the expression, that mx double dot is equal to eE, which is a function of time.

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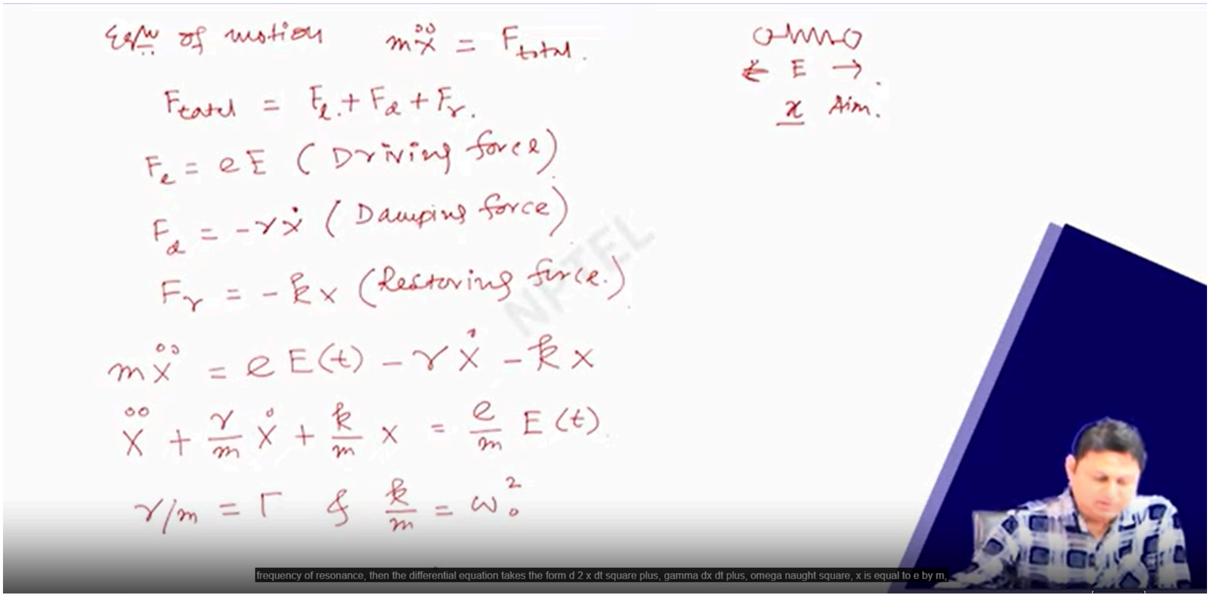

By the way minus gamma x dot and then minus k of x I divide everything m and put this term x and x one side. So I can have x double dot plus gamma divided by m x dot then plus k by m x is equal to e by me which is a function of time. So just rescaling this say gamma by m, if I write big gamma and k by m, if I write this as omega naught square this is the characteristic frequency of resonance, then the differential equation takes the form d 2 x dt square plus, gamma dx dt plus, omega naught square, x is equal to e by m, e function of time. So, this is the differential equation we have to solve. This is an inhomogeneous differential equation, but it is e0 e to the power of i omega t. So, we assume that if that is the electric field or the driving field then the corresponding position I can write also x 0 e to the power i omega in the same form assuming that the dipoles are also vibrating with the frequency omega. So, inserting this solution, assuming this is a solution, this is a differential equation we can simply have x naught is equal to e by m e 0 whole divided by omega 0 square minus, omega square plus, i gamma omega. This is a very straightforward calculation. You just need to put the solution here in the form x equal to x 0 to the power i omega t and the rest of the thing will come automatically. Now, the point is what is the dipole moment? So, from here I can find the dipole moment is small p is equal to e into x and this is the x we have. So I can write p is equal to e x is equal to e square e divided by m, this expression we have whole divided by omega naught square minus, omega square plus, i gamma omega. I simply write e here and x here because if I multiply both sides with e to the power my i omega t e to the power i omega t, then this e 0 will be simply e as a function of t and this x will also become a function of t. Well, the polarization P will be simply a dipole moment multiplied by N, where N is a number of dipoles per unit volume. So, n is a number of dipole per unit volume that makes the polarization, p is equal to p into n small, p we know and if I put this small p, I will going to get p equal to ne square divided by m e whole divided by omega 0 square minus omega square plus i gamma omega. That is essentially equal to epsilon naught susceptibility of first order and E. Now, if I compare these two equations then we can simply get the first order susceptibility which is a function of frequency by the way is simply e square then n divided by M epsilon naught is also divided whole by omega 0 square minus, omega square plus, i gamma omega that is the term we have when we find out the susceptibility.

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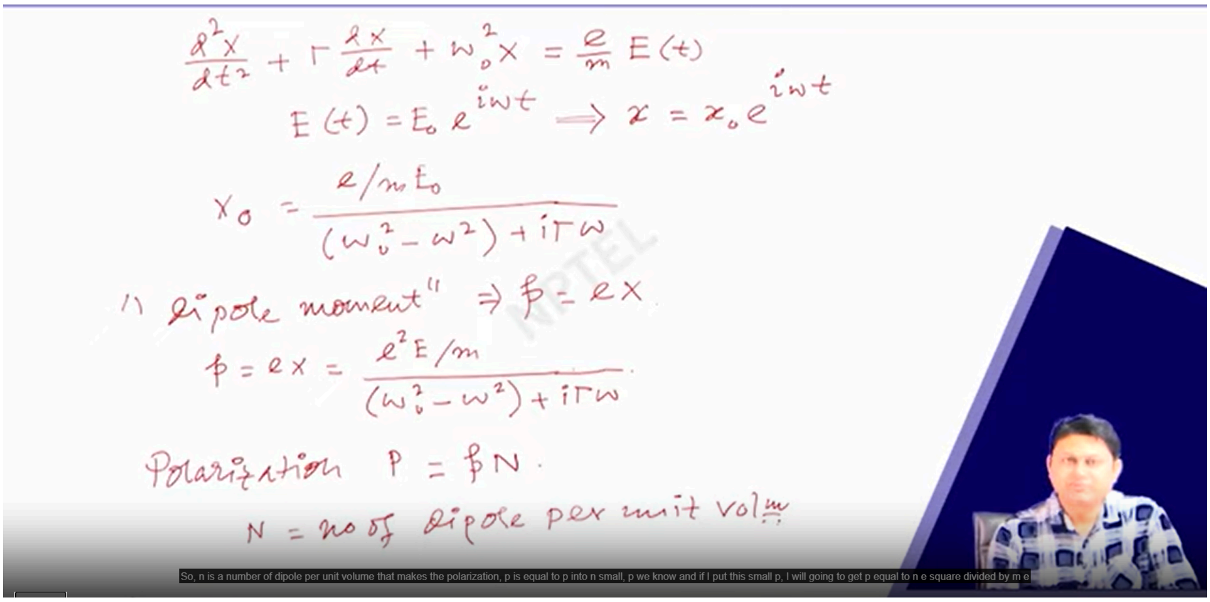

Now, the susceptibility can be related to the refractive index. That is our goal to find how the refractive index is a function of frequency. Since susceptibility is a function of frequency, refractive index I can write 1 plus susceptibility function of frequency which is 1 plus this entire term e square m divided by m epsilon naught we write omega p square this is called the plasma frequency. So, in terms of plasma frequency we have an omega p square, whole divided by omega square minus, omega square then plus, i of gamma omega. Now, it is interesting to note that this is a complex quantity and in order to remove the complex quantity so complex quantity is sitting here actually in the denominator. So the standard way we multiply everything with the complex conjugate and write in x plus i y form. So here also we are going to do that. So n square is 1 plus omega p square and we need to multiply omega 0 square minus omega square minus, i gamma omega and the denominator will be simply omega 0 square minus, omega square whole square of that, then plus gamma squared omega square that will be the denominator and I can write it. Now, okay, I can write it now 1 plus, omega p square, omega 0 square minus, omega square whole divided by omega 0 square minus, omega square whole square plus, gamma square omega square and then I have one term, minus of i, then omega p square gamma omega whole divided by omega 0 square minus, omega square whole square plus, gamma square omega square. So the point is we have a complex term and you should know that this real part contains the refractive index and the complex part that is coming in the refractive index is due to the absorption that we have. Now, if we consider that gamma is equal to 0 that means we are dealing with the problem far away from the absorption region. This is far away from the resonance of the system. Then simply if I put gamma equal to 0 in this equation then we can simply have n square of omega is equal to 1 plus, omega p square whole divided by omega 0 square minus, omega square. So that is the expression we have here and this expression suggests that the refractive index is essentially a function of frequency. Now we can do one thing, later maybe, we aren't able to do today because of the lack of time that we replace all this omega into the frequency term, maybe we can put it as homework. Now what can you do? Students you can do that omega, you put as 2 pi by lambda and if you do that 2 pi lambda into c actually then you can find that if we replace this omega into 2 pi c by lambda then n whatever the function of omega we have it become a function of lambda. And our goal here is to let me go back. So, what was our aim? Our aim here is to find out how the refractive index is a function of lambda. So we have an expression here and in the next class maybe we are going to discuss how exploiting this expression we have a nature of n that is the nature of refractive index and how this refractive index varies with respect to lambda and then find out the value dn d lambda because in the group velocity equation. Let me go back to the group velocity equation we had this term dn d lambda. So if we manage to get n as a function of lambda then from here also we can find out the derivative of n with respect to lambda and once we know the derivative of n with respect to lambda, for a given lambda then we can in a position to find that what is the group velocity of the system. Note that since n is a function of lambda and this value depends on the lambda itself it is in this value it is not a constant. We are going to see that this value is not a constant, rather this value is a function of lambda itself. So, that means Vg will be a function of lambda as well. It is already shown here that Vg is a function of lambda because it is sitting here. That means different frequency components will travel at different velocities and that is why what we have is a dispersion of the system. So, when the light wave with different frequency is propagating inside a medium what happens that because of the fact that the refractive index is a function of lambda it creates the issue that the group velocity of the different frequency component is also moving in a different velocity and we will see the expression of we see that the dispersion is happening and we find rigorously today that using the Lorentz model using how one can figure out that how the refractive index become a function of lambda. Okay with this note I would like to conclude today's class, in the next class we will discuss more about how the refractive index will be a function of lambda. We are going to explicitly do that but today I already gave you some homework. I hope you can do this homework and you can find out how the refractive index is a function of wavelength and if I plot how the plot will look like. In the next class we will understand that if a system is having a dispersion when the optical wave with different frequency is propagating to this system how this optical wave is the dispersion is happening for this optical wave. So with that note I would like to conclude here. Thank you very much for your attention and see you in the next class.

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