Foundation of Classical Electronics
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Lecture-71
Tutorial 2 (Electrostatic)

Hello student to the foundation of classical electrodynamics course. So, we have already completed our theory part. So, in last couple of lecture we will be going to discuss about few problems. So, today under tutorial 2 we are going to discuss few problems related to electrostatics.

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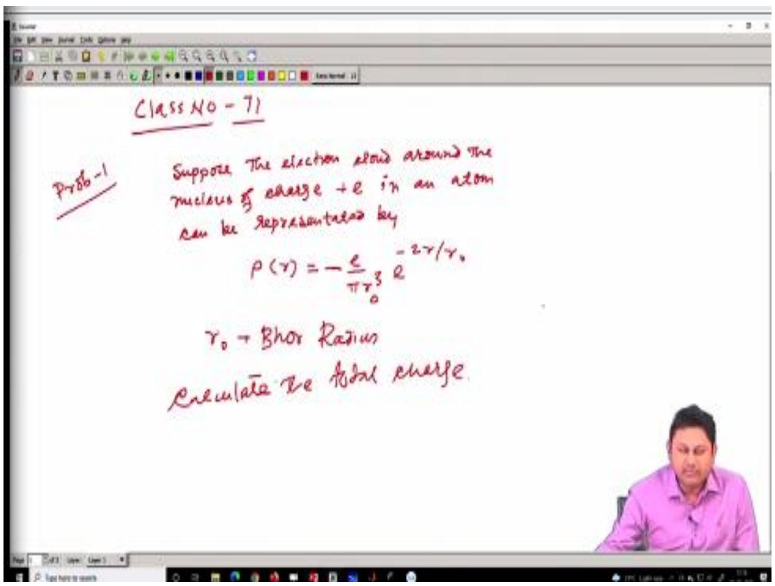

So, today we have class number 71. So, let us directly jump to the problem, so the problem 1 states that suppose the electron cloud around the nucleus of charge +e in an atom can be represented by this following charge distribution. So, the charge density is represented in this way $-\pi$ r^3 and then r_0^3 and then $e^{\frac{-2r}{r_0}}$, where constant r_0 is Bohr radius. So, the question is calculate the total charge, so the charge distribution is given, we need to calculate the total charge that is the problem. So, let us directly solve it the solution.

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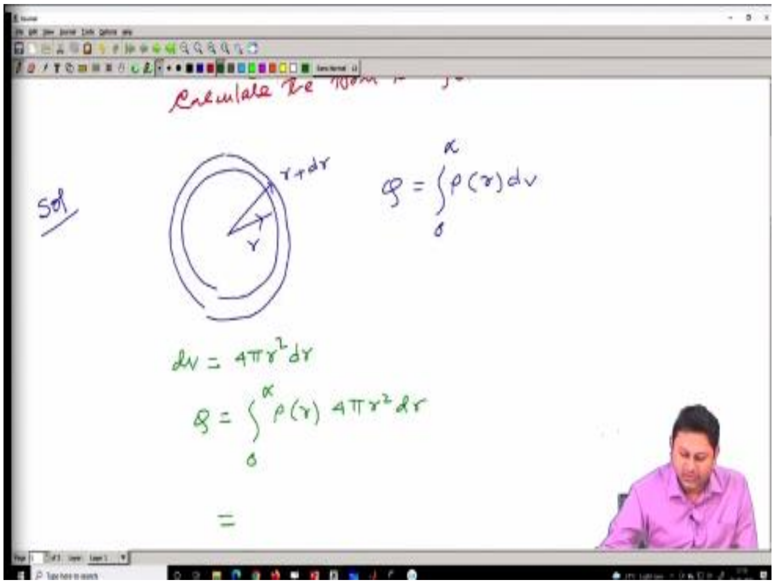

So, since the charge distribution is given and this charge distribution is a function of r, so we can take a small section of this charge distribution because. So, this is r and from here to here it is r + dr. Now total charge Q, I can write it as the integration of the charge density over the volume and this volume can goes from 0 to infinity in principle. So, here my dv is how much? dv is $4\pi r^2$ dr and then my Q is 0 to infinity ρ(r) and then this $4\pi r^2$ dr. So, now if we put the value of the ρ, which is this one which is given, so $-\frac{e}{\pi r^3}$.

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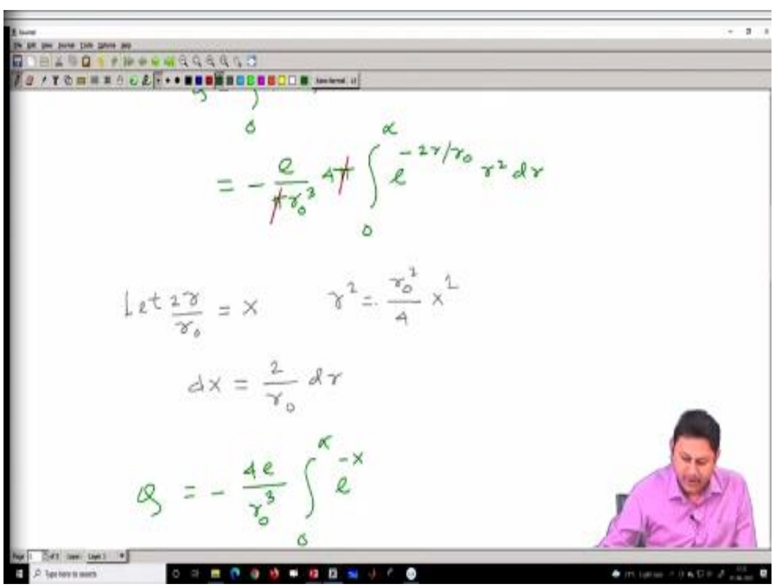

So, this quantity I can take it outside, so I simply have $-\frac{e}{\pi {r_0}^3}$ and then we have 4π and then we have the integration 0 to infinity $e^{\frac{-2r}{r_0}}$ and then r^2 dr that should be our integration. Now this integration is a special kind of integration, so we will be going to discuss, that that is why I put this problem here. So, let $\frac{2r}{r_0}$as a new variable say x, so that r^2 is simply $\frac{{r_0}^2}{4}$ and then x^2.

So, if that is the case then we can have, we have just changing the variable we can write my dx as $\frac{2}{r_0}$ dr, I am going to replace this dr here and then my integration in terms of, so few terms is going to cancel here let me cancel this. For example this π and π will cancel. And we eventually have my integral Q = $-\frac{4e}{{r_0}^3}$ and then $\int_0^{\infty} e^{-x}$ because e to the power we have $-\frac{2r}{r_0}$, so that quantity I have –x.

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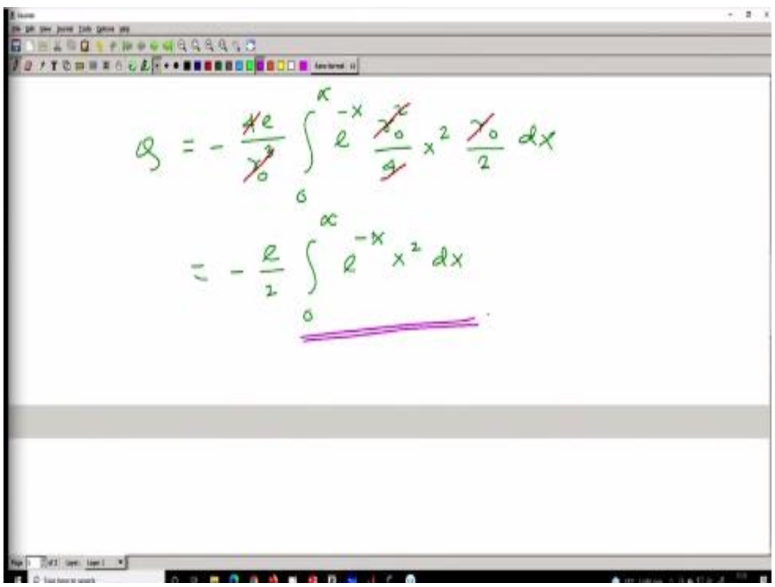

And then r^2 I just replace here the value of r^2, which is $\frac{{r_0}^2}{4}$ x^2 and dr I replace which is again $\frac{r_0}{2}$ and then dx. So, this integral, so let **let** everything outside, so we have -4e and we have here r^2 here r, so this quantity should cancel out and these 4 also going to cancel out. So, we have simply $\frac{e}{2}$ at the outside, we should have $-\frac{e}{2}\int_0^{\infty} e^{-x}$, then we have x^2 dx. So, this is a very special kind of integral called the gamma function. So, let me write it here how the gamma function works.

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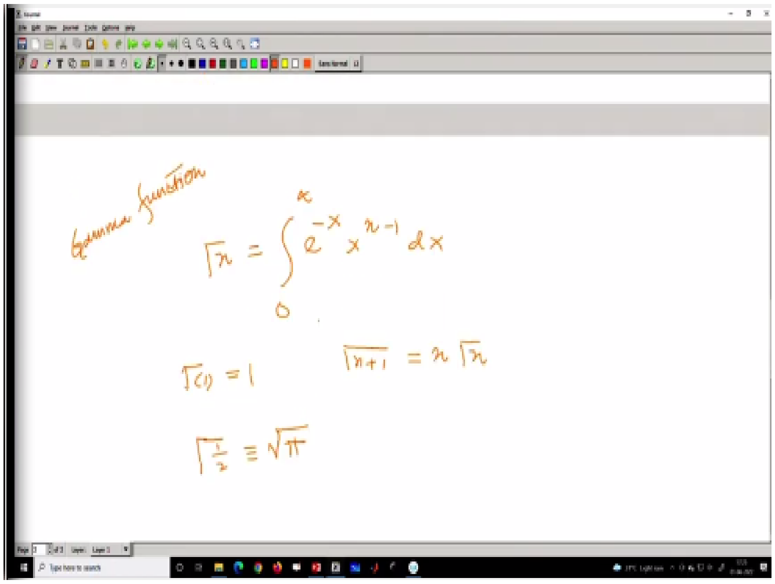

So, this is called the gamma function, what is gamma function? The gamma function normally defined at gamma n and it is defined like 0 to infinity e^{-x} x^{n-1} dx that is called Γn. Now there are some properties, for example gamma of 1 that value should be equal to 1. In general we have Γ(n + 1) = n Γn.

And also one useful value is $\Gamma\frac{1}{2}$ that value is equal to $\sqrt{\pi}$, very important quantity in many cases in physics we encounter this value $\Gamma\frac{1}{2}$. But here if I consider with these expressions that this quantity e^{-x} x^2 dx 0 to infinity if I tally with this then the value of the n here is 3.

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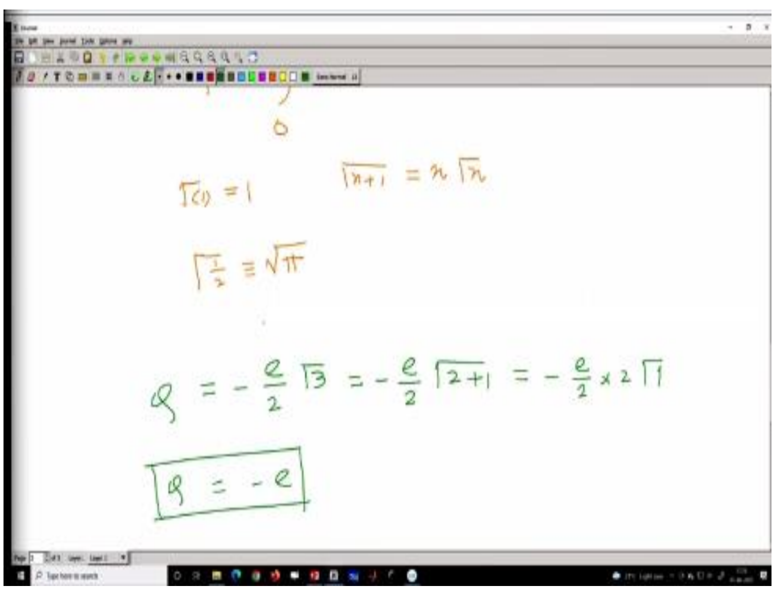

So, eventually my Q is simply $-\frac{e}{2}$ and then we have Γ3. Γ3 I can write it as Γ(2 + 1). So, that thing is $-\frac{e}{2}$ × 2 and Γ1. And Γ1 is 1, so eventually we have this value is –e, so the total charge should be -e and that is expected. Because if you look carefully the problem it says that suppose the electron cloud around the nucleus of the charge +e.

So, charge of the nucleus is given as +e. So, the total charge distribution for the electron cloud has to be -e to neutralize that atom and we are having the same result here, so that is my problem 1. So, let us go to problem 2.

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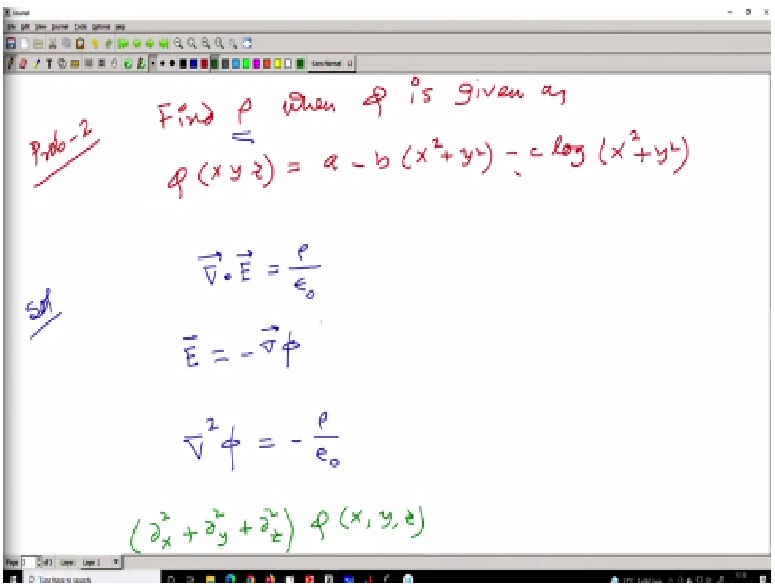

Problem 2, so the problem 2 is this find ρ when potential ϕ is given as ϕ, which is the electrostatic potential is a function of space x y z is given as a - b (x^2 + y^2) - c log (x^2 + y^2), where a, b, c are constant. So, the question is simple that the potential is given and we need to find out the charge density. So, here we are going to use this solution. So, we will be going to use the expression that $\vec{\nabla} \cdot \vec{E}$ we know that is $\frac{\rho}{\epsilon_0}$ and then we know that my e is minus of this quantity.

So, if I put there then we have a Poisson equation and which suggests that we have $\nabla^2\phi = -\frac{\rho}{\epsilon_0}$. So, eventually we need to find out this the explicit form of ϕ is given, I just need to find out what is the value when we apply this Laplacian over this ϕ, that is all. So, this operator is simply ∂_x^2 + ∂_y^2 + ∂_z^2 in shorthand notation this is the way we represent the operator here and that should be operated on the function ϕ. So, now if I go back to the function it is $a - bx^2$, so I just do the operation here.

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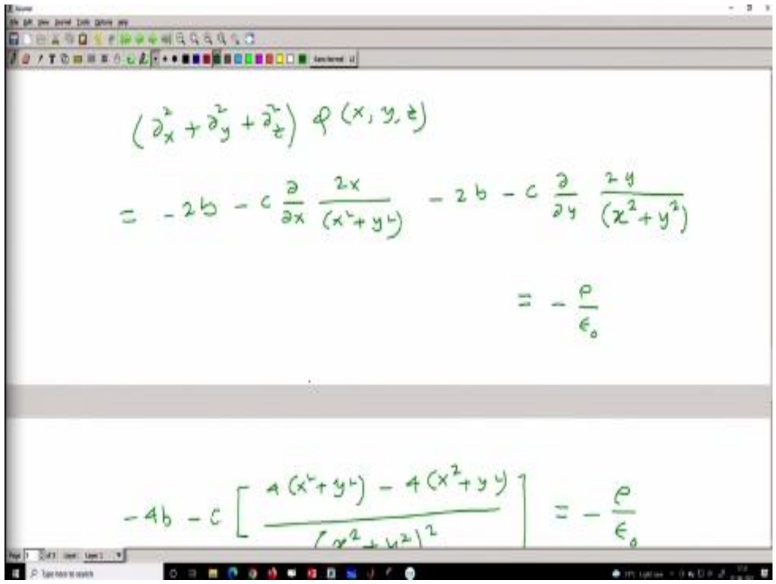

And first we have -2b then we have -c and $\frac{\partial}{\partial x}$ it will be going to operate over $\frac{2x}{x^2 + y^2}$ because this is the way it is here, so I need to make a derivative with respect to x. So, here for log we will have this and then for y we have - 2b and then - c again $\frac{\partial}{\partial y}$ and then again we have $\frac{2y}{x^2 + y^2}$ and that quantity is equal to $-\frac{\rho}{\epsilon_0}$.

Now from here I can calculate ρ because the left-hand side and if I execute the left-hand side then the left-hand side will be something like this. So, let me complete that. So, we have - 4b and then if I take c common, c then we have something like $4(x^2 + y^2) - 4(x^2 + y^2)$ seems they will cancel out divided by $(x^2 + y^2)^2$ and the right-hand side we have $-\frac{\rho}{\epsilon_0}$. So, this quantity associated with c will be going to cancel out, you can check it; I am not doing the explicit calculation here.

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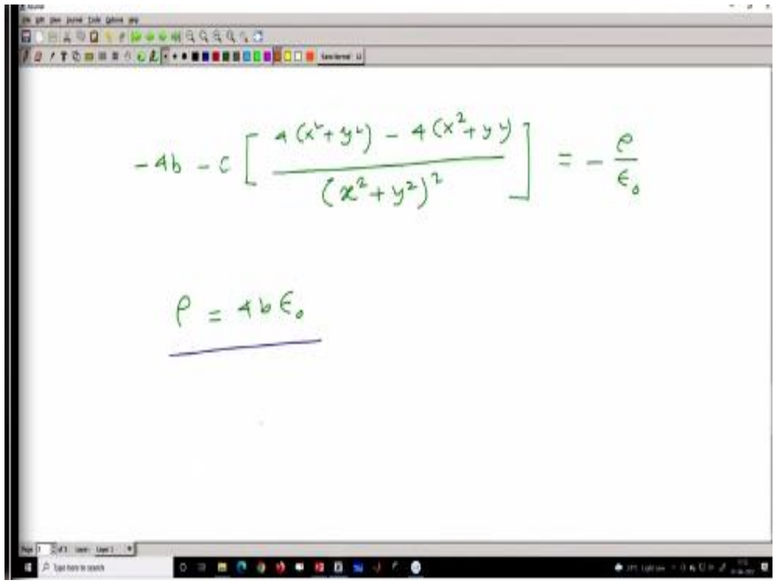

So, then my ρ simply becomes $4b\epsilon_0$ that should be our result. So, what is the strategy of this problem? When you are asked to find out ρ by giving ϕ in explicit way, you just need to use the Laplacian equation that is the strategy we are using. We use the Laplacian equation, find out the left-hand side and that should be the value of the ρ**.** Then after that maybe we will go to another problem. The problem is again related to the charge density.

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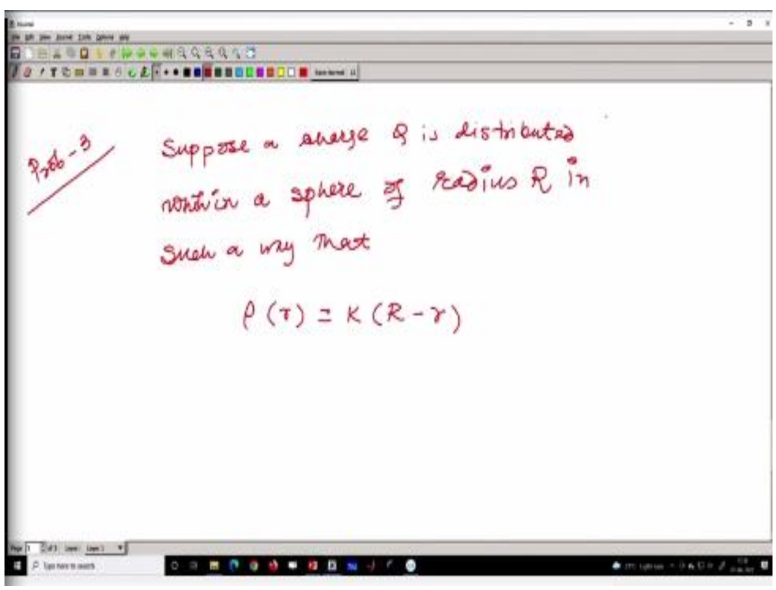

So, this is problem 3, so the problem is suppose a charge Q is distributed within a sphere of radius R in such a way that the charge density at a certain distance is given as this or ρ is not distributed uniformly, so the variation of the ρ is given, it is like the problem we had in the as a problem 1. So, the Q is distributed in such a way that the charge density is not uniform.

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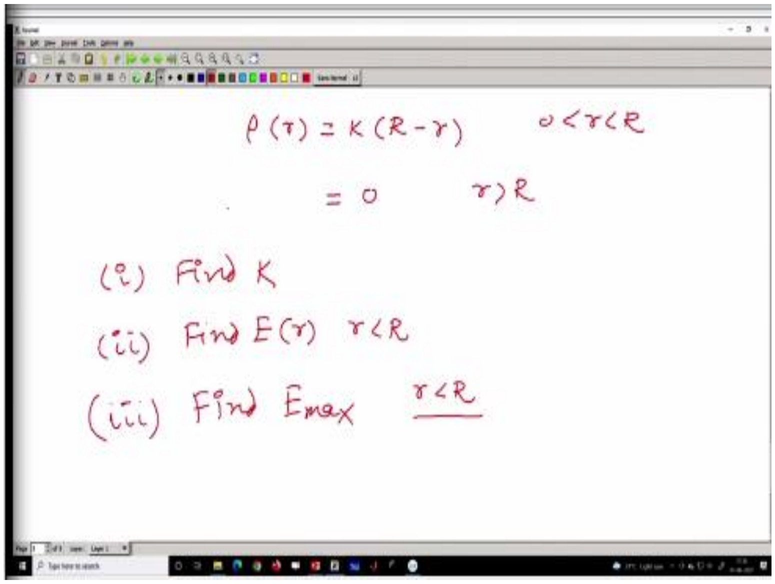

And it is distributed in this way from the origin to certain r, this is this and beyond that it is 0 when r is greater than R. Now the charge distribution is given the question that is asked is 1, first thing we need to find out find K, find the value of K in terms of total charge, so that will see. Second question is find E electric field at some point r where r is less than R, third point is find E_{max}. So, there should be a maximum field then find out where the field will be maximum for r less than R again this condition. So, here we find E(r) and now we need to find out what is the maximum r. And these are the 3 problems we need to do.

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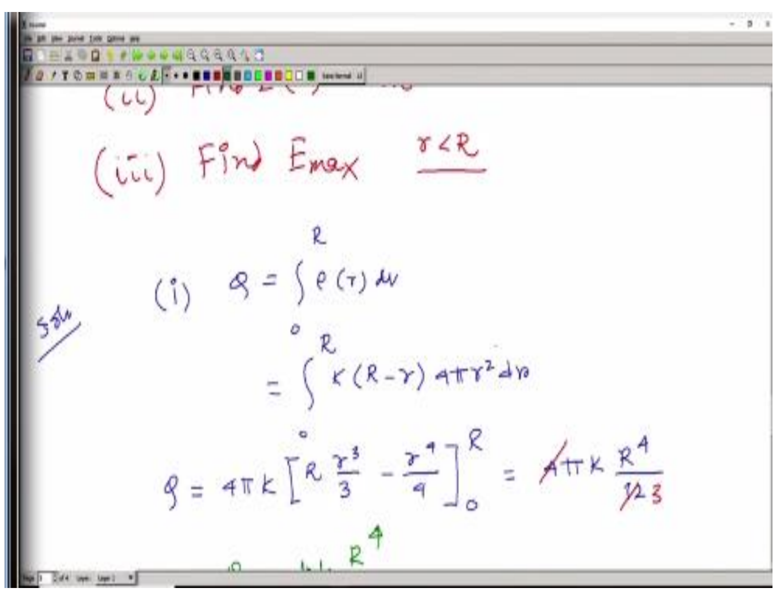

So, the strategy is straightforward, the solution for the problem 1 is this, Q should be equal to 0 to r and ρ(r) dv, so that we know. And ρ is given here; the charge density is given here, so we will be going to exploit that expression to find out what is the total charge? So, it is 0 to R and then it is K(R – r) and since ρ only depends on r, so the dv I should write simply $4\pi r^2$ dr, exactly like the first problem, this is r dr.

So, if I integrate that my Q should be simply 4πK and then the integration in this limit is simply R and then let me do the integration first $\frac{r^3}{3}$ - $\frac{r^4}{4}$ from 0 to R, that is the limit. And that value is simply if I put that value R and this is, so it should be $\frac{R^4}{3}$, so okay, so it should be 4πK and that value is $\frac{R^4}{12}$ and this 4 will be going to cut with the 12 and we will have 3 here.

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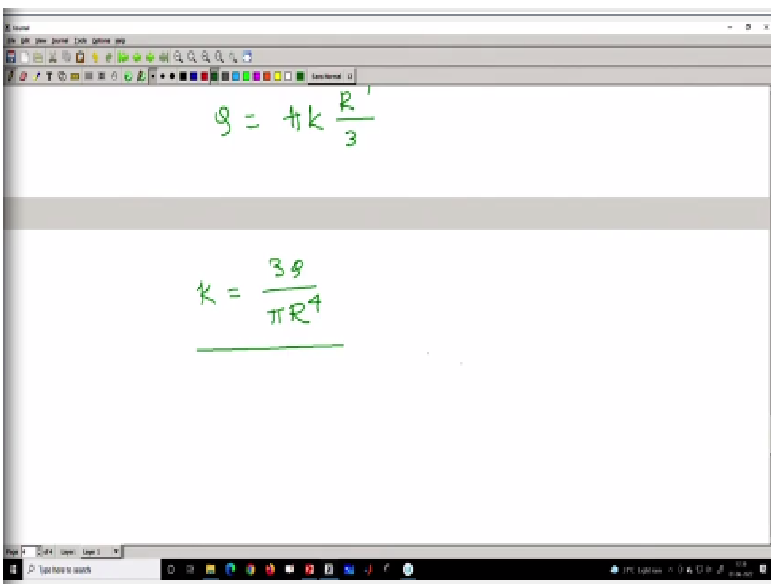

So, my Q what I find is simply πK and then $\frac{R^4}{3}$. So, from here I can find out what is the value of the K that is the problem 1. In terms of total charge my K should be $\frac{3Q}{\pi R^4}$ that should be my first answer. What about the second one? What is the electric field? So, I will go to use the Gauss's law here simply.

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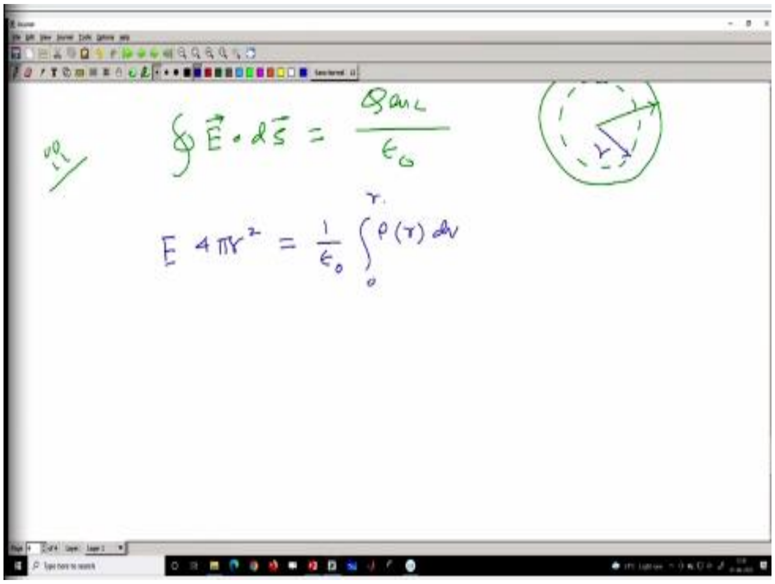

Problem 2 is I will be going to use line integral total surface integral ds = $\frac{Q_{enc}}{\epsilon_0}$ because I need to find out the field for this spherical distribution where the ρ is changing with a function of r. So, I can take a Gaussian surface here and calculate the electric field that is a simple strategy. So, at R from here to here I can have it should be r because this is R is the radius, so it should be r. So, E then $4\pi r^2$ that should be the charge enclose, so charge enclosed I need to calculate, so it should be ρ at that point over dv 0 to r.

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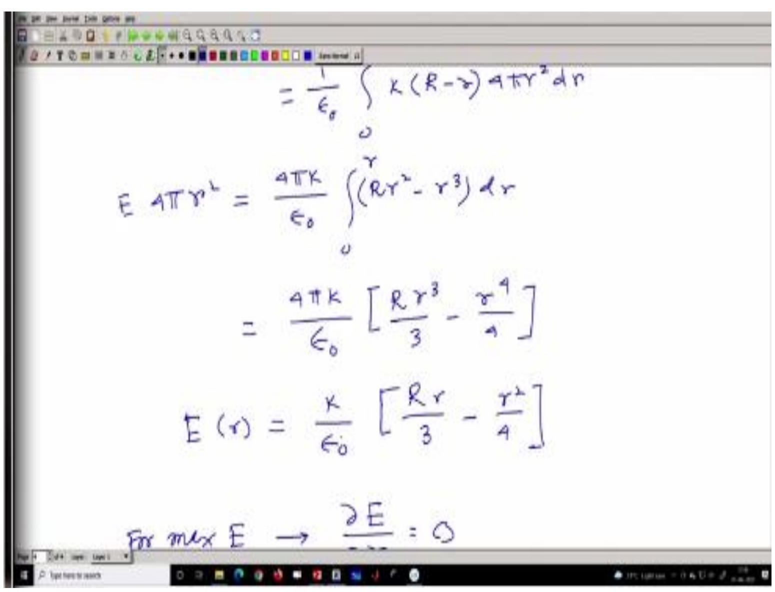

So, that quantity $\frac{1}{\epsilon_0}$ and ρ is given, so it is integration 0 to r, K(R – r) that is the way it is changing. And dv again I should have $4\pi r^2$ dr, so if I do this integration then let me write the left-hand side it will be E $4\pi r^2$ that is equal to $\frac{4\pi K}{\epsilon_0}$. And the integration 0 to r, the thing I need to do is $(Rr^2 - r^3)$ that I need to integrate.

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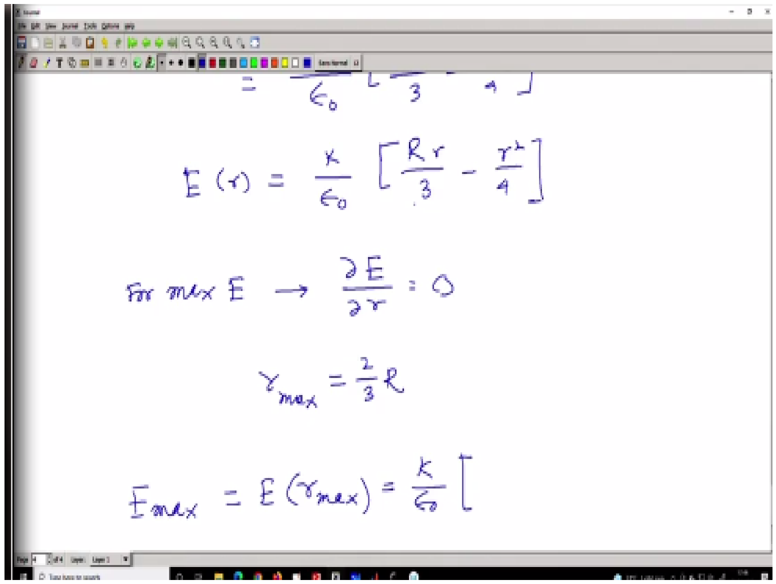

So, that quantity I simply find out as $\frac{4\pi K}{\epsilon_0}$ and this is R, then $\frac{r^3}{3}$ - $\frac{r^4}{4}$, that is all. So, my E at any point r should be the 4π, 4π will cancel out, it should be $\frac{K}{\epsilon_0}$ and then we have $\frac{Rr}{3}$ - $\frac{r^2}{4}$. So, this is my E and the third point is for maximum E what we do? We will calculate this quantity.

So, this quantity give us a value of r at which the E should be maximum and if I calculate it is easy to find that r_{max} where these max is simply $\frac{2}{3}$R. So, that means if I put R as $\frac{2}{3}$R here then I am going to get the maximum E.

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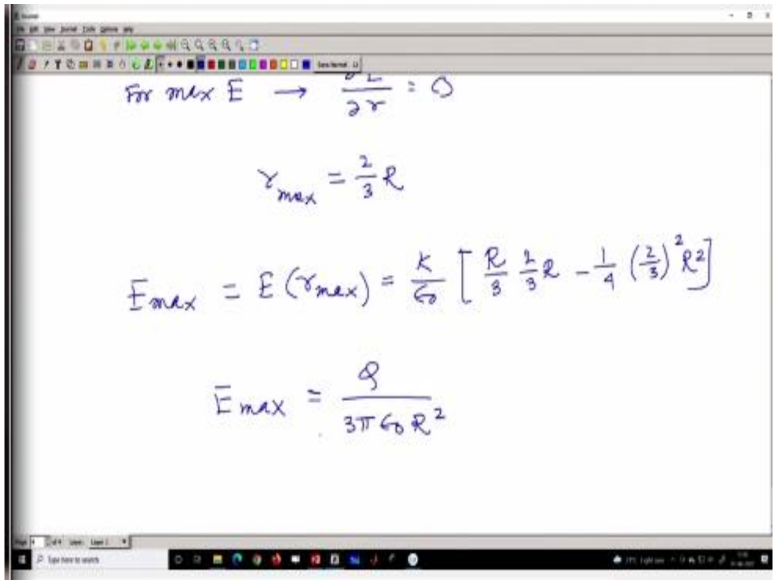

So, maximum E, E_{max} should be simply E at where at the position of r_{max}, so that is $\frac{K}{\epsilon_0}$ and then we have the value here, so $\frac{R}{3}$, so I simply have $\frac{R}{3}$, then $\frac{2}{3}$R and then $-\frac{1}{4}$ then $(\frac{2}{3})^2$ and then R^2. So, eventually I have E_{max}, if you just calculate that you will be going to get this result in terms of total charge because K is already there, we know what is the value of the K.

So, if I put the value of the K it should be $\frac{Q}{3\pi\epsilon_0 R^2}$, I am not doing the rest of the part because this is a simple algebra. So, I believe you can figure it out that what is the value of the E_{max} when you put this. Now let us go to problem 4.

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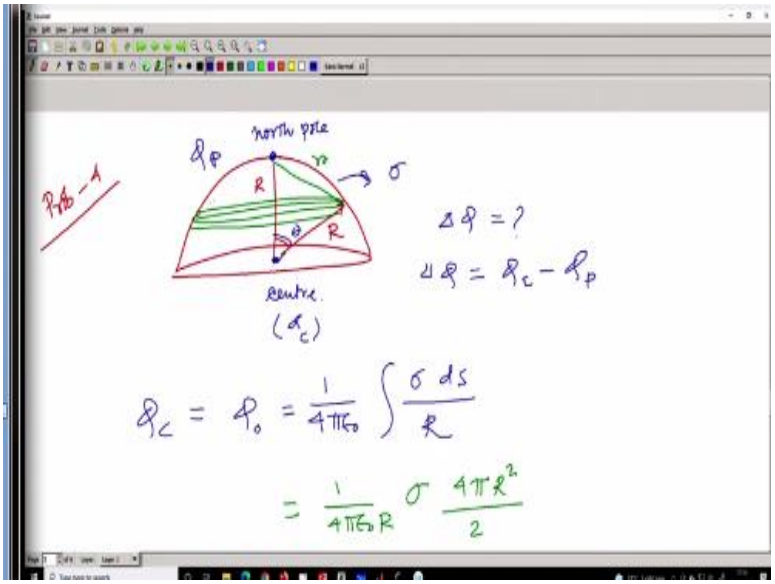

This interesting problem, the problem 4 is let me draw that, then you will be going to understand what is the problem. I have a half hemisphere, a hemispherical bowel like this of radius R, so we have a radius here and that value is R. So, we have 2 points here, so this is the pole like this, so I am having I have one point and this is another point. So, the surface charge density here is σ the surface charge density and the question is the potential difference between this north pole, this is north pole and the center, this is center.

So, the problem is $\Delta\phi$, what is $\Delta\phi$? The potential difference between the center minus the north pole say ϕ_p, so this is potential here is c and here the potential is p. So, what is the potential difference? When there is a surface charge distribution is there and σ is a surface charge distribution. So, let us find out what is ϕ_c first? ϕ_c is ϕ_0 or that is the reference say $\frac{1}{4\pi\epsilon_0}$ and according to the formula we simply have this. Because this is over the surface all the points are from same distance, so this central point is the distance is same, so R is same, so let me take a so okay, I will do that later.

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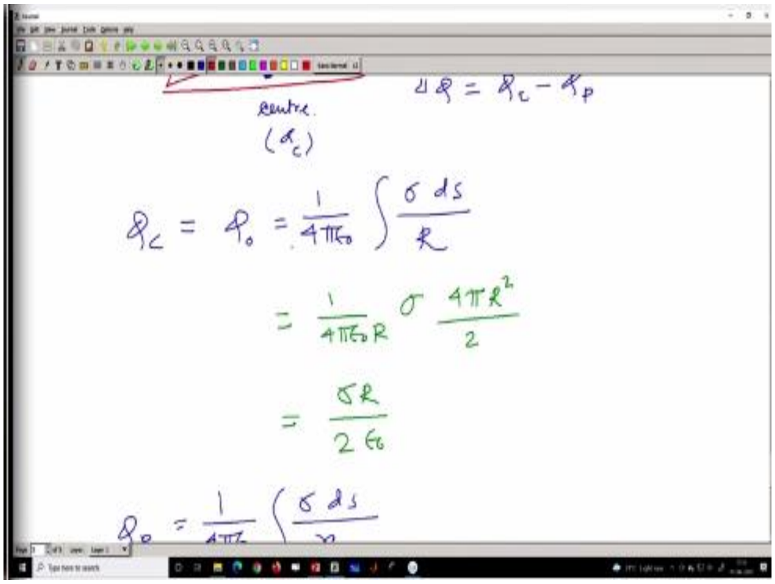

So, that value simply I have $\frac{1}{4\pi\epsilon_0 R}$, then σ and $\int ds$ is simply $\frac{4\pi R^2}{2}$ half of the sphere. So, this quantity I simply have σ $\frac{R}{2\epsilon_0}$. Now the interesting part is the next one how to find out the potential at north pole? Say in order to do that let me draw a section here, a small section like this and from here to here suppose this distance is r, now this r is changing every time. So, that is the trick we are having here, so we need to take care of that issue.

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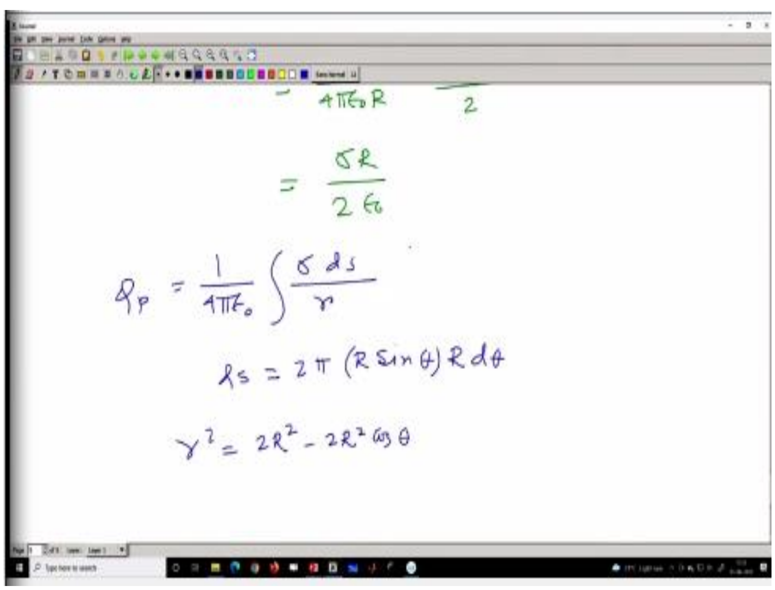

So, the ϕ pole should be $\frac{1}{4\pi\epsilon_0}$ and then the charge over the small surface area, which is defined by this ring here divided by r. Now here ds is simply 2π R sin θ and then R dθ because this angle if it is θ then if I calculate this surface area then this surface area should be this one, R sin θ because I am taking from here to here this distance and then dr and then R dθ is this strip, this portion of the strip.

So, it should be the surface area of this small section, what is r? So, there is a relation, so because if I calculate r in terms of this, this is r and this is also r from here to here this is also r and the angle between these 2 is θ. So, then I can simply have my r^2 as 2 R^2 – $2R^2$ cos θ, it is simply R^2 + R^2- 2R × R cos θ. The angle between the 2 vector is θ and the magnitude of 2 vector is R. So, the other arm if I want to find out I simply use the vector law and this is the way one can find, so this is my r^2. So, I am going to put this here and I will get the result.

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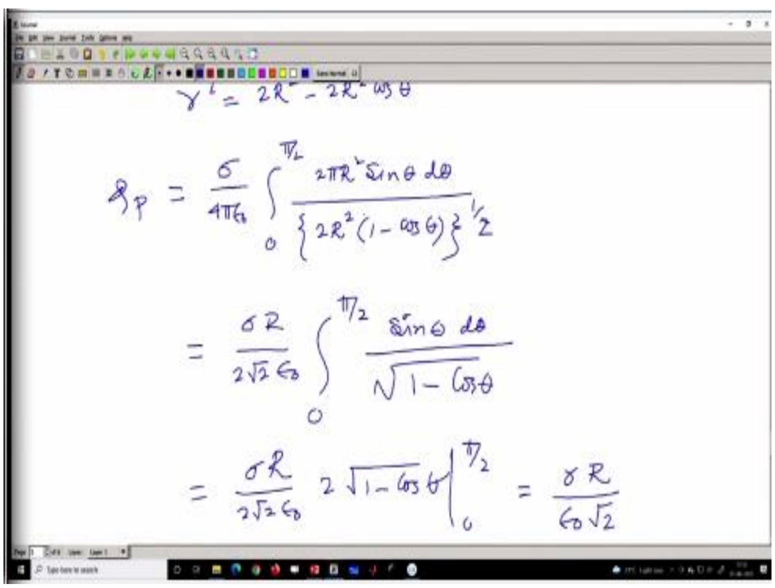

So, ϕ_p then simply equal to $\frac{\sigma}{4\pi\epsilon_0}$ 0 to $\frac{\pi}{2}$ and then we have $2\pi R^2$ sin θ dθ, I am just writing ds here. And below we have R, so we have $\{2R^2\ (1 - \cos\theta)\}^{1/2}$. So, simplifying this we have $\frac{\sigma R}{2\sqrt{2}\epsilon_0}$, 0 to $\frac{\pi}{2}$ and then we have simply $\frac{\sin\theta\, d\theta}{\sqrt{1-\cos\theta}}$.

So, that thing is because I can write sin θ as a dcos θ and then the rest of the thing I calculate. So, it should be simply $\frac{\sigma R}{2\sqrt{2}\epsilon_0}$, then we have $2\sqrt{1-\cos\theta}$ with the limit 0 to $\frac{\pi}{2}$ and that value is simply $\frac{\sigma R}{\sqrt{2}\epsilon_0}$.

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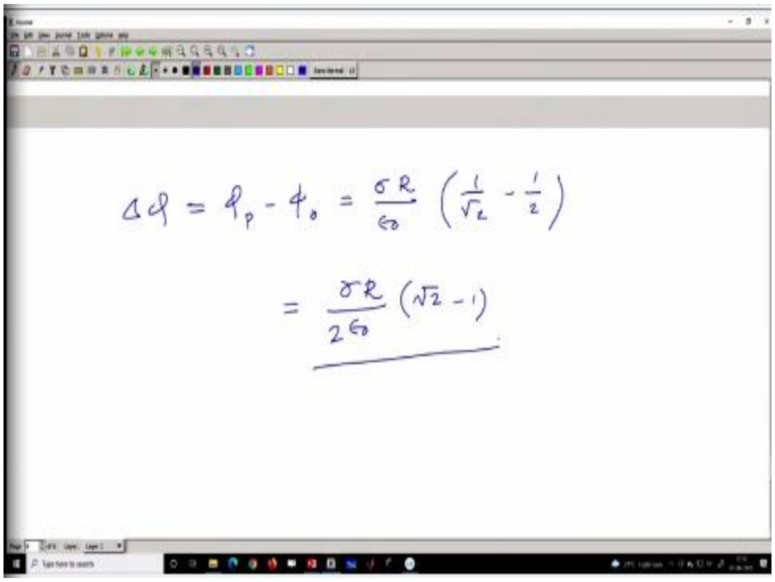

So, then my $\Delta\phi$, which was there in the problem is simply ϕ_P - ϕ_0 that is equal to $\frac{\sigma R}{\epsilon_0}$ $(\frac{1}{\sqrt{2}}$ - $\frac{1}{2})$ that should be our result. You can simplify this expression it is $\frac{\sigma R}{\epsilon_0}$ and then how much? It is $(\sqrt{2} - 1)$, if I multiply 2 here it should be 1, it should be the power of 2, so I need to have 2 here, yeah, it should be something like this.

So, that is the difference between the potential for this system, the trick here is to calculate the potential at the north pole where the distance is changing continuously. Now go to the next problem, the next problem is something like this.

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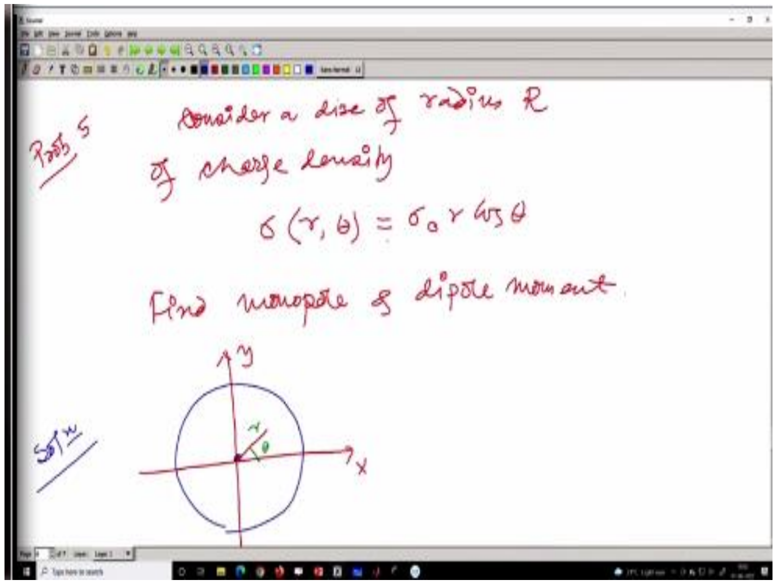

Problem 5, and say the problem is consider a disc of radius R of charge density, the charge density of the disc is given. Now the charge density here is a function of r and θ, which is say σ_0 r cos θ, this is the way the charge density is changing. So, the problem is find monopole and dipole moment of the system.

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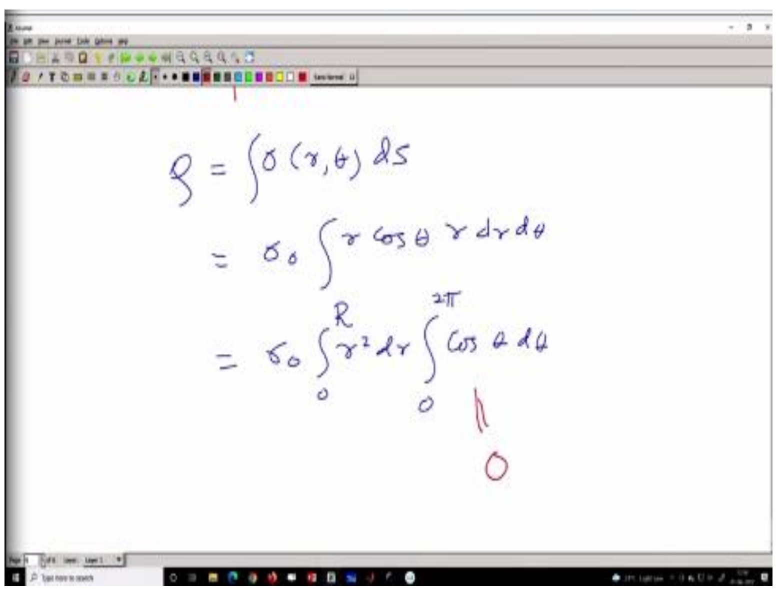

So, solution, what the problem says is that like this, I have a disc here, this is the center of the disc and from here to here suppose I am having a coordinate system say this is x and this is y. So, at this point this is say r and θ, the value of the surface charge density is given as this, this is a function of r and θ, so r is changing also σ will be going to change, θ change, σ is also change, so I need to take care of that issue.

So, monopole moment means total charge. So, total charge we know how to calculate for the surface charge density it is $\sigma(r, \theta)$ over the entire surface ds. So, it is σ_0 and then I should have r cos θ, how it is distributed? r cos θ. So, it should be r cos θ and ds is a surface element in polar coordinates, which is r dr dθ and that is all. So, if I calculate σ then it will be 0 to R for r^2 dr 1 integral and another is 0 to 2π for cos θ dθ, so the second integral will be going to vanish because this has to be 0. So, if this is 0 then we simply have Q = 0.

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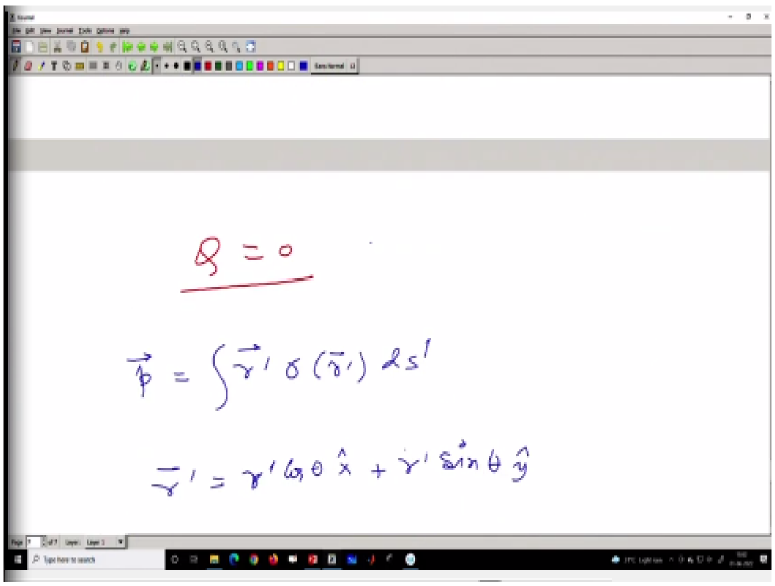

So, monopole moment will not be there, so the total charge distribution is such a way that the charge distribution is here in such a way that total charge is 0. So, the positive and negative charge they are equal, so there is no neat total charge. What about the dipole moment? For dipole moment $\vec{p}$ we know our expression it should be $\vec{r}\,'$ $\sigma(\vec{r}\,')$ and ds'. So, this is by definition the dipole moment.

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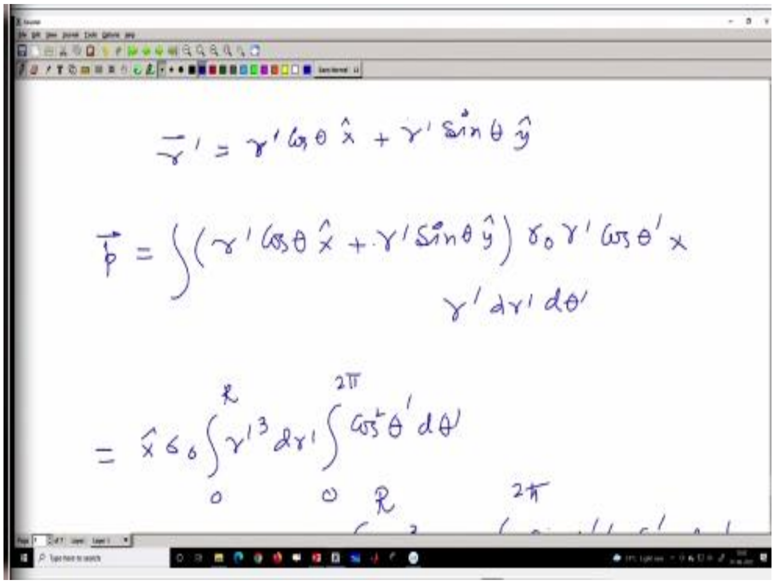

$\vec{r}$' here is the location and the point and this is r' cos θ $\hat{x}$ + r' sin θ $\hat{y}$. So, $\vec{p}$ then simply integral of this, so I have r' cos θ $\hat{x}$ + r' sin θ $\hat{y}$ and then I have the $\sigma(\vec{r}')$ cos θ' that is the surface charge density. And then we have the surface element, which is r' dr' dθ'.

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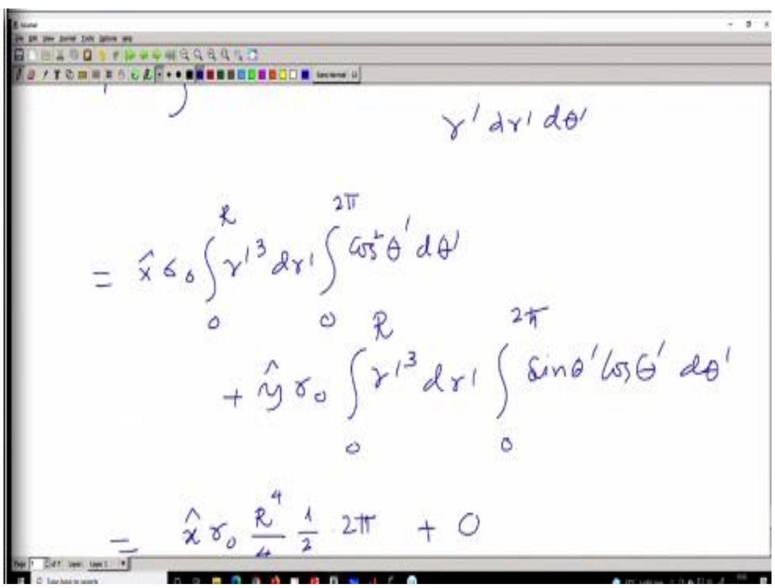

Now if I calculate separately for x and y components, the x component if I calculate σ_0 I can take outside it should be 0 to R and then r'^3 dr'. And then we have 0 to 2π and we have $\cos^2$ θ' dθ'. And y component will be $\hat{y}$ σ_0 $\int_0^R r'^3$ dr'. But here we have 0 to 2π and then we have sin θ' cos θ' and then dθ'.

So, we can see that this quantity should be 0 and eventually what I am getting in on the x component, so $\hat{x}$ σ_0 it should be $\frac{R^4}{4}$ and then this quantity is $\frac{1}{2}$ 2π. And y component will be 0, if you do the calculation you will find that.

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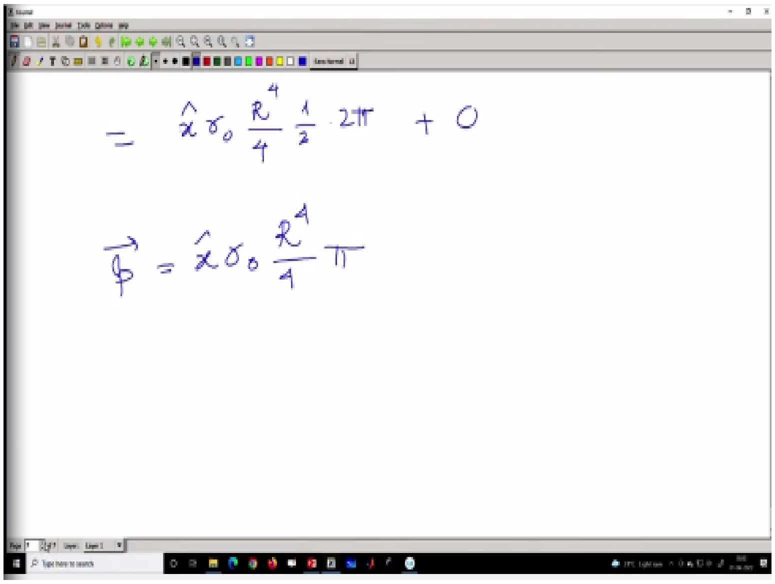

So, eventually my $\vec{p}$ should be $\hat{x}$ σ_0 $\frac{R^4}{4}$ π, so this is the way we can calculate when this is a distribution of the charge you can calculate the monopole and dipole moment by exploiting the basic expression and we will be going to get this. After that we will just go to the final problem.

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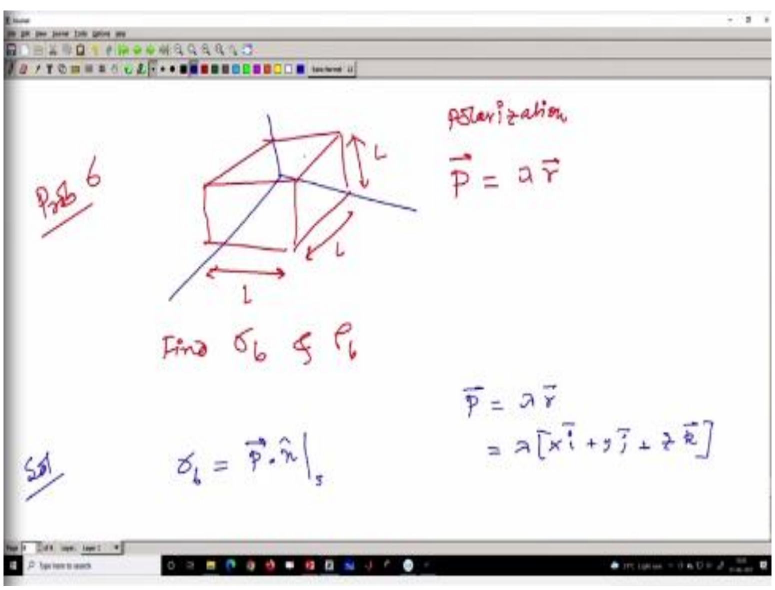

So, problem 6, and the problem 6 is saying that we have a dielectric, so I am not writing the problem just so people going to understand that because I am saving some time. So, we have a dielectric block here with the length L both the sides and the $\vec{P}$, the polarization of this dielectric is not uniform, this is a function of $\vec{r}$ like this, this is the polarization, polarization is a function of $\vec{r}$. Now the question is find bound surface charge density and bound volume charge density.

So, let us solve this problem. So, we have a dielectric cube where the polarization is a function of $\vec{r}$ like this. And now we need to calculate the surface charge bound charge density and volume bound charge density and also we need to show that if I add these 2 charges then it should vanish. Because there should not be any net charge.

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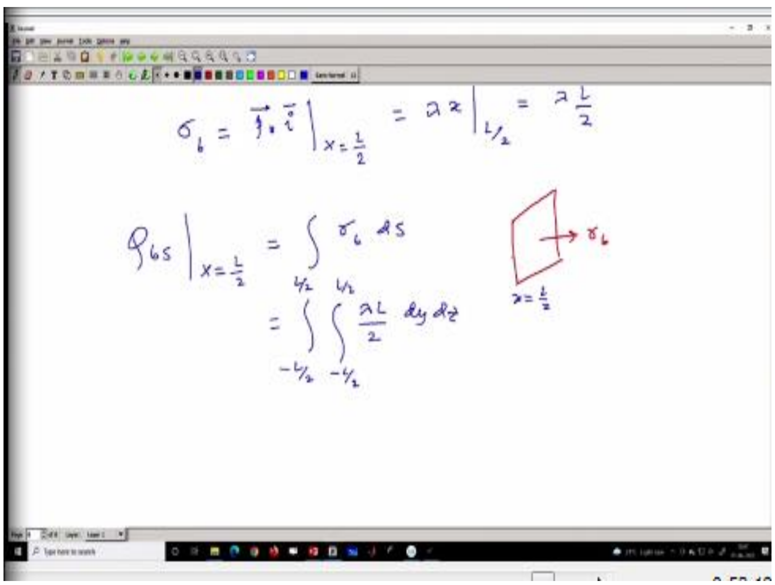

So, we know how to calculate the surface charge density if the polarization is given and for okay let me write down the polarization here first. Polarization is $\lambda\vec{r}$, which is lambda x$\hat{\imath}$ + y$\hat{\jmath}$ + z$\hat{k}$ and the polarization for any surface is $\vec{P} \bullet \hat{n}$ that is where we calculate on top of the surface. So, here for one surface I can simply calculate the surface say at x = $\frac{L}{2}$ if I consider my coordinate at the center here.

So, this is the coordinate at the center of this block then at x = 2 it is dot $\hat{\imath}$ calculated at x = $\frac{L}{2}$, so this quantity is simply λ x divided calculated at x, $\frac{L}{2}$, so it is simply λ $\frac{L}{2}$, this is the surface. Now there should be if I want to calculate this is the charge density the total surface bound charge for surface if I want to calculate at $x = \frac{L}{2}$.

Then I should simply write σ_b over ds for 1 surface, say this is one surface of this block, σ_b is known. If I want to calculate what is the total charge? I need to integrate it over the surface. So, that quantity that is at $x = \frac{L}{2}$ point. So, that quantity if I calculate for surface it is $-\frac{L}{2}$ to $\frac{L}{2}$, $-\frac{L}{2}$ to $\frac{L}{2}$, σ_b I already calculate this is $\lambda\frac{L}{2}$, ds is dy dz because this x surface, so ds is dy dz.

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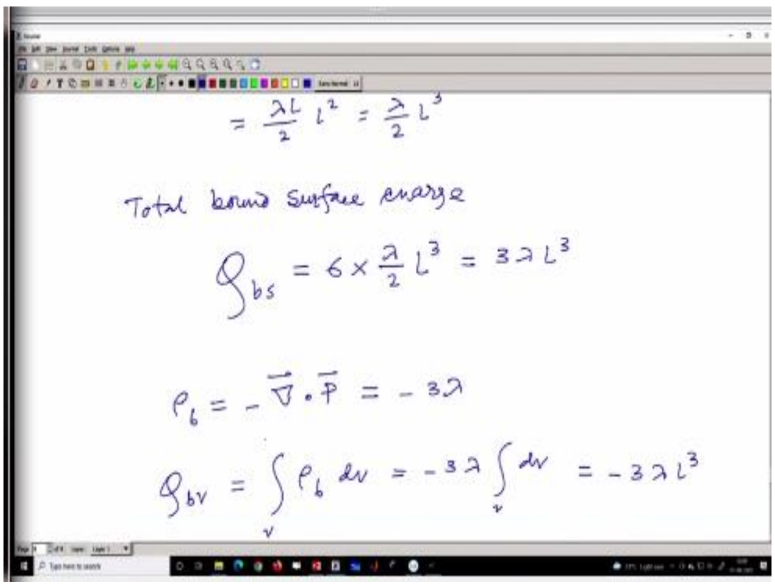

So, that quantity is simply $\lambda \frac{L}{2}$ and L^2, so it should be $\frac{\lambda}{2}$ L^3, that is for one surface. How many surfaces are there that 6 surfaces are there, so the total bound surface charge Q_{bs} for total surface bound charge should be 6 multiplication of that quantity. Because there are 6 surfaces all the surface you will be going to get the same value, so we will get 3 λ L^3. What about the volume bound charge?

So, we know the volume bound charge ρ_b is $-\vec{\nabla} \cdot \vec{P}$, which is simply -3λ. So, the total bound charge then how much? Q total bound charge for volume should be volume integration ρ_b dv. Now ρ v is $- 3\lambda$ and then integration of dv over this Q, the volume of the Q is L^3, so it should be $- 3\lambda L^3$.

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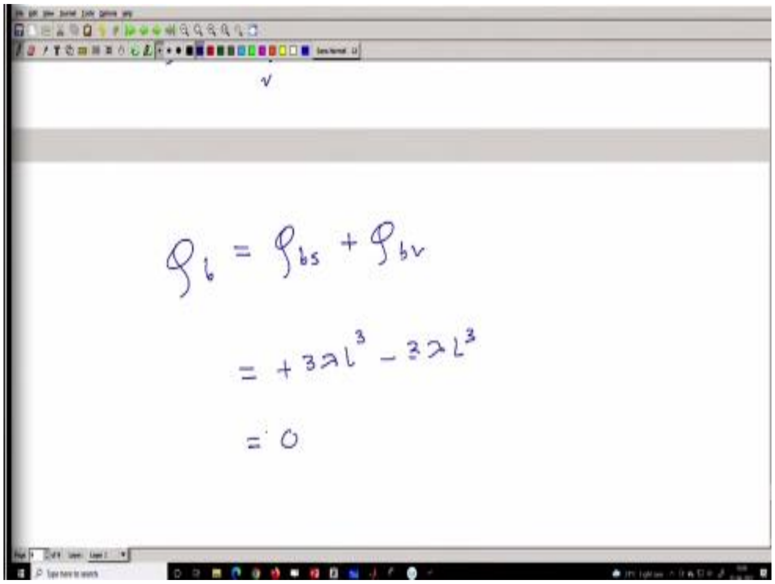

Now interestingly if I try to find out what is the total bound charge then it will be the combination of the bound charge for the surface and the bound charge for the volume. So, this quantity is $+3\lambda L^3$ but this quantity is $-3\lambda L^3$, so they will be going to cancel out and we are going to get 0, which is expected, there should not be any bound charge here. So, we are now running out of time, so with that note I will like to conclude today's class.

So, today we cover almost all kind of problems with the limited time. So, I believe you will go to practice different problem from the books and you can able to do this problem by your own. So, we will have another class and that should be our last class where we discuss some problem related to magnetostatics. So, with that note I will conclude here, thank you very much for attention, see you in the next class.