Foundation of Classical Electronics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology-Kharagpur

Lecture-64
Maxwell's Equation (Contd.)

Hello student to the foundation of classical electrodynamics course under module 4 today we have lecture 65 where we will continue our discussion on Maxwell's equation.

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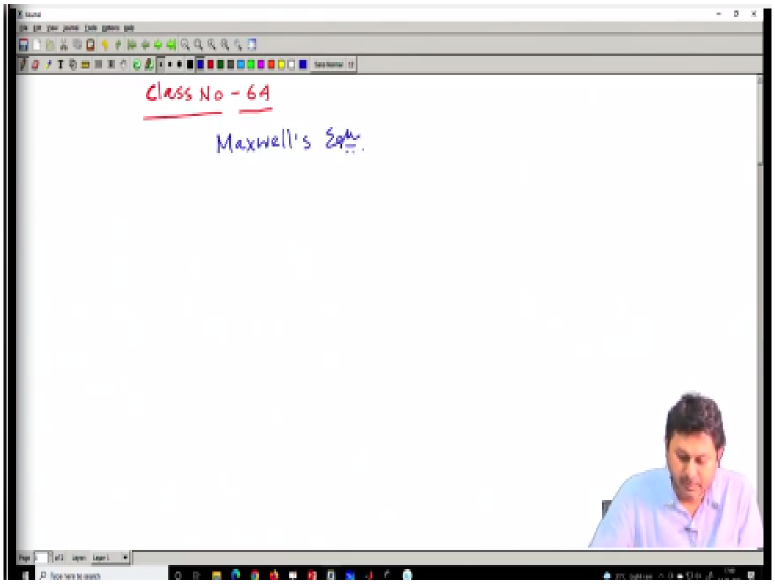

So, today we have class number 64 and our discussion on Maxwell's equation will going to continue, so Maxwell's equation.

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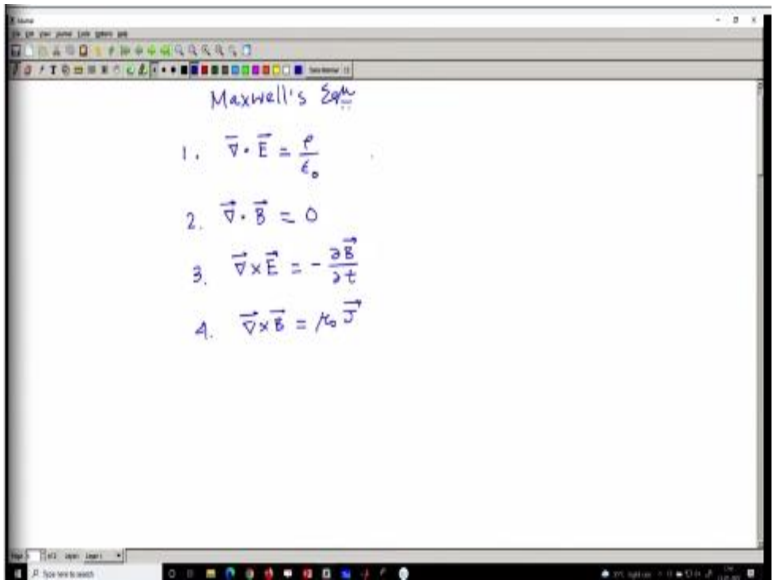

Let me write down the 4 equation that we discussed the first equation is a $\vec{\nabla} \cdot \vec{E}$ that is $\frac{\rho}{\epsilon_0}$ that was the Gauss's law. Second equation was divergence of the magnetic field $\vec{B}$ that is 0 telling that the magnetic monopole does not exist. Third equation was $\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$ Faraday's law and fourth equation was $\vec{\nabla} \times \vec{B}$ that is $\mu_0 \vec{J}$ and that is Ampere's law. And we find that there is a serious problem in equation 4, what was that?

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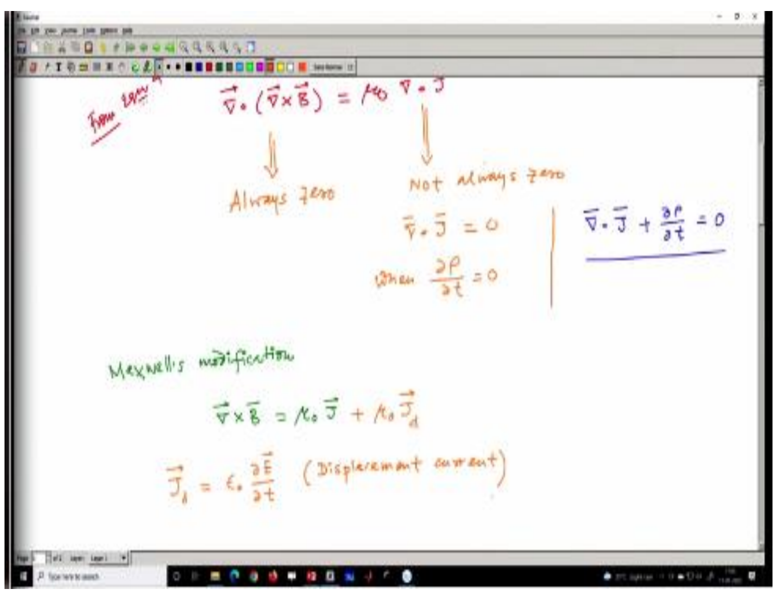

So, if I take the curl from equation 4 what we get? If I take the divergence both the side then I should have $\mu_0 \vec{\nabla} \cdot \vec{J}$. Now the left-hand side is always 0, these is unconditionally 0, so it is always 0. But this is 0, this is not always 0 and it is 0 only for the condition of the steady current, that is when $\frac{\partial \rho}{\partial t}$ is 0. Because these 2 I should write it here that the 2 source term ρ and $\vec{J}$ are associated with this very important continuity equation.

So, they are not independent rather they are related with this equation. So, the $\vec{\nabla} \cdot \vec{J}$ 0 is eventually means $\frac{\partial \rho}{\partial t}$ is 0. Now obviously some modifications were required. So, the Maxwell make this modification, so today we will be going to discuss this. So, Maxwell's modification and the modification is this, the original equation was $\vec{\nabla} \times \vec{B}$ is $\mu_0 \vec{J}$ but Maxwell's added a new term that he says I am adding a term called $\mu_0 \vec{J}_d$.

But this is the Maxwell's contribution where $\vec{J}_d$ is $\vec{E}_0 \frac{\partial \vec{E}}{\partial t}$, how this term appears? I like to discuss this today's class and it is called the displacement current. So, now let us have this term and check that everything is now work properly or not. This equation 4 is now identically left-hand side and right-hand side is valid or not then we will be going to discuss about this $\vec{J}_d$ term.

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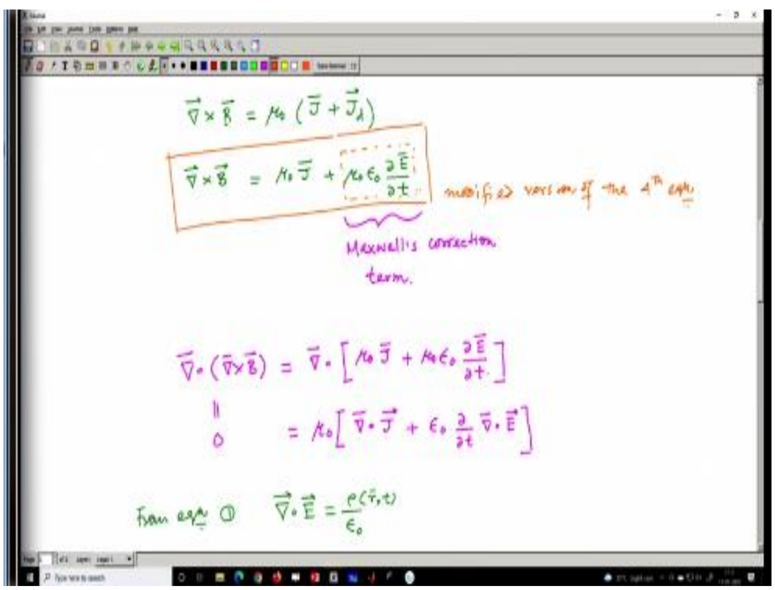

So, we have now after adding this we are now having $\vec{\nabla} \times \vec{B}$ is now we have $\mu_0 (\vec{J} + \vec{J}_d)$ that is the modification we are having and $\vec{J}_d$ is this. So, I can write that the $\vec{\nabla} \times \vec{B}$ is $\mu_0 \vec{J}$ with the addition of $\mu_0\ \epsilon_0 \frac{\partial \vec{E}}{\partial t}$. So, if I go back to the Faraday's equation you can see that we had a similar looking equation here where the $\vec{\nabla} \times \vec{E}$ is associated with the time rate of change of magnetic field. Here we have a similar looking thing the $\vec{\nabla} \times \vec{B}$ is now somewhere depends on the time derivative of the electric field t. So, this is the new term that we are having right now.

Now let us go back to the case that whether the divergence of both the side of the equation is satisfying or not. So, that is the modified version of the fourth equation with Maxwell's correction term and the Maxwell's correction term is this, this is Maxwell's correction term. Now let us check, now we have $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{B})$ in the left-hand side and that should be identically 0 without any condition unconditional this is 0. What about the right-hand side?

The right-hand side we should have divergence of first term we have $\mu_0 \vec{J}$ and then the Maxwell's modification $\frac{\partial \vec{E}}{\partial t}$, so that we are having. So, left-hand side is identical is 0, right-hand side we are going to check what is going on? Now this quantity I can have like μ_0 then $\vec{\nabla} \cdot \vec{J}$, μ_0 I can take common and then we have + ϵ_0 and $\frac{\partial}{\partial t}$, I can switch the operator, divergence operator I can put inside the derivative because they are independent to each other and then I am having this one.

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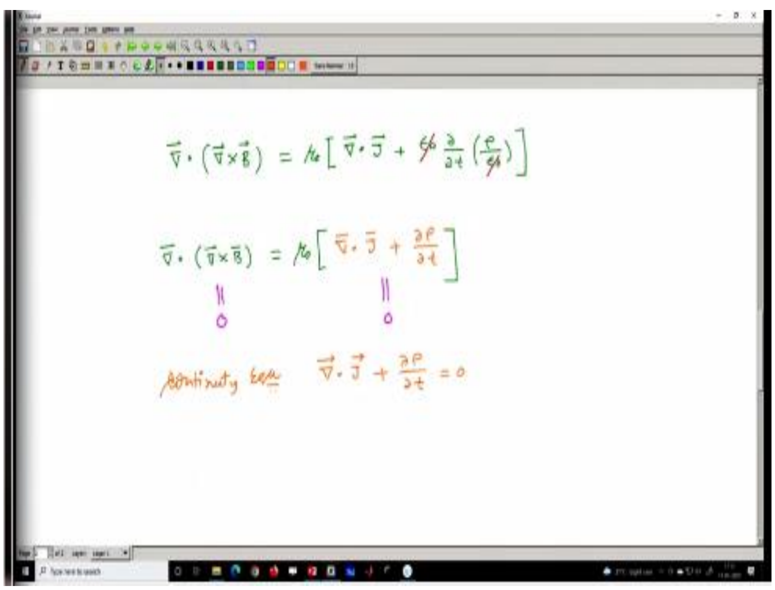

Now what is $\vec{\nabla} \cdot \vec{E}$ from equation 1, which we already wrote here the very first equation the $\vec{\nabla} \cdot \vec{E}$ is $\frac{\rho}{\epsilon_0}$. So, if I write that part what is $\vec{\nabla} \cdot \vec{E}$ that is $\frac{\rho}{\epsilon_0}$, this ρ has to be a function of space as well as time? So, I should now have if I put there in this equation. So, I can have that $\vec{\nabla} \bullet (\vec{\nabla} \times \vec{B})$, which is 0 on the right-hand side we have μ_0 then $\vec{\nabla} \bullet \vec{J}$ + ϵ_0 and then I have $\frac{\partial}{\partial t}$ of this quantity.

So, I eventually have $\frac{\partial}{\partial t}(\frac{\rho}{\epsilon_0})$. So, now ϵ_0, ϵ_0 will be going to cancel out and eventually the thing we are getting is very interesting and I should write it here. Like $\vec{\nabla} \bullet (\vec{\nabla} \times \vec{B}) = \mu_0$ then I write here the equation in different colour that + $\vec{J}$ + $\frac{\partial \rho}{\partial t}$. Now this is continuity equation. And we know that this equation is always 0, so the left-hand side is always 0, now after this modification done by Maxwell very important modification this is the modification we are looking for.

So, after adding this term what happened that the left-hand side and right-hand side now both become identically 0, unconditionally 0? Because continuity equation I should write equation saying that $+\frac{\partial \rho}{\partial t} = 0$, that is the continuity equation and it is there. So, that means a change of electric field induces a magnetic field that is the physical outcome we are having.

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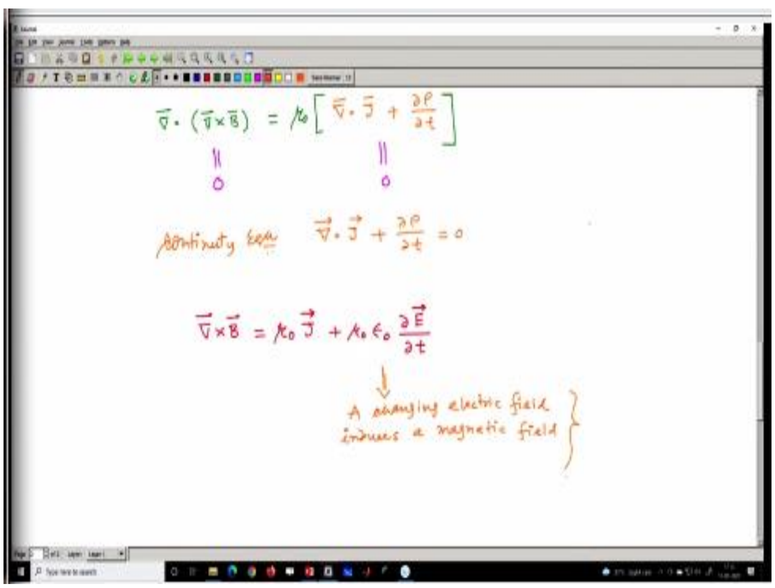

So, finally we are having the equation that $\vec{\nabla} \times \vec{B}$ is $\mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$ and that basically this term is saying that a changing electric field induces a magnetic field. Now let us try to understand physically the concept of this displacement current that we put here. We put here and as a adhoc like addition and then we find that after putting this addition we have nicely matched left-hand and right-hand side when we take a divergence both the side.

So, the left-hand side is unconditionally 0 and right-hand side also leads to equation, which is unconditionally 0. So, now we will be going to understand the concept of displacement current.

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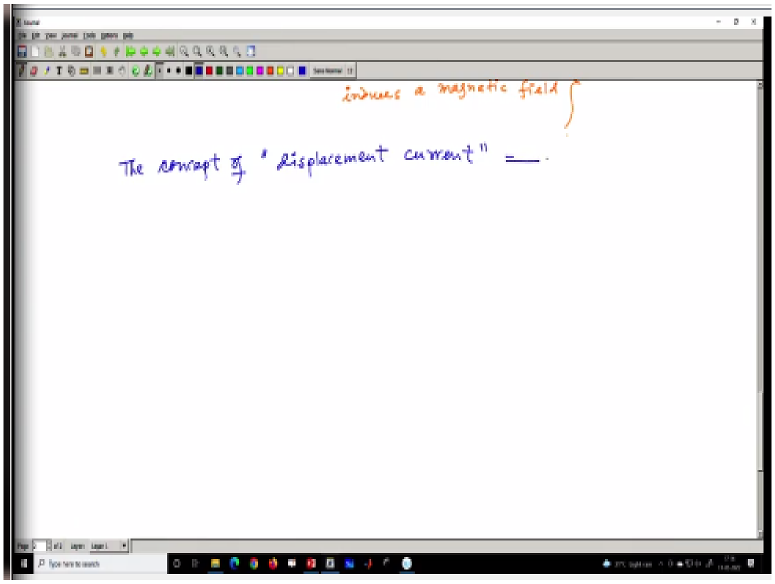

Now one can understand this is a very standard way to understand this displacement current and that is the capacitor problem. So, let us consider a model where a capacitor is charged.

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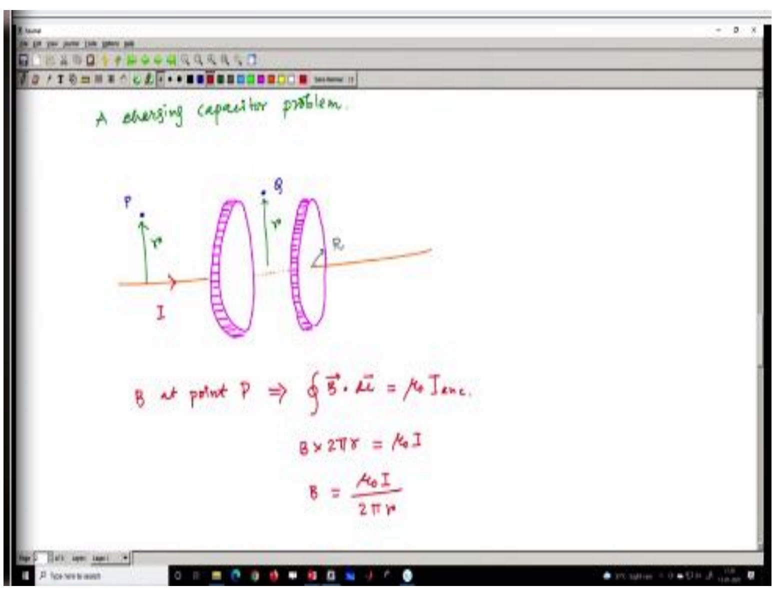

So, charging capacitor problem, what we have in charging capacitor problem? Suppose I have a 2 capacitor plate like this, so this is one capacitor, we are having this is a capacitor plate, circular capacitor plate say, so they are now in charging condition. So, let us define few things, let us define this radius to be R and let us say the current that is flowing through this capacitor is I, this is the amount of current that is flowing.

So, what happened that when the current is there? So, this capacitor will going to be charged up. Now the point is if I try to find out the $\vec{B}$ at point P, so here at this point suppose I want to find out the magnetic field. And we know that when we have a current carrying wire it should produce some magnetic field around it. And if this length is say r, then what should be the magnetic field that is produced here in this region? So, if I calculate so suppose this is a point P, so this is my point P.

So, I should find out that magnetic field $\vec{B}$ at point P is how much? I can use the Ampere's rule that the $\oint \vec{B} \bullet d\vec{l}$ = μ_0 and the current enclosed that is my rule. So, here the amount of current enclosed is I and the dl is 2πr, so I simply have B multiplied by 2πr = μ_0, the current that is flowing through the wire and I have B = $\frac{\mu_0 I}{2\pi r}$, that should be the value of the magnetic field here in at point P.

Now if I want to find out the magnetic field in between this region, so I want to find out now the magnetic field in between this region here say a distance r and let us define a point here Q. Then what should be the value of this? Let me, this is not a continuous, so I should make a dotted line here and from here it is r. So, what should be the field? So, $\vec{B}$ at point, so let us first try to calculate one by one.

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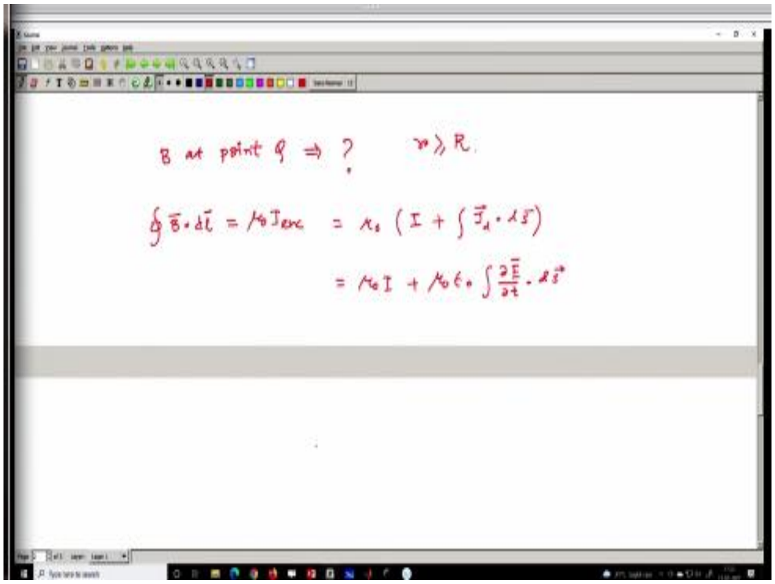

So, I want to calculate here, $\vec{B}$ at, the question is what is the magnetic field at point Q? What is this value? Where r is say greater than equal to R, outside so this is r, so r is greater than the radius of this circular plate. So, if I calculate I should get $\vec{B} \cdot d\vec{l}$ and that should be equal to μ_0 $I_{enclose}$. And now you can see that there is no current that is enclosing. So, in principle the current is here and then it will going to charge up the capacitor, so there should not be any current.

So, that means in principle it should be 0, so if it is 0 then obviously there should not be any magnetic field present here, that is a little bit weird. Because here we are having the magnetic field and this side also I suppose to get the magnetic field because the current will be there but in between the capacitors certainly the magnetic field drops, certainly in the magnetic field 0. So, obviously this is not feasible, obviously there should be some issue related to that and that we are going to resolve here.

Because we mention a displacement current, so that means when the current is there the plate is gradually charged up. And before going to a saturation state gradually it will go to charge up, so the electric field in between this region is a time varying, there is a time varying change of the electric field. And as a result what happened that I should get the displacement current. And this displacement current should be one should take account, so let us take account both the current now.

So, it should be μ_0 and consider this I current is also there which it should be I plus the displacement current. So, displacement current density because I am taking the current, so I should write current density dot $d\vec{s}$ that should be my displacement current. And this quantity is μ_0 I, if I is there say in general I am just writing then μ_0 ϵ_0, I should integrate it and it should be $\frac{\partial \vec{E}}{\partial t} \bullet d\vec{s}$. Because that is the way I define displacement current. Now, so electric field $\vec{E}$ is here, so I should know what is the electric field because we are dealing with the capacitor.

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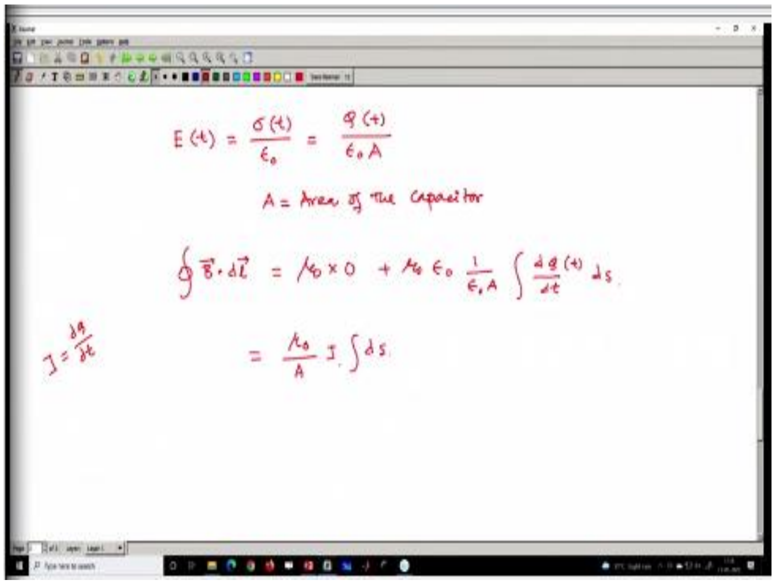

So, the electric field $\vec{E}$ here is which should be a function of time should be the surface charge density, which is gradually increasing. So, I should write this σ as a function of t divided by ϵ_0. That quantity σ is again the total charge, which is also changing with respect to time divided by ϵ_0 and the area of this capacitor. So, A here is area of the capacitor. So, what should be the value of the $\vec{B}$? So, $\vec{B}$ that is simply if I want to find out $\vec{B}$ then I should use this expression.

So, I should have the $\oint \vec{B} \bullet d\vec{l}$ and then I should have μ_0 and I am saying the current is 0, so I should have 0. Because I current is not there, this is my general expression, so I current is not there. So, rest of the term if I write it should be plus μ_0 ϵ_0 then $\frac{1}{\epsilon_0 A}$. And then we have $\int \frac{dQ(t)}{dt}$ and then ds.

So, this portion is 0, so ϵ, ϵ going to cancel out, so we have $\frac{\mu_0}{A}$ and this quantity $\frac{dQ}{dt}$ is my current that is already flowing. I, which is simply $\frac{dQ}{dt}$, this is the current that is flowing through the circuit, so I should have simply I here and then integration ds, this integration ds is over this capacitor area, which is also known. So, eventually, so I should write this I as I_c to distinguish whether this is the current that is flowing through the wire, so I_c should be this.

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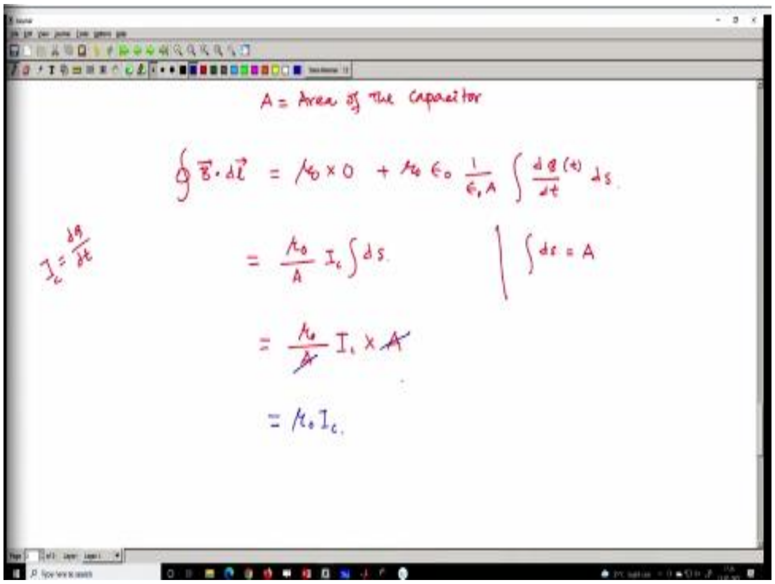

And ds is A, so integration, note, $\int ds$ is A, so I should write it, so it is simply μ_0 A and then I_c multiplied by A again, A A will going to cancel out and eventually we have μ_0 I_c. Exactly the same value one supposed to have.

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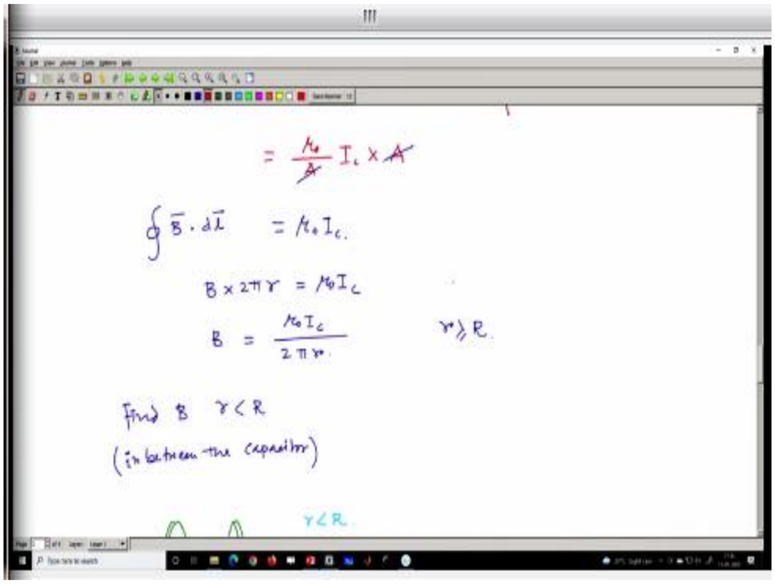

So, now the left-hand side again, left-hand side is $\oint \vec{B} \bullet d\vec{l}$ and I want to find out at r. So, it is B multiplied by $2\pi r$, which is μ_0 I or I_c and that value, so that means B from here is simply, so I should write μ_0 a little bit lower because I did not put the denominator here. So, it is simply μ_0 I_c, so B is now μ_0 I_c or I divided by $2\pi r$, that is the value, that is eventually the same value that we are having.

So, this is the value when r is greater equal to R. So, that means whatever the magnetic field I am getting here the same magnetic field one should get by considering the displacement current concept.

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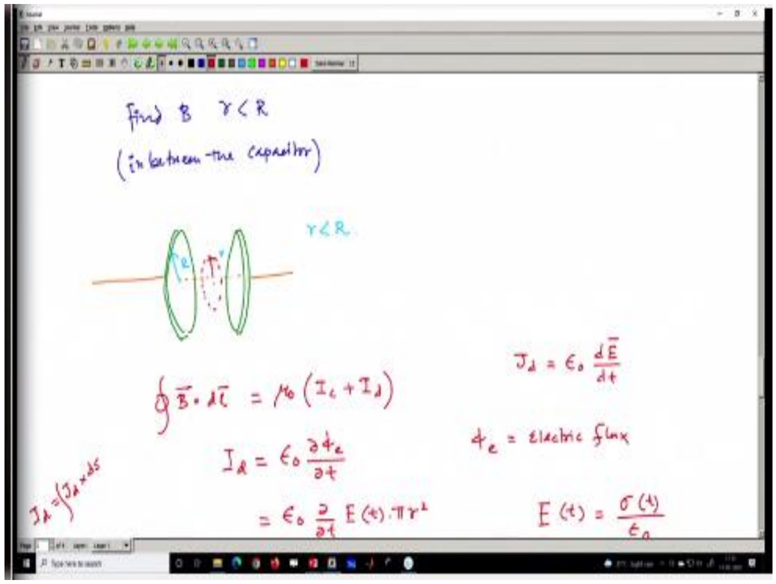

Now another question is there what should be that the find B when r is less than R? That means in between the capacitor.

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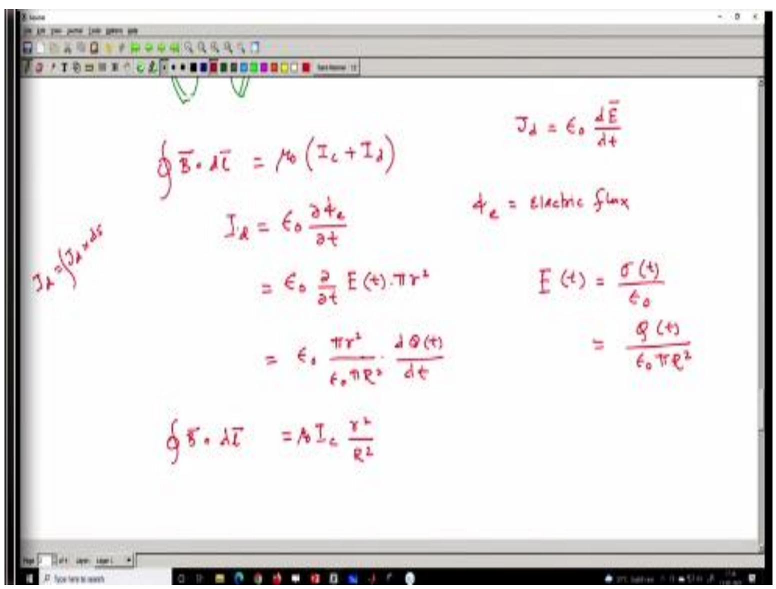

So, if I draw quickly what we had, so we had 2 capacitor like this and suppose this is the, so current is flowing like this, so I want to find out, so what we want is? We want to find out a region here, if I make a circle, so note that this is R and it is r, so r is less than R. Again if I should calculate like this, so $\oint \vec{B} \bullet d\vec{l} = \mu_0 (I_c + I_d)$, in general I_d is a displacement current, I_c is the current that is conducting, I_d is ϵ_0 and then displacement current is del.

So, this displacement current, so J is μ_0, so I should now I_d in I should write in terms of flux, so it should be the electric flux divided by t where ϕ_E because I had $J_d = \epsilon_0 \frac{dE}{dt}$, E is an electric field and if I want to find out in terms of flux then it should be the total current is J multiplied by the, so I_d here is J_d multiplied by the area ds and if I integrate it over. So, this area if I multiply with the electric field, so electric field multiplied by the area should be the corresponding flux.

So, that is why I can write it in the form of flux, so it is electric flux. So, here I should write it is ϵ_0 and then $\frac{\partial}{\partial t}$ and I should have E here flux in terms of electric field multiplied by the area and this area should be πr^2, that should be πr^2. And then I should have E, let me find out E then, E here which is a function of time is $\frac{\sigma(t)}{\epsilon_0}$. So, that quantity I should write in terms of Q, Q as a function of t divided by ϵ_0 and the area. And this area has to be the total area, so I should have πR^2.

So, here it is $\epsilon_0 \frac{d}{dt}$ of that thing, I should write as πr^2 and here we should have ϵ_0 multiplied by πR^2. And then I have dQ, which is a function of t and dt. So, this eventually gives me I_c multiplied by $\frac{r^2}{R^2}$. So, from here, so left-hand side is still have $\vec{B} \bullet d\vec{l}$, so if I want to execute the value of $\vec{B}$.

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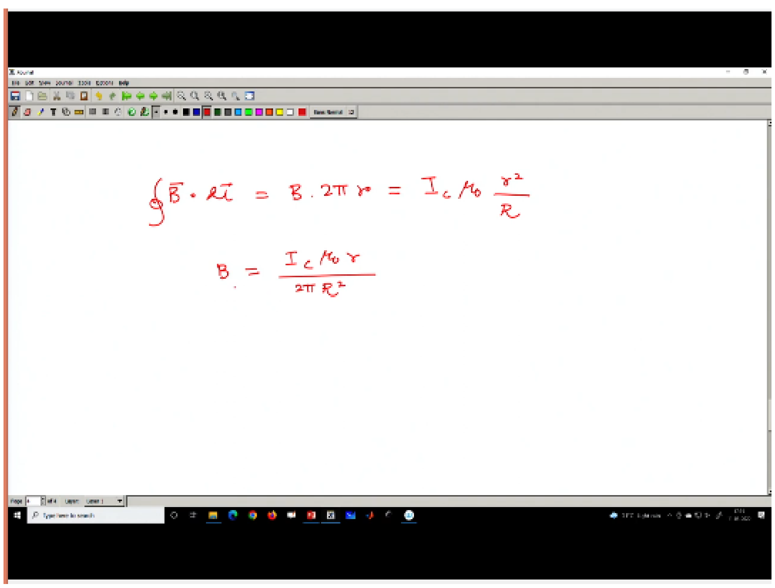

It should be $\vec{B}$, so $\oint \vec{B} \bullet d\vec{l}$ can be written as B into $2\pi r$ and that is I_c and then it should be μ into this quantity. I should have a μ here because $\vec{B} \bullet d\vec{l}$ is μ_0 and this quantity I_d I am calculating so far, so I_d multiplied by μ is there and I_c is 0 in between. So, it is $2\pi r$ I_c and then $\mu_0 \frac{r^2}{R}$. So, I should have B is simply I_c and then $\frac{\mu_0 r}{2\pi R^2}$.

So, this is the way the B will going to change, B as a function of r if I plot. So, you can see that in this point B 0 and gradually if it goes then this value is gradually linearly increasing at exactly at this point, it should be exactly at this point at the edge of this, it should be $\frac{\mu_0 I_c}{2\pi R}$. And when it reaches the higher value then accordingly it will going to change. So, anyway the point is we have an understanding now that when the capacitor is charging.

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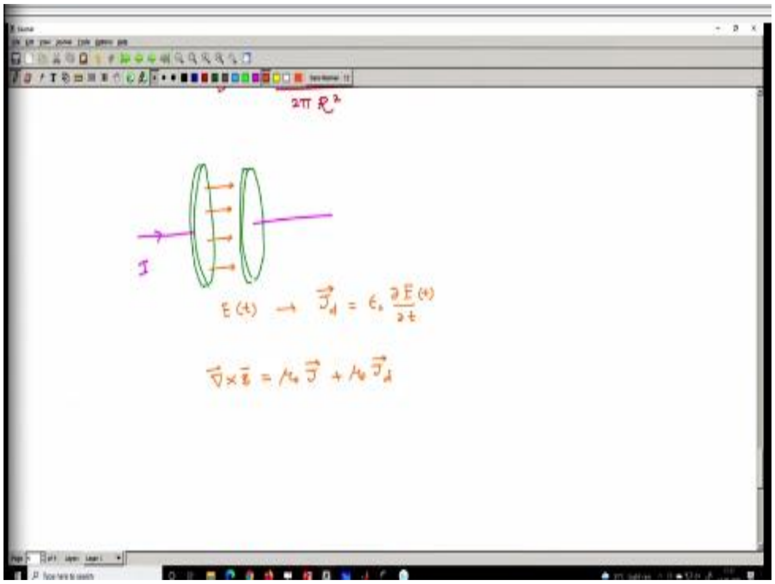

So, let me summarize, so we had 2 capacitors here like this. So, 2 plates are there in the capacitor and they are charging by a current I. So, when they are charging there is a change of the electric field, so we had electric field in between this region and this electric field is a time varying electric field. And this time varying electric field gives rise to something called the displacement current density J_d, which is simply ϵ_0 and the rate of change of the time varying electric field.

And this basically gives rise to the magnetic field with this formula. The magnetic field not only now produced by the normal current but also there is a contribution of the displacement current that is producing the magnetic field. And the displacement current is associated with the rate of change of the electric field that one can understand how it is produced in between this region in between 2 capacitors when it is charging.

So, this is the way one can understand the displacement current. So, today I do not have much time, so I like to conclude my class here. So, in the next class I will discuss more about the Maxwell's equation in detail and try to understand these constants, not constants rather I should discuss about the gauge the constant and how to put this constant. And then understand that how one can write the electric and magnetic field, how the information from the Maxwell's equation can be extracted in the form of vector and scalar potential. So, with that note I would like to conclude, thank you very much for attention and see you in the next class.