Foundations of Classical Electrodynamics
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Lecture-61
Electromagnetic Induction

Hello student to the foundation of classical electrodynamics course. So, today we will be going to start our module 4 that is the last module of this course. And today is lecture number 61 and in this lecture we will discuss about the electromagnetic induction.

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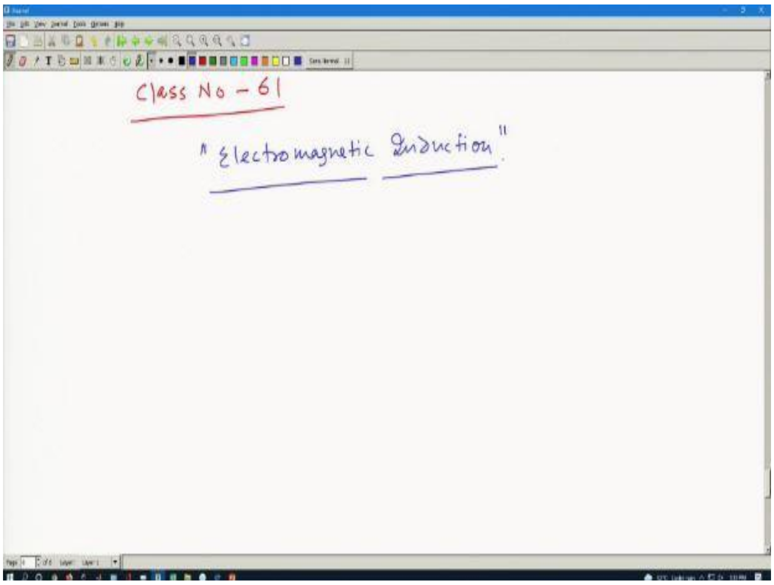

So, we have class number 61 today. And today our topic is electromagnetic induction. So, this is the first time we mention the terminology electromagnetic. So, far we are discussing about the electrostatic and magnetostatic, but never mention anything about the electromagnetic. So, now in this section we will discuss everything in terms of electromagnetic taking the consideration of the electromagnetic theory.

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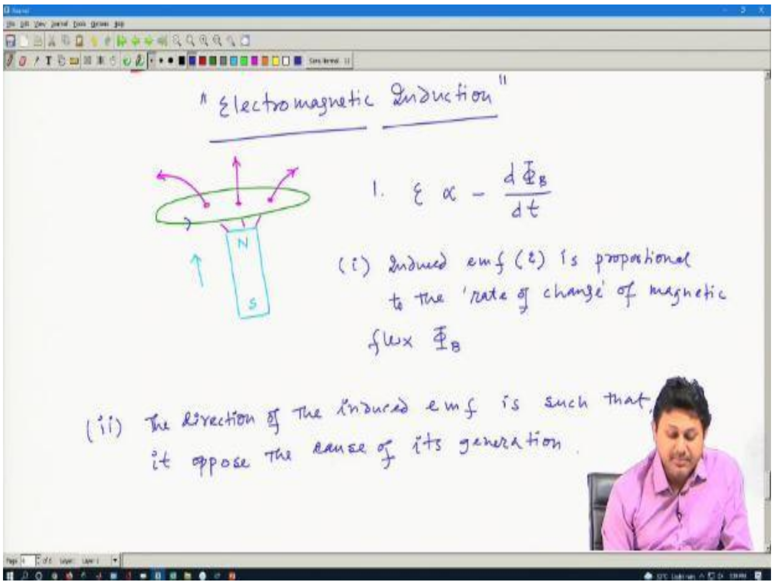

So, the first step of these problems starts with this very well-known problem well-known phenomena rather. So, suppose I have a current loop and I have a bar magnet. So, this is my bar magnet N and S. And I am throwing the bar magnet through this loop. So, the bar magnet is moving along this direction and because of that what happened. If I push through this bar magnet then there will be a flux change, because the bar magnet should have some magnetic field line and that will be going to cross this closed loop, whatever the loop is there.

So, we should have a field line, those will be going to cut this loop. Let us draw in this way. So, suppose this is the way they are going to cut this loop. So, there will be a change of the field line, there will be a change of the flux and it will be going to change with respect to time, because these things are in motion. So, that means if I write, so what happened? So, let us first discuss what happened then I will be going to write these things.

So, because of the change of this field line there will be induced EMF. The EMF will be going to induce. And that EMF should be proportional to the rate of change of these field lines, so, in other way the magnetic flux. So, the rate of change of magnetic flux leads to the generation of some kind of EMF and one can expect some kind of current here, because of the EMF. So, the first thing the expression that should I write here.

Is the generated EMF is proportional to $-\frac{d\phi_B}{dt}$, rate of change of flux it is proportional not only that there is a negative sign. So, one by one I should write. So, this is the expression. So, first thing I should write that some EMF will going to induce that give rise to some amount of current alongside this direction. So, the induced EMF is proportional to the rate of change of magnetic flux, which is ϕ_B here.

The rate of change of magnetic flux that is the first observation we had. What about the second one the negative sign. The second thing is the direction of the induced EMF is such that it opposes the cause of its generation. So, I am throwing a magnet here and what happened, that some EMF will going to produce here. And we know that when we have an EMF some current will flow and this flowing current can produce its own magnetic field.

And the magnetic field should be such that it will be going to oppose the motion of these bar magnet. Why it is that, because of the energy conservation. If that is not the case, then what happened? If we just throw and the EMF that is generate in such a way that the magnetic field that is produced due to the current, of this wire can help this magnet to pass through the system.

Then we can have more and more flux change here and more and more EMF. So, that means we are gaining a much amount of energy without doing any work. So, that is the violation of the energy conservation. So, that means in order to generate some EMF we need to do some work. That means we need to throw this magnet, against the magnetic field that is produced by the cause of the rate of change of this magnetic flux inside this wire.

Inside this wire what happened, there will be a current and that current can produce a magnetic field itself. And the direction of the magnetic field should be such that it opposes the motion of this magnet. So, that means we need to do some kind of work that is the physical significance of having this negative sign.

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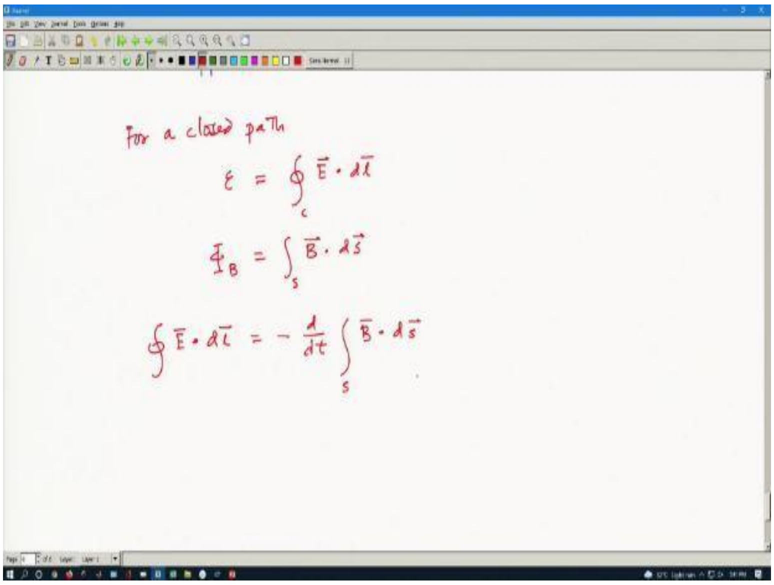

Well for a closed path, so, now let us do some mathematical stuff here. So, for a closed path, so the EMF that is generated I can write because, this is a path is closed that whatever the electric field integrated over the line. And this is over the closed path c and what about the flux? My flux ϕ_B, rather magnetic flux is integration of $\vec{B} \cdot d\vec{s}$ magnetic field into area. This is over the surface.

Now according to this law I have $\vec{E} \cdot d\vec{l}$, it is EMF, which is equal to $-\frac{d}{dt}$. Let us consider this proportionality constant to be unity. Then it should be the surface integral of $\vec{B} \cdot d\vec{s}$. I am writing everything in the mathematical. So, this is also mathematical form, but writing in terms of electric and magnetic field. So, you can see that, there is already a relationship of the electric and magnetic field. That is the first time we are having this relationship.

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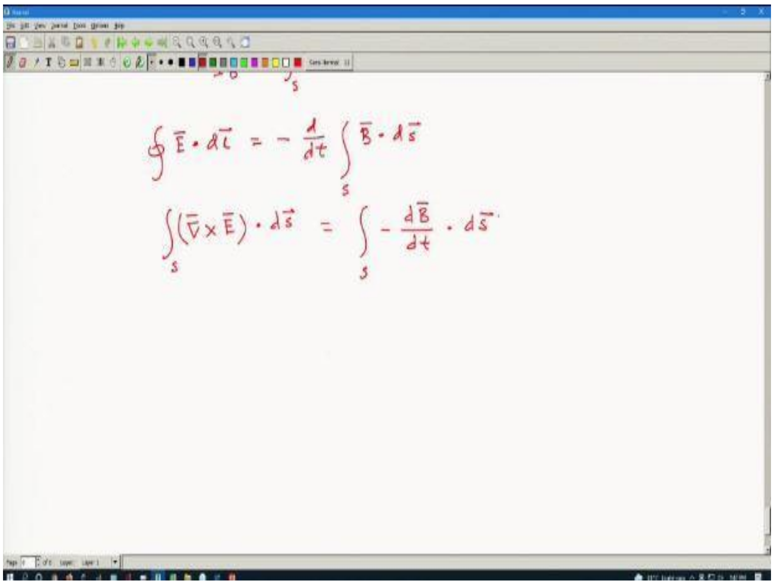

So, this I can write using the Stoke's law like $\vec{\nabla} \times \vec{E}$ and then dot d$\vec{s}$ over this surface integral that the close circle is enclosing this surface and the right-hand side I have the surface integral $-\frac{d\vec{B}}{dt} \cdot$ d$\vec{s}$. So, this is true for any kind of surface, any kind of loop.

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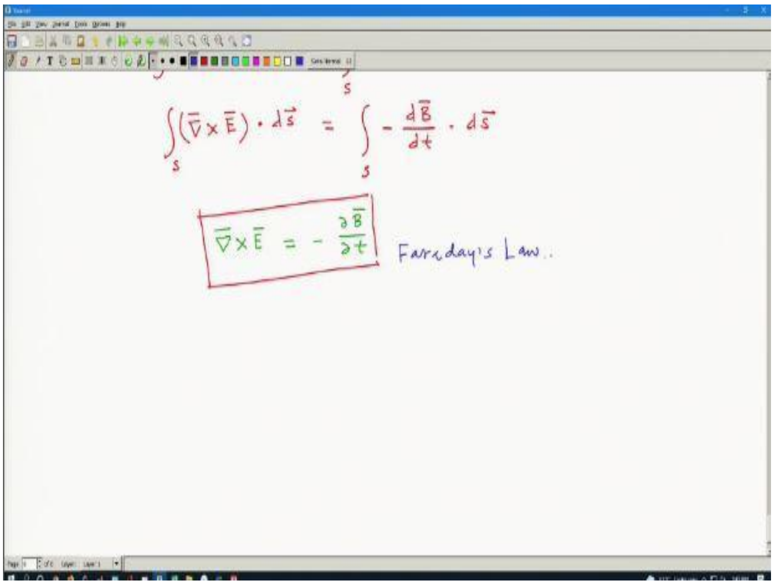

So, I can write from this expression. I can write a very important equation that $\vec{\nabla} \times \vec{E} = -\frac{d\vec{B}}{dt}$. Mind it in electrostatic, we find that $\vec{\nabla} \times \vec{E}$ was 0, but here this is not the case. We find that $\vec{\nabla} \times \vec{E}$ is not zero. This is $\frac{d\vec{B}}{dt}$, but here we have something extra and that is the corresponding magnetic field. So, this gives rise to a new law and that is the famous Faraday's law.

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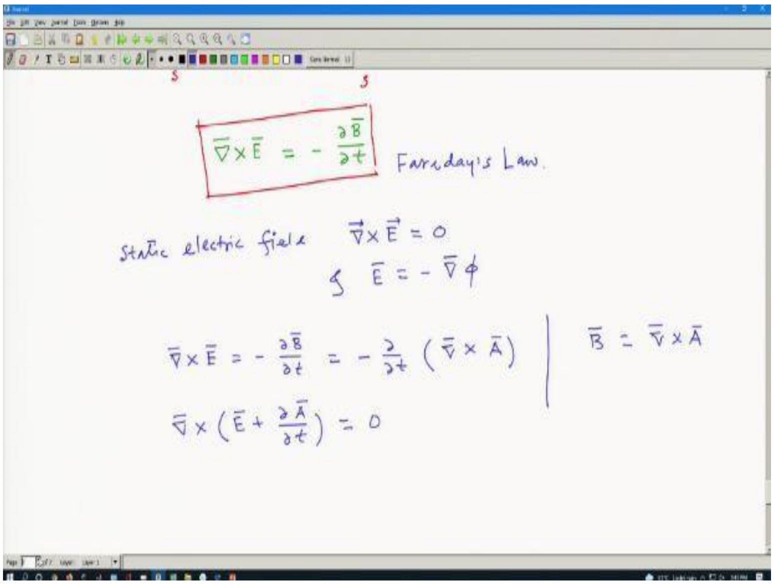

So, as I mentioned that for static electricity, so let me put this again here. For static electricity, what we get is $\vec{\nabla} \times \vec{E}$ to be 0 that was our expression for static electric field. And now so that and because of that what we get that $\vec{E}$ can be written in terms of a potential and that potential is $-\vec{\nabla}\phi$. This is a scalar potential scalar and I can write $\vec{E}$ in terms of the scalar potential with the gradient of that scalar potential with the negative sign and that is the way we define $\vec{E}$.

Now from Faraday's law what we get? That $\vec{\nabla} \times \vec{E}$ is no longer zero rather it is $-\frac{d\vec{B}}{dt}$ and that is again $-\frac{d\vec{B}}{dt}$, I can write in terms of the vector potential $\vec{A}$, so it is $\vec{\nabla} \times \vec{A}$, as $\vec{B}$ can always be written in terms of $\vec{A}$ like $\vec{\nabla} \times \vec{A}$. $\vec{A}$ is a vector potential. So, that gives us $\vec{\nabla} \times (\vec{E} + \frac{d\vec{A}}{dt})$ that to be 0. So, previously we had $\vec{\nabla} \times \vec{E} = 0$.

Now we are having an expression, where it says that $\vec{\nabla} \times (\vec{E} + \frac{d\vec{A}}{dt}) = 0$. So, now I can write that $\vec{E} + \frac{d\vec{A}}{dt}$ t to gradient of some scalar field. So, that means that gives that leads to an expression.

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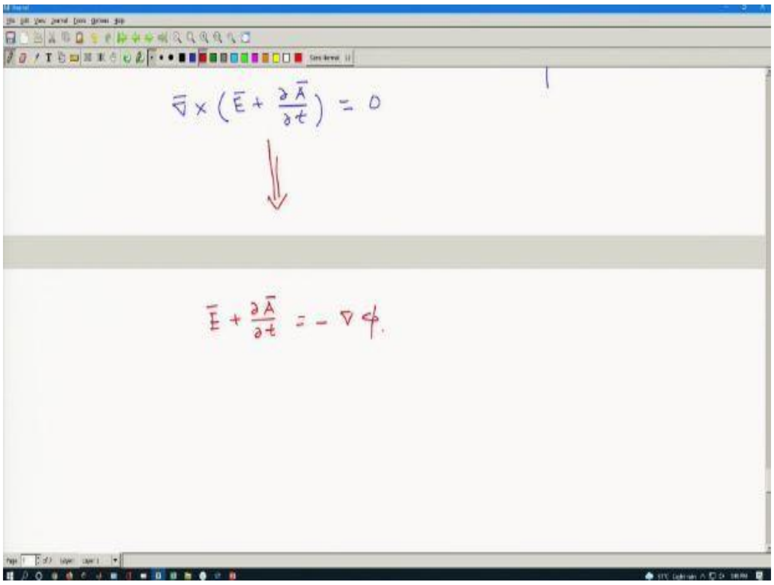

So, these things can leads to an expression that $\vec{E}$ + $\frac{d\vec{A}}{dt}$ this quantity, now can be written as a gradient of a scalar field ϕ.

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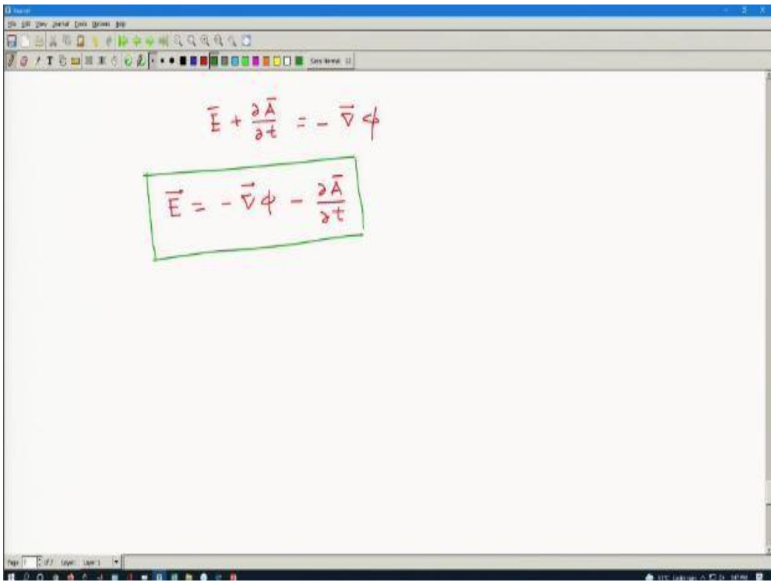

And then the electric field $\vec{E}$, now modified previously for electrostatic I have $\vec{\nabla}$ϕ. Now when we have the magnetostatic I mean the magnetic field here**.** So, it gives me another term like this. So, now $\vec{E}$ is constructed not only scalar potential, but also a vector potential. So, previously we know that it can all only produces from a scalar potential, but in general this is the expression of the $\vec{E}$ in terms of scalar potential ϕ and the vector potential $\vec{A}$**.** Now let us try to find out few values, I mean if you do some few examples to understand this, whatever the discussion we had so far.

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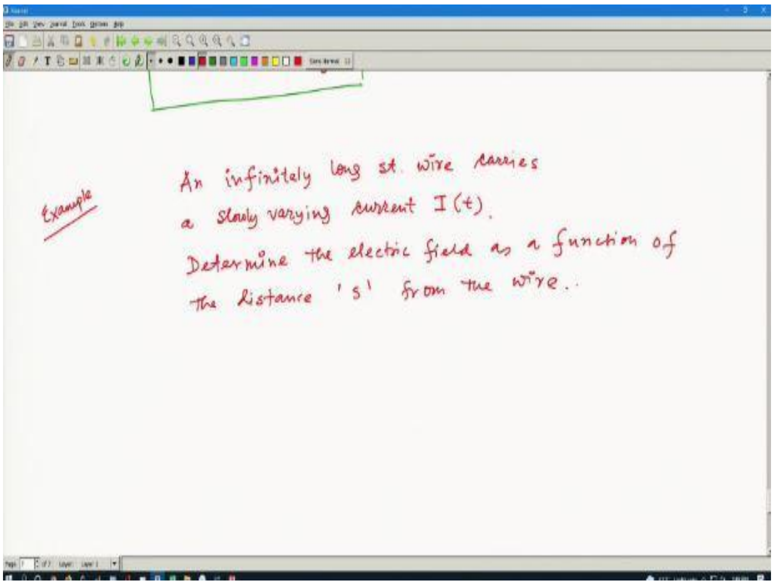

So, the example is an infinitely long straight wire carries a slowly varying current I(t). The question is determined the electric field as a function of the distance s from the wire. So, this is a problem very straightforward problem. So, let us try to understand this problem.

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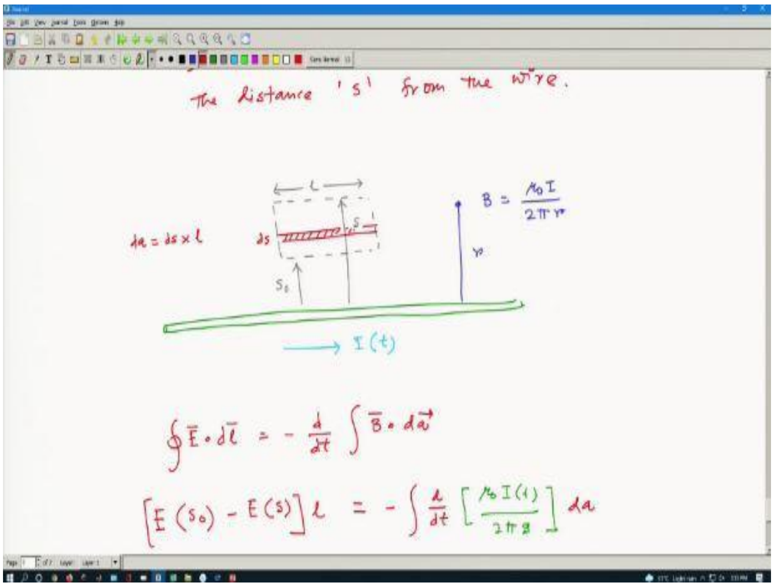

Suppose, I am having a current carrying wire here infinitely extended current carrying wire and then the current that is flowing through this wire I that varies slowly with respect to time. So, if that is the case what happens? It produces a magnetic field here and that magnetic field also should vary with respect to time. And we know what is the amount of magnetic field is going to produce at some distance.

So, let us first have a loop here a rectangular kind of loop with say length l and from here to here say distance is say s_0 and from here to here it is s. The B that is produced at some point we know let me do it here. At some point here the B that is produced is $\frac{\mu_0 I}{2\pi}$. If this is r, then it is r. So, the question is what should be the amount of the electric field there not the magnetic field.

So, we can we need to extract the information of the electric field from the expression where we have the relationship with $\vec{E}$ and $\vec{B}$, so, that is the problem here. So, we know what is the magnetic field here and also the relationship with the electric field and magnetic field and that relationship if I exploit here. So, it is $\oint \vec{E} \bullet d\vec{l}$ so the loop is already there.

So, whatever the EMF effect will going to generate, because of the change minus of the change of the flux. And here we should write $\vec{B} \bullet d\vec{a}$ or $d\vec{s}$. Let us write $d\vec{a}$ because, s we are already going to use. So, the electric field that is produced here if I just simply integrate this close line integral. So, the electric field at the point s_0. This is a dot product so perpendicular direction will simply cancel out.

And the electric field at s and the length is l, so it is multiplied by l. On the other hand, I should have $-\frac{d}{dt}$ and the value of the $\vec{B}$ is this one. Here so let me write it. This is μ_0 I that is a function of t and then 2π. Say since, we are having a distance s. So, we need to calculate the distance at s. So, the function of s, so, I should put it as s and then surface da.

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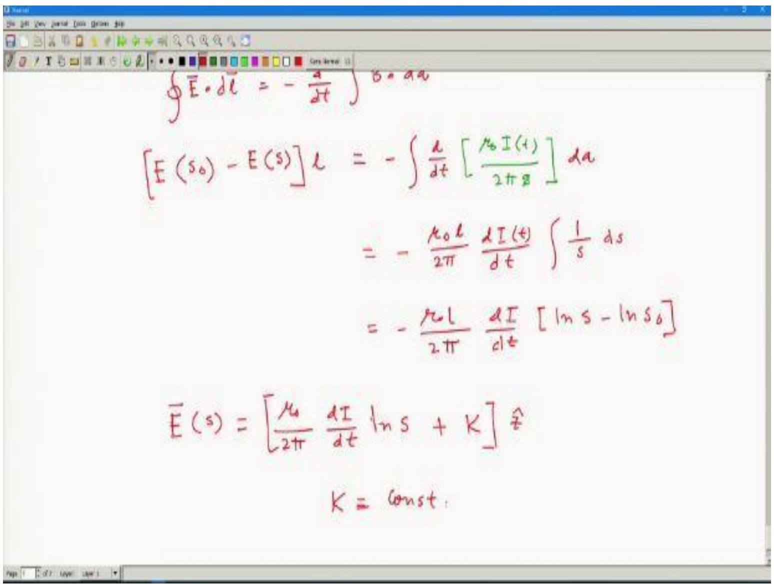

So, what we have here, then so this quantity if I write. So, μ_0 is constant it can come out. Then what else the area, so da is the area. So, area is length into distance length. So, l into s. So, that means I have $\frac{\mu_0 I}{2\pi}$, which should be here and the rate of change of current that also I can take outside the integral. Inside the integral I have $\frac{1}{s}$ and the area is l into da small amount of area if I take here like a small amount of area at a distance.

So, from here to here is s and this is the area s is a variable here. So, this is the area and this is my ds. So, it should be simply ds, the area here is ds multiplied by, so, what is the area? da here is ds multiplied by the length l, this length. Now if I simply integrate then I am going to get the result. So, it should be $-\frac{\mu_0 I}{2\pi}$ and then $\frac{dI}{dt}$ and it seems to be a log function.

So, I should have simply $\ln s$ - $\ln s_0$ the reference distance. So, in the left-hand side I have $[E(s_0) – E(s)]$ l and in the right-hand side I have this quantity. So, eventually this $\ln (s_0)$ and E(s) E at s_0 it is I can put it in a constant. And I can simply write the variation E as a function of s, should be simply l going to cancel out. So, I can have simply $\frac{\mu_0}{2\pi}$ and then the change of the current, the explicit form of the current how it going to change is not given.

So, I simply have this. And then I have ln s and a constant K. And E will be along z direction. So, let us put in this way, where K is a constant. K is something, which is constant**.** So, this is the way one can calculate by exploiting this Faraday's law the electric field. Another example I can give another very straight forward example.

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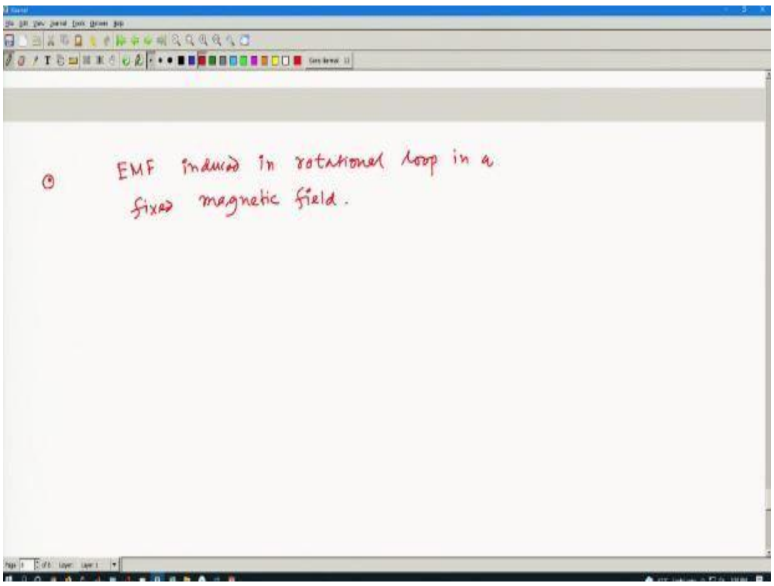

Like to find the EMF induced in rotational loop in a fixed magnetic field. So, a very common problem.

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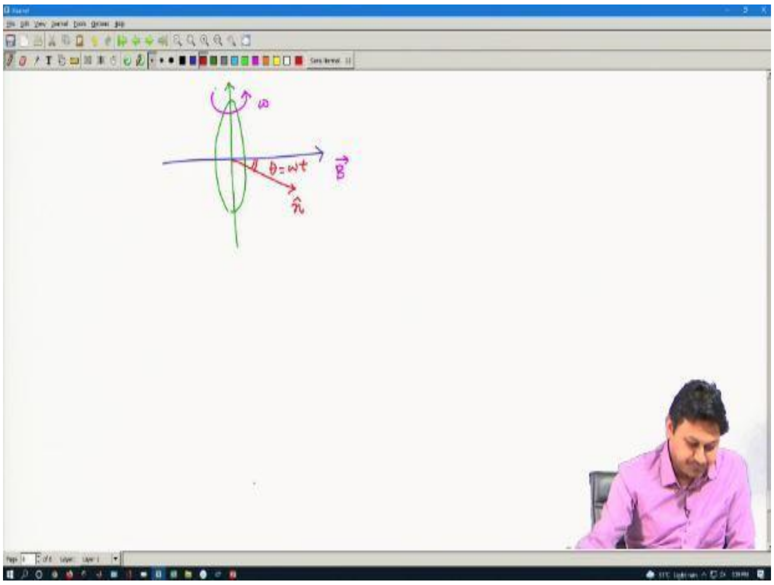

So, suppose I have so what is the definition? What is the problem? The problem is I have a magnetic field here fixed magnetic field, but I am having a ring here, which is rotating. Suppose this is the axis through which it is rotating, it is rotating in this way. So, it is rotating and this is the amount of the B. So, when it rotates what happened? That every time there is a change in the flux is rotating with respect to time.

Suppose, it is rotating with a angular frequency ω. So, when it is rotating what happened? That every time there is a constant change of the area that leads to constant change of the flux and that is why EMF will going to generate. So, if this is my B at any moment, the area of the say this is the direction of the area at any instant. And this is area vector $\hat{n}$ and this angle at that moment is θ and that instant it is ωt, because ω is angular frequency.

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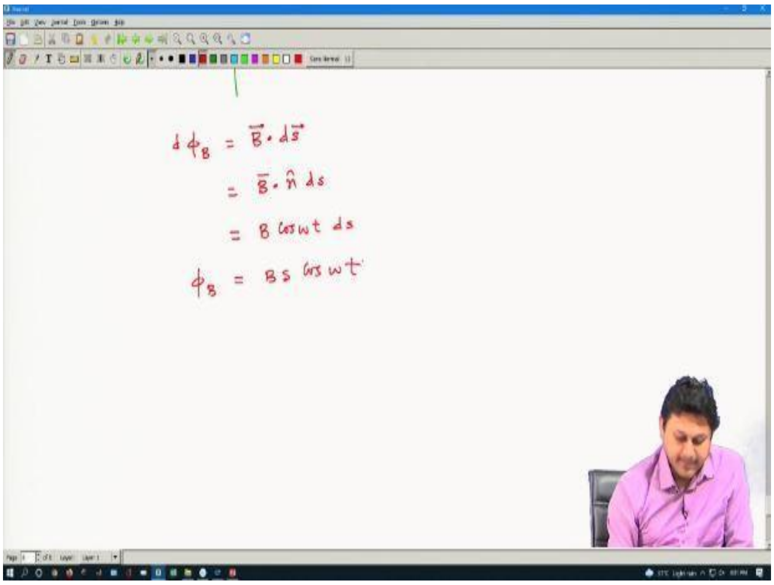

So, the ϕ_B for that system ϕ_B the magnetic flux will be $\vec{B} \bullet d\vec{s}$. So, that means it is $\vec{B} \bullet \hat{n}$ ds. So, $\vec{B} \bullet \hat{n}$ ds is simply the angle between $\hat{n}$ and $\vec{B}$ is ωt. So, it should be B and then it is cos ωt. So, this then multiplied by ds. So, if I want to find out what is my total then I need to integrate over this entire surface. So, it should be B multiplied by the surface say s and cos ωt. So, what should be the value of the EMF?

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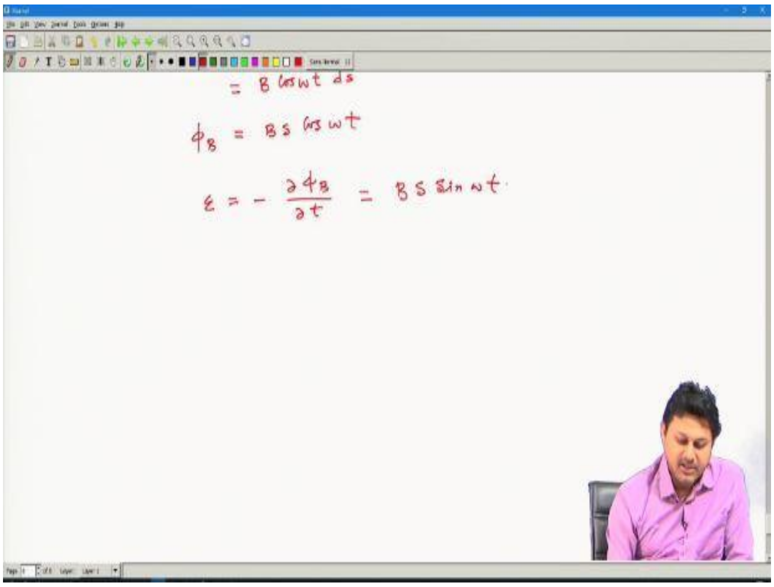

The EMF we know it should be minus of del total flux del t. So, I should have an expression like B s and then sin ωt. So, there will be a sinusoidal variation of the electric field. Next another problem I can consider before ending let us consider that as well.

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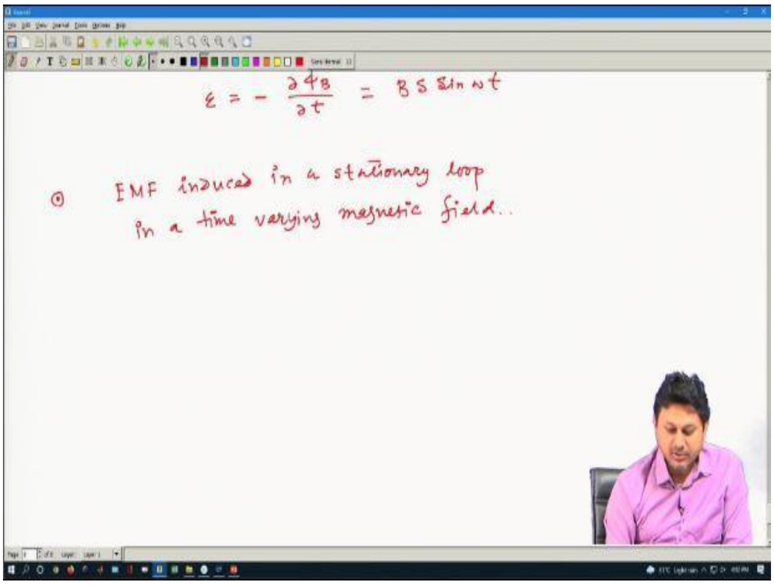

And that is the EMF induced in a stationary loop in a time varying magnetic field. A similar problem but, here only thing is that previously the loop was rotating and now the magnetic field is changing with respect to time.

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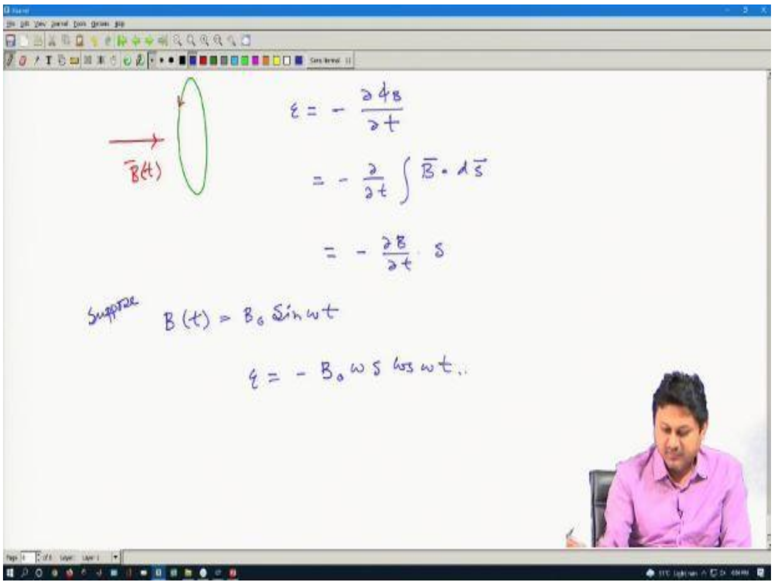

So, again so the loop if I draw is this. And I now have my magnetic field and that is a function of time that makes the flux as a function of time. So, this is the initial problem we have. We had suppose the electric field is generated in this way and the current is generated in this way. So, the EMF that we should have is $-\frac{\partial \phi}{\partial t}$ and that thing simply $-\frac{\partial}{\partial t}\int \vec{B} \bullet d\vec{s}$.

So, s is the area, which is not changing. So, I simply have $-\frac{\partial B}{\partial t}$ multiplied by the total area s. Now if the explicit form is given. Suppose, B as a function of t is given as some B_0 and sinusoidal function say sin ωt. Then I can really find that the EMF that is generated is simply -B_0 ω s and then cos ωt. So, these few are very, very simple and straight forward problem.

May be if I have some time I will like to do few more problem in the tutorial mode that we are planning. So, here today we are going to discuss about the electromagnetic induction. And the next class we will continue our topic, continue our discussion on electromagnetic induction. So, we will discuss about the self inductance and mutual inductance etcetera. So, with that note I like to conclude here. Thank you very much for your attention and see you in the next class.