**Foundations of Classical Electrodynamics**
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**Lecture – 06**
**Line, Surface and Volume Element (Cont.,)**

Hello students to the foundation of classical electrodynamics course. So, we are now still doing the module 1 with the mathematical preliminaries. So, today we are going to understand more about the line, surface and volume element in different coordinate system. So, last day we did it for Cartesian coordinate system. Now, apart from Cartesian coordinate system how line, surface and volume elements are there for cylindrical and spherical coordinate system, how we define that we are going to understand today.

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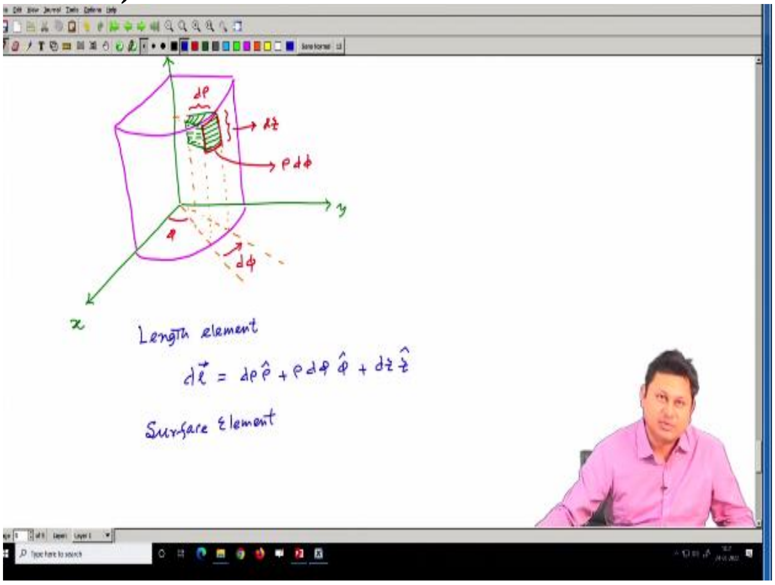


So, today this is class number 6. So, we already defined the line element in Cartesian coordinate system line, volume and surface element in Cartesian coordinate system. So, now we will do that first for cylindrical coordinate system how we define it? So, my system is now in cylindrical coordinate system, first we need to draw the coordinate system itself and then just draw two lines here. Then I can join this line from here to here. What I am doing is I am making a volume element here first.

So, this shaded one is I believe you can understand that this is the volume element I am just drawing first, which I can cut from this cylindrical system. So, this is a volume element that I just cut it from here. So, now this is my say x, y and z now, I define this coordinate here so, this is my $\varphi$. If this is $\varphi$ from here to here this is my d$\varphi$ and these from here to here this is my d$\rho$ and this slide implement is d$z$.

So, what is this value? This from here to here, since I am having this arc and this is $\rho$ so, it is simply $\rho$d$\varphi$. So, I almost define you know all the sides, all the length and everything. So, now I am in position to you know simply write down what is the length element, what is the volume element and what is the surface element? Once you draw this then things are very easy and straightforward.

So, my drawing is not that professional because I am drawing this using my hand. In the books, you will go to find a better drawing, but I believe you can understand that what I am trying to do here. The length elements simply dl should be the different length component that we are calculating. So, d$\rho$ what are the lengths here? So, d$\rho$ $\hat{\rho}$ + $\rho$d$\varphi$ $\hat{\varphi}$ + d$z$ $\hat{z}$.

So, this is simply the length element and in order to find you need to find first volume and then the different elements of these volumes different side length of this volume will give you this value. After having the length element then we need to find the surface element, what is surface element here? So, there are 3 surfaces you can see.

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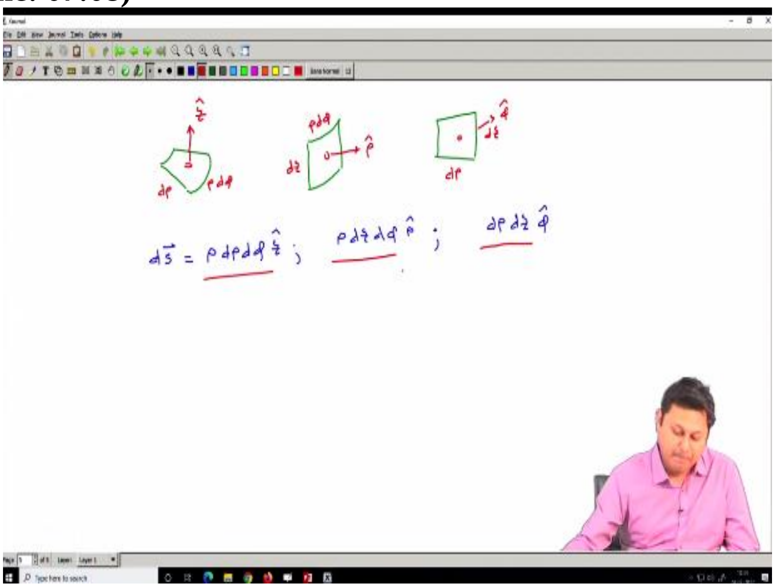

These 3 surfaces if I draw separately so, one surface like this, another surface is like this and another surface is like this. So, the 3 surface I draw whatever the chunk I have here, so, this is one surface, this is one surface, this is another surface and this is another surface. These 3 surfaces I am talking about. So, here along this is my $z$ direction so $z$ unit vector should be this.

What is this value? this is d$\rho$ and what is this $\rho$ d$\varphi$. What about this one? This one is $\rho$ d$\varphi$ along this we have $\rho$ unit vector and this length is my dz.

What about this one? So, this is d$\rho$ and this is dz and along this direction it is the $\varphi$ unit vector. So, for these 3 surfaces I know these lengths so, I can find out what is my you know surface elements, so, ds surface elements here is $\rho$ d$\rho$ d$\varphi$ with z unit vector. In this case, it is you know $\rho$ dz d$\varphi$ along $\rho$ unit vector and in this case, it is simply d$\rho$ dz along $\varphi$ unit vector. So, these are the 3 surface elements one can have here. One is this, another is this and another is this in 3 different directions.

Mind it, you need to check the dimensionality. $\rho$ d$\rho$ this is the surface should be length square. In all cases, you will find that these lengths square dimension is valid so, please check it. What should be the volume element?

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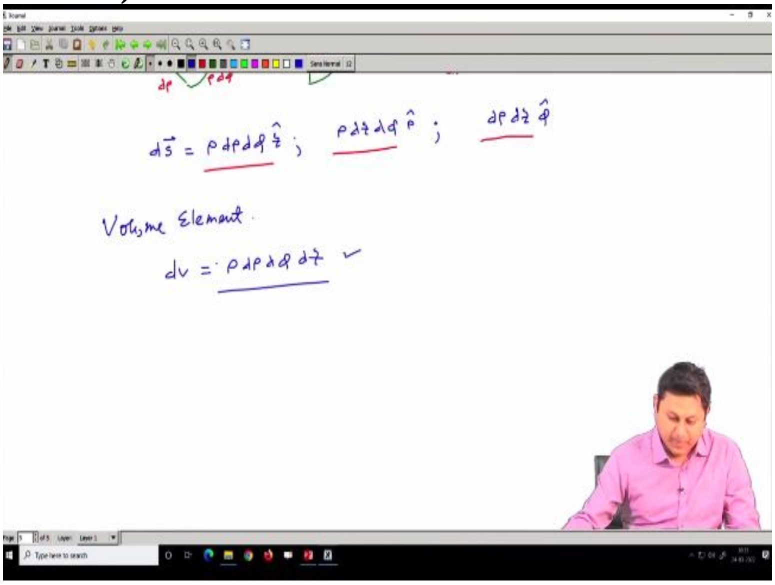


Because I already I mean have the volume elements so it should not take much time to understand that the volume element dv is simply the multiplication of all these things $\rho$ then d$\rho$ then d$\varphi$ and dz because all these 3 elements I know d$\rho$ dz $\rho$ d$\varphi$ if we multiply all together then what we are getting is the volume element. So, this is the volume element for cylindrical system.

So, for cylindrical system we managed to find out what is the volume element, what is the length element and what is the surface element. The main thing is that you need to draw this properly. Once you draw then you will readily understand that how these volume elements are calculated?

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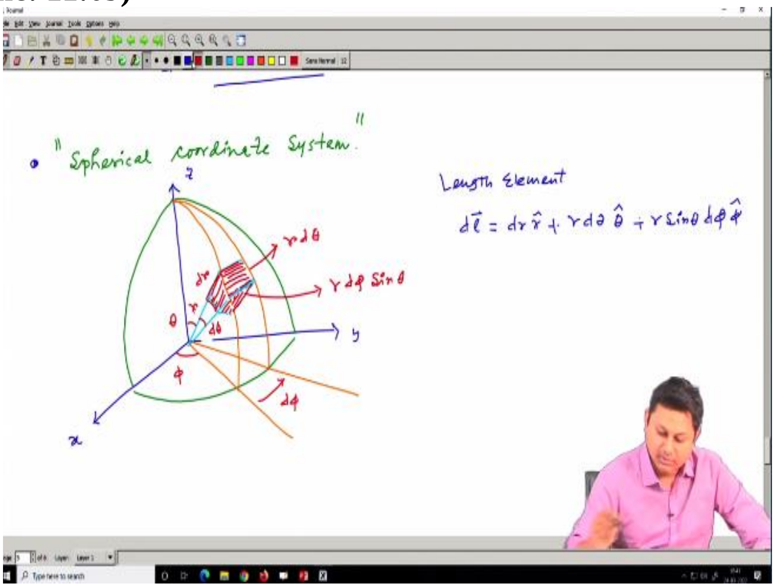


The next thing that we will do is to find out the same thing the surface, length and volume element but this is for spherical coordinate system. For spherical coordinate system we need to figure out all these 3 and again the drawing is important. Once you draw properly then you can. So, let us try to draw this here how one can find the volume element in spherical system. Suppose this is x, y and z. For spherical system these are the spherical surfaces I am cutting that.

We have placing over that and now I am going to draw this because I am making a cut on that and then I should draw from here to here. So, I am eventually having so let us erase this part so I have a volume like this. So, this is the volume element I am talking about here. This is a 3-dimensional figure so it is difficult to draw by hand but I believe you understand that this is a volume and I cut a piece from that. I am just cutting a piece from that, that is all.

Now it is time to define this so this is my again $\varphi$, this is d$\varphi$. This is r so this portion is dr and this is dr so this portion is these things is r d$\varphi$ and r d$\varphi$ is this region and r d$\theta$. So, another r d$\varphi$ not r d$\varphi$, this is r d$\varphi$ then one should have a sine component as well as sin$\theta$ because I am having a $\theta$ here r is this one, but I am having a $\theta$ here also $\theta$ and this tiny thing is d$\theta$. So, this quantity this length should be r d$\theta$.

So, all the length elements in now in my hand, so, once we have this, it is easy. So, first let us find out what is dl the length element? The length element dl will be simply because these 3 components are known. So, I can have dr and r unit vector plus I can have r d$\theta$ then it should be $\theta$ unit vector and plus r sin$\theta$ this is a projection of r over this. So, this is the length I am talking about this length should be r sin$\theta$ d$\varphi$ with $\varphi$ unit vectors.

So, this is the length element we got here. Now, what should be this volume and surface element that we want to find out?

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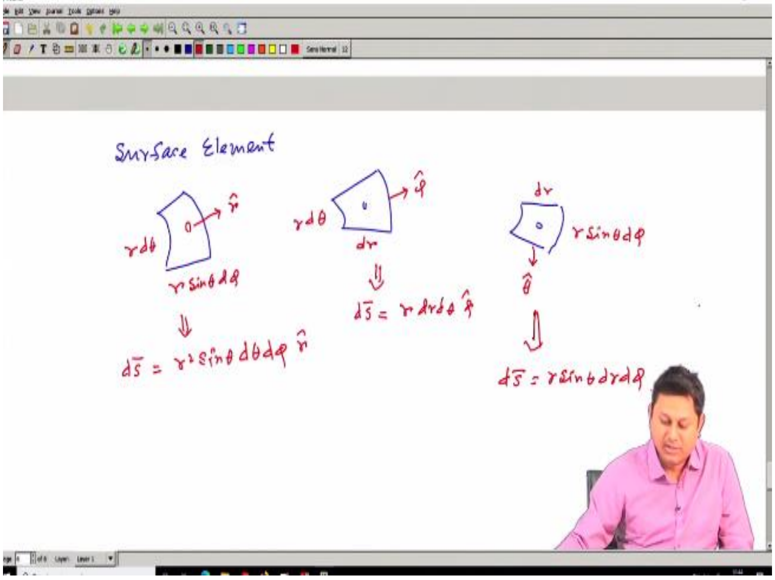


What should be the surface element like before? I need to draw the surface again so one surface should look like this. And this is r unit vector and this thing is r d$\theta$ and this thing is r sin$\theta$ d$\varphi$. Another surface one can consider like the side one and along this direction this is the $\varphi$ unit vector. So, this length is dr and this one is r d$\theta$ and another surface again one can there should be 3 surface the bottom one.

And in the lower side that $\theta$ unit vector goes this is you know this is one is dr. And dr is this one, this is dr and this one is r sin$\theta$ d$\varphi$. So, now you can calculate the surface elements. So, ds here ds should be $r^2$ sin$\theta$ d$\theta$ d$\varphi$ with unit vector r. In this case, ds will be r dr d$\theta$ with $\varphi$ unit vector and this case ds will be r sin$\theta$ dr d$\varphi$. These are the 3 surface elements one can have from this segment.

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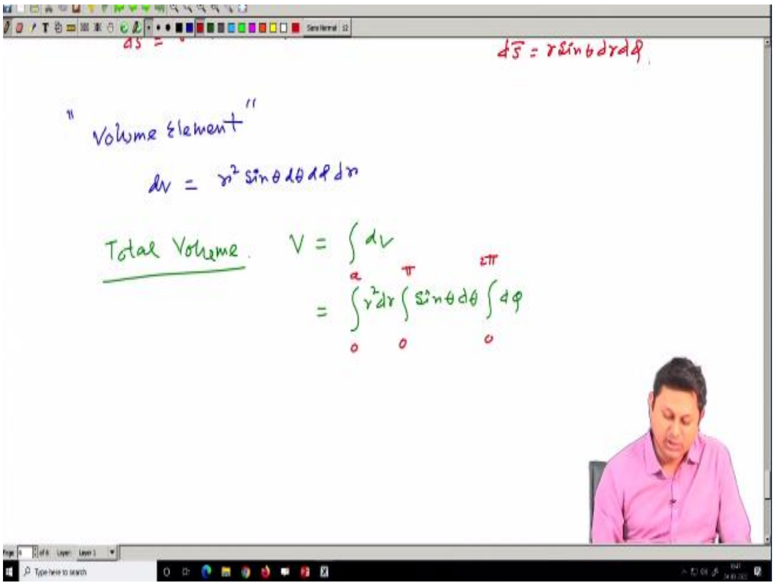


Now, a volume element is straightforward so volume element is dv then I have all the line element here all the line element along different directions I just multiply all together like we have in Cartesian coordinate system volume is dx dy dz here I have dx, in place of dx I am having dr, in place of dy I have r $d\theta$, in place of dz I have r $\sin\theta$ $d\varphi$, so it just simply multiply all these things.

If I do then simply, I get $r^2 \sin\theta\, d\theta\, d\varphi$ dr that should be the volume element. Very important term because we will be going to use this volume element in several cases. Now, the thing is, this is some set of homework I like to give and this homework is that find out what is the total volume because you know the volume element here for Cartesian coordinate system, for spherical coordinate system and for cylindrical coordinate system.

For example, for spherical coordinate system, you know the volume element now, if I want to find out the total volume then the total volume V will be the integration of dv and this integral, so this event is known so I know what is the values. So, it is $r^2$ dr and then integration of $\sin\theta$ $d\theta$ and then integration of $d\varphi$. Now, if I want to find out the total volume for a sphere, I know the limit also. In sphere should have some radius.

So, the integration should be from say here 0 to the radius a if it is a radius of a. $\theta$ I know what is the range of this $\theta$ it is 0 to $\pi$ and I know what is the range of $\varphi$ it is 0 to $2\pi$. So, if you calculate that you will get some value so, try to find out what value are getting it should be the usual 4/3 $\pi$ $r^3$ and you should get it from this expression itself. So, now, we will be going to jump to a more important thing after having these preliminary ideas about the volume element, surface element, all these things different coordinate system.

Now, we will go to more serious things like line, surface and volume integral. So, this is really important here in this course,

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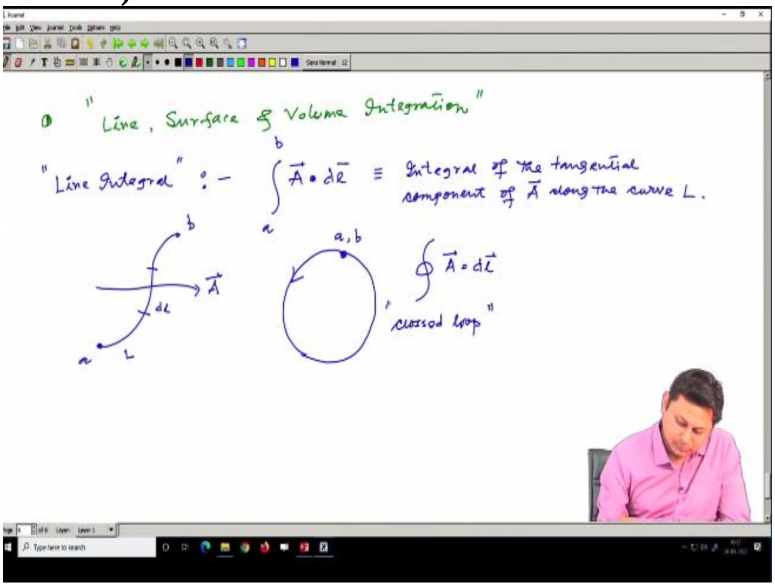


So, let us start that. So, we are starting a completely new topic here, which is the line, surface and volume integration, how to make a line integration, surface integration and volume integration let us introduce these today and may be in the next class we will discuss in detail. So, let us start with line integral. What is the meaning of that? So, the line integral normally we define like a vector field so, normally I write like A dot dl.

So, I am having a line element here suppose going from a to b line and having a line element here, which is dl say and also, I am having a vector field here A. So, what I am calculating is these things over the given line say I am calculating over the entire line this quantity A dot dl. So, eventually is the integral of the tangential component A along the curve L. So, if this is curve, say l over l I am integrating.

So, this quantity, which we call the line integral is the integral of eventually it is the integral of the tangential component because eventually we are making a dot product here mind it. So, the tangential component of A vector along the curve L so I am having a curve, this is my initial and final point and I am making a line integral and line integral is defined like A dot dl. So, what we are doing? We are just integral this so this is A vector.

And if what is the component along these, I am just calculating at each point and then I integrate that is the thing. Interesting observation is the line can also be closed. I can start from here and I come back to this point. So, line so here, I am starting from a and I can come back to same point b, where a and b at the same points. Suppose this is the path I am following. So, then, I have a special notation I have a closed line, this is the notation normally we use.

And this is saying that it is A dot dl I am calculating but I am calculating for a closed loop, because the initial point a and final point b are same this is a special implication is there, we will be going to discuss that and this is for closed loop this is called closed loop.

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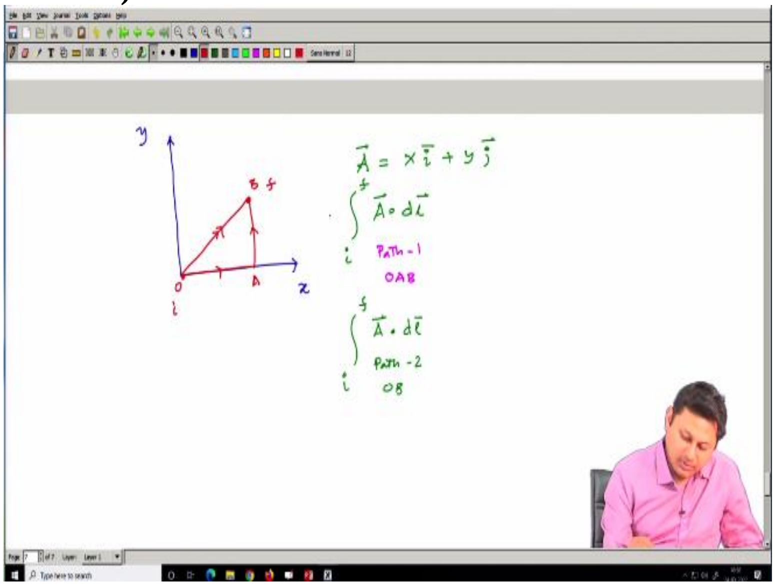

Now, what are the examples? The examples are very well-known the for example, work done when you calculate the work done by a force W is a total work done and how we calculate? We calculate like F dot dl. So, what we are calculating is calculating the line integral from 1.2. So, now, mathematically the force in Cartesian coordinate system if I expand this, so, force is $F_x$ with the unit vector i, $F_y$ unit vector j and $F_z$ unit vector k and dl this is the line element again and we know that how to define the line element in Cartesian coordinate system? This is dx i + dy j + dz k. So, how you calculate F dot dl it is a dot product so, eventually I am going to have like $F_x$ dx + $F_y$ dy + $F_z$ dz. Now, if I want to calculate the line integral say I want to calculate this line integral from one point to another point and this is F dot dl. So, this simply tells us that I can calculate x1 to x2 if these 2 point like that, so this is 1 point and this is point 2.

The coordinate here is x1, y1, z1 this is x2, y2, z2 then the integration should be executed in this way dx I just do that then I should write y1 to y2 $F_y$ dy plus z1 to z2 $F_z$ dz so, this is the way we calculate the line integral. So, Next, I mean today I suppose to do a problem but this problem is relatively a big problem. So that is why may be I am not going to finish this problem in today's time.

So next day, we will start from this work done, not work done rather this line integral concept that what is line integral, and how to execute that for a physical problem for example, I mean you can do that I am just giving you one example, here whatever the time left, but I can do a more rigorous problem, suppose I am having a coordinate system, 2 dimensional let us make like simple x and y. And I am giving a path here, suppose the path is from here to here, and then goes here, and then come back.

So, my path is if this is 0, or O, this is A, B so, I am going in one case, I am going in this path from O to A and then B. So, this is my final point, and this is my initial point. So, this is my initial point and this is my final point. But I am going in two different paths, one is going from here to here and another is going to here, going this point from here to here. And A vector field is given suppose this A vector field is given, simply say xi + yj.

Now the question is what should be the value so simple, I am giving you this as a homework that find out A dot dl integrating initial point to final point and the path is I should also mention the path because there are 2 different paths already. So, path 1, this is for path 1 and you need to go from say O to A, and then B, this is one path. Then again, execute the same thing initial point to final point A dot dl.

And now I am giving another path say path 2, which is directly from O to B. So, this is one problem, and this is another problem, but execution will be same. So, how you do that I am giving you the hint, I am not going to do the entire problem, I am just giving you the hint. So, A dl here you need to first execute what is A dl?

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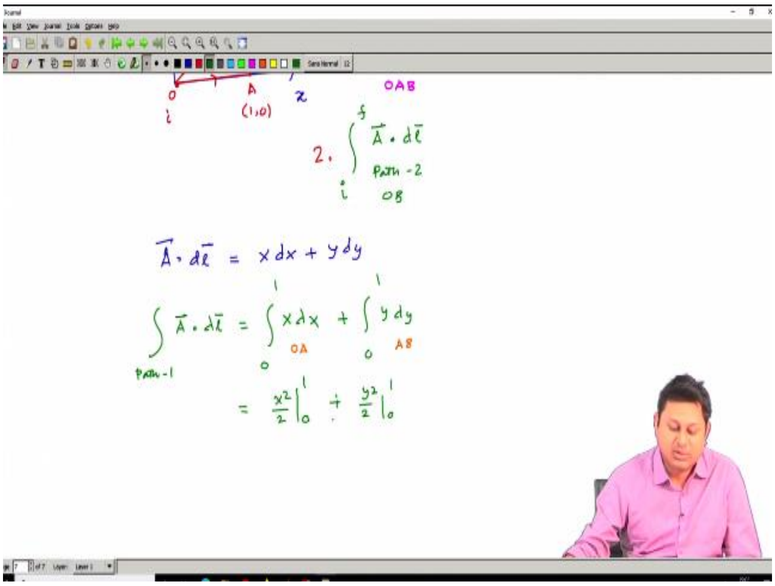


A dl is A dot dl, which is a 2-dimensional problem. So, it is simply x dx + y dy that is all. So, when you have x dx + y dy, then you should put it and then one by one execute for example, here the path I need to give the value also, otherwise I will not be going to so, this is say from here to here, this is this point here the coordinate of this point A is 1 0 and coordinate of this point is 1 1. So, this length is 1 and this length is 1.

So, when you execute for path 1, you can see that when you go from O to A, there is no change in y, only change in x. So, the dy term will not be there and you just need to execute this integration x dx over this length and your limit should be 0 to 1. When you go from here to here on the other hand, your x will not be going to change, your y is changing. So, eventually you get the value. So, let me do that quickly it will not take much time.

So, integration for path 1, A dot dl is essentially if I now start putting for this length, I have 0 to 1 x dx, dy will not be there 0 and for other case, it will be 0 to 1 y dy. And this value is x square divided by 2 with the limit 0 to 1 this is the path for OA and this is the path for AB. And then for other case, it should be y square divided by 2 and 0 to 1. So, whatever the value is there, you can find out it is seems to be you get the value 1 that should be the case. I am just giving this arbitrary problem.

But the procedure you need to think carefully that when you go this path, A dot dl you need to execute and you first find out what is A dot dl. You know that line element we just learnt that A is a given vector. So, you just find out what is A dot dl for example, here it is x dx + y dy because my vector is like that x i + y j. And then you will get. The next another trick of this problem is I mean how to find from O to B.

Because here in 1 case, the x is not changing in other case, y is not changing, but in this case, x and y both are changing. So, in this case, the trick is you can see the coordinate is 1 1 that means the change of x and y is same, so, x = y in the entire equation, if you put x = y, then your integration. For example, dy you just replace it is x is y here, because it is changing like that. So, the integral should be just 2x dx and then you integrate and change of x the value of the change of x is 0 to 1.

So, you just integrate and you will be going to get the result that is all. So, in the next class, maybe, we will be going to discuss in detail about a more critical problem. And we will execute that how to calculate the line integral. And also, in the class we calculate the surface integral for a given problem and the volume integral. So, with that note, I like to conclude. Thank you for your attention and see you in the next class.