Foundations of Classical Electrodynamics
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Lecture-49
Charge Particle in Magnetic Field

Hello student to the foundation of classical electrodynamics course. So, today we will have lecture number 49 and we will be going to discuss the charge particle in magnetic field. So, eventually today we start our module 3, which is magnetostatics.

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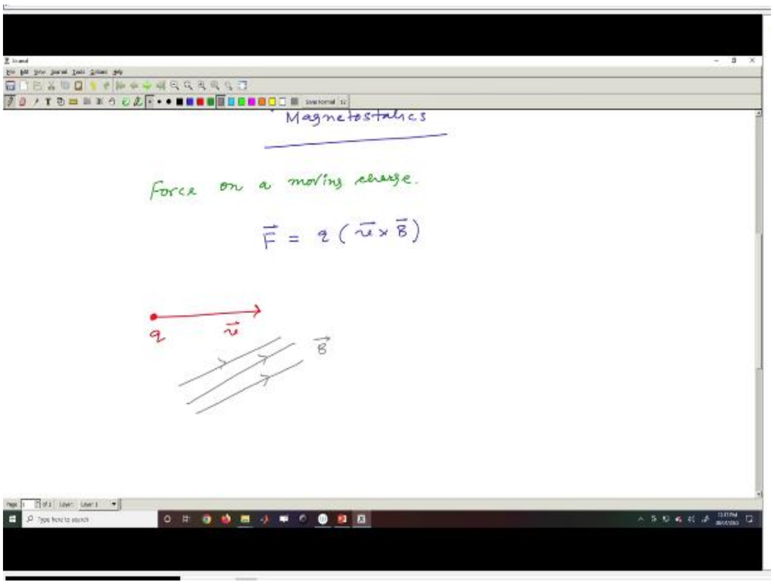

So, we have class number 49 today and today we have the first class of the module 3, which is magnetostatic. So, mainly today we are going to understand this magnetic field force on a charged particle. So, the starting point of this we are going to discuss in detail in later class also. So, let us start with force on a moving charge. So, that we know that $\vec{F}$ = q $\vec{v}$ × $\vec{B}$. This force is due to the application of the magnetic field.

So, force on a moving charge that means a charge particle that is moving say this is a charge particle q that is moving with a velocity and that is moving say some under given magnetic field. Suppose this is the direction of the magnetic field $\vec{B}$ in same plane say. Then the force will be q into $\vec{v}$ × $\vec{B}$ on this moving charge particle.

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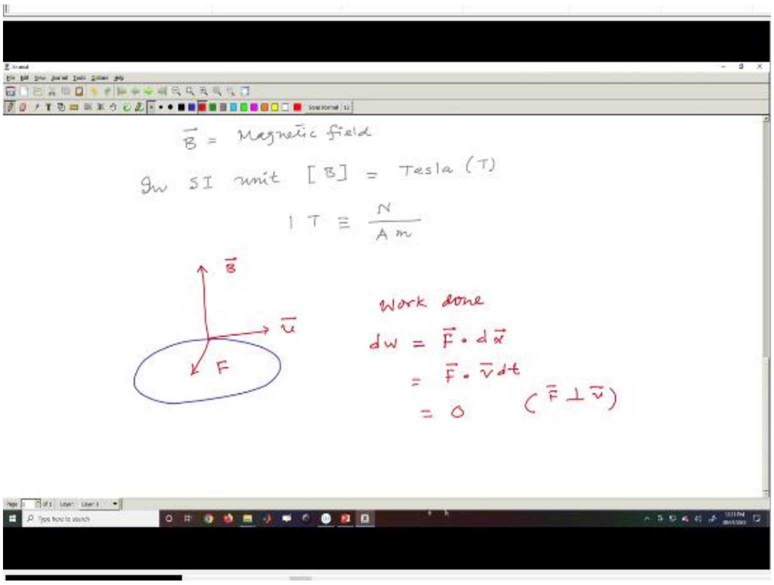

Now $\vec{v}$ is a velocity, $\vec{B}$ as I mentioned the magnetic field. So, in SI unit the dimension of $\vec{B}$ or the unit of $\vec{B}$ is tesla and 1 tesla is equal to Neutron per ampere meter. So, let us schematically talk that suppose I have a magnetic field in this direction and $\vec{v}$ is this direction. So, no this is the magnetic field and this is the velocity. So, the force will be simply is as a cross product. So, the force will be simply the perpendicular direction like this my drawing is bad. So, I should remove this part first.

So, this is the force because that should be perpendicular to $\vec{v} \times \vec{B}$. So, $\vec{v} \times \vec{B}$. So, this is the direction the force should apply and as a result we will have some motion and that we will be going to calculate today. So, let us quickly calculate what should be the work done? Work done dw = $\vec{F} \cdot d\vec{x}$ in one dimension. So, it is $\vec{F} \cdot \vec{v}$ dt I can write in $\vec{v}$ dt but $\vec{v}$ and $\vec{F}$ these are perpendicular. So, we will be going to get this as 0 because force will be perpendicular to the velocity. So, magnetic force that means this magnetic force does not do any work.

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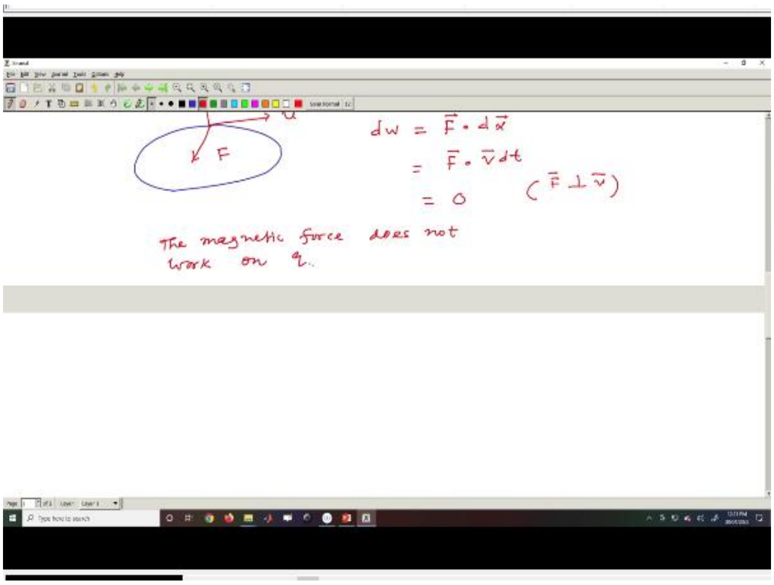

The magnetic force does not work on q, but we can have a generalization expression for the force, which is called the Lorentz force because the electric field and that electric field and magnetic field the charge particle will be going to experience a force.

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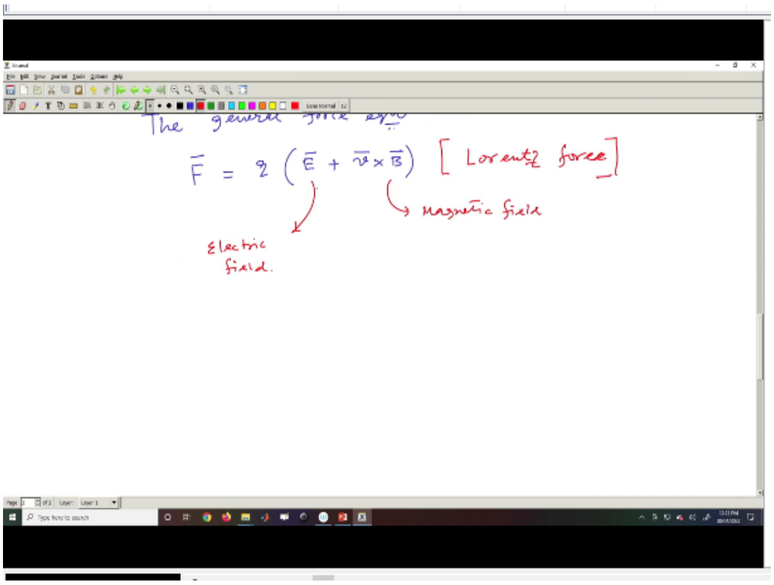

So, the general force equation is $\vec{F}$, which is q ($\vec{E}$ + $\vec{v}$ × $\vec{B}$). So, this portion is due to electric field, the force due to electric field and this is the force due to magnetic field and you can see that the force due to magnetic field will exert on the charge particle q if it is in motion, some velocity should be there. If it is a static charge then there should not be any force due to the magnetic field.

So, considering that fact we can also find few things these are known. So, I am not going to spend much time. So, this force is called Lorentz force. So, all these things are known to you I believe. So, that is why it is just a recap of whatever you already know.

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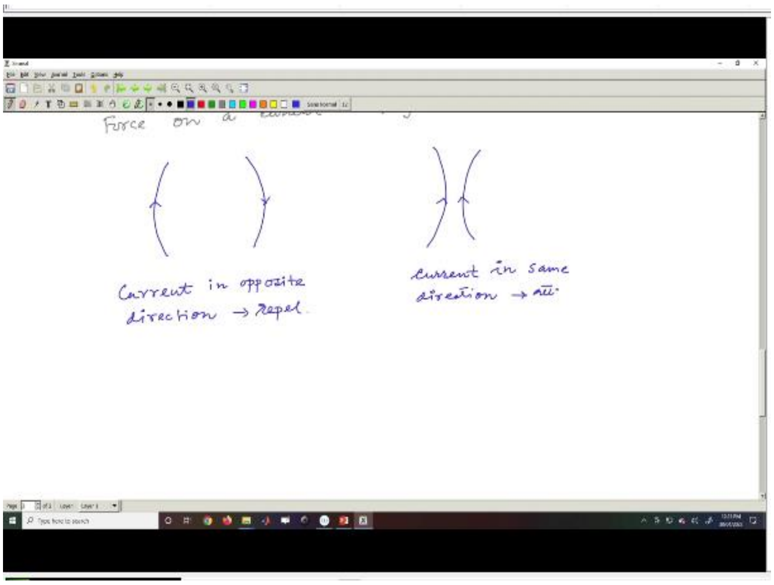

Next thing that we like to understand is force on a current carrying wire. So, when the wire is carrying some current. So, then only the charge particle is having some velocity and that gives rise to some magnetic field. So, we will discuss this in detail. So, let us first draw when the current is moving in opposite direction they will repel. So, current in opposite direction it repels.

On the other hand, current moving in the same direction it results some attraction, why they are repelling, why they attract we will be going to discuss in detail later I am just discussing the phenomena. So, here what happened that. So, let us do few calculations.

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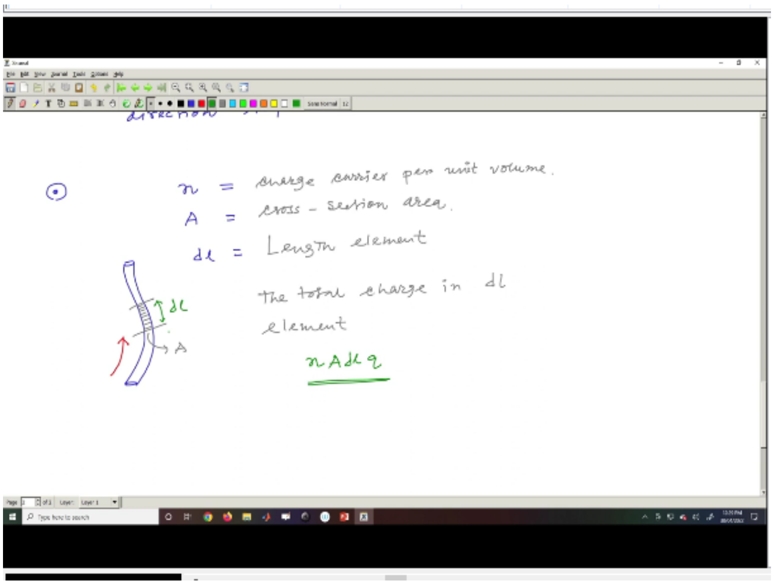

So, suppose n is a charge carrier per unit volume. So, whenever we have a wire. So, suppose this is the wire. So, the charge carriers are moving here like this. So, n is the charge carrier per unit volume what about A? A is the cross sectional area of the wire. So, I draw that wire here. So, the cross section area. So, if I make a cross section here this point. So, this is the cross section area and dl the length element.

Let us make symmetry here. So, this is length element. So, the total charges then why we are doing the total charge in this region I can calculate the total charge in dl element then it simply the volume whatever. So, from here to here this is dl. So, that should be the volume A dl and then the number of charge carrier and each charge carrier having charge q. So, that is the amount of the total charge. That is we are having in the element dl.

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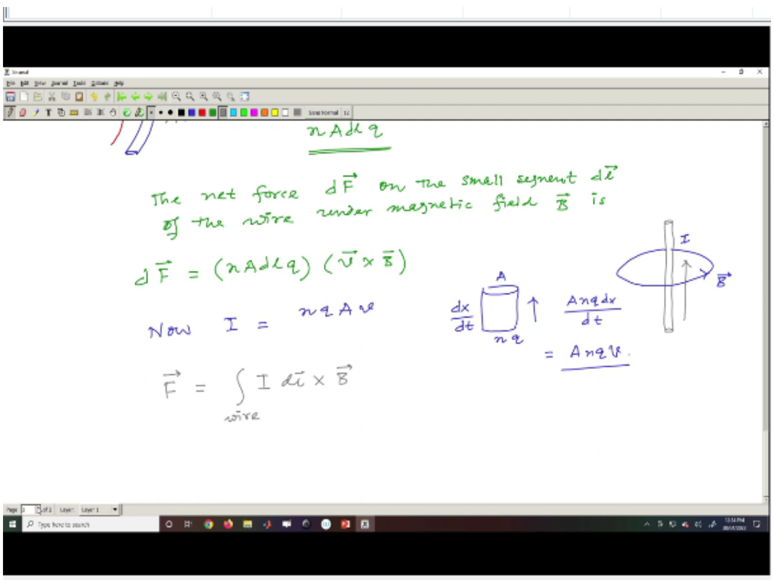

So, the net force if I need to calculate say d$\vec{F}$ due to this small section d$\vec{F}$ on the small segment d$\vec{l}$ of the wire under magnetic field $\vec{B}$ is how much? d$\vec{F}$ is equal to the amount of the charge n A d$\vec{l}$ q and then the velocity cross $\vec{B}$. So, what happened is? So, when two wires is there. So, you should first appreciate this fact that whenever a wire carrying some current because of that we had an associated magnetic field. This following a thumb rule we can calculate by using the Ampere's law.

What should be the amount of the magnetic field? So, that is due to the carrying of some current. So, that we know. So, that means we had a magnetic field and then this amount of magnetic field, which is already there I place another wire having a charge density or amount of charge per unit volume is n. And then the amount of force will be this. Now we will be going to calculate this.

Now I it is the current is charge divided by time. So, amount of charge we have n q and A and now this is the charge flow number of charge per unit volume q and that amount with the velocity gives the total amount of current because this is charge divided by time. So, n q A v is eventually if this is my section and I am flowing that. So, this is area A, n is here, total charge is q and this tiny amount of distance it is moving per unit time. So, the amount of charge that is flowing per unit time that is the current is simply the total charge multiplied by x.

So, this is like A n q dx that is the volume divided by dt. So, which is A n q v. Now my total force in terms of this that I calculated here is simply integration of the current and then I have dl length that is experiencing the force cross $\vec{B}$. That is over the entire wire total force, I need to put a vector sign as well. So, for closed loop. So, this is the expression of the force whatever the force we are talking about just now this should be the expression of the force. Now what happened if I have a closed loop?

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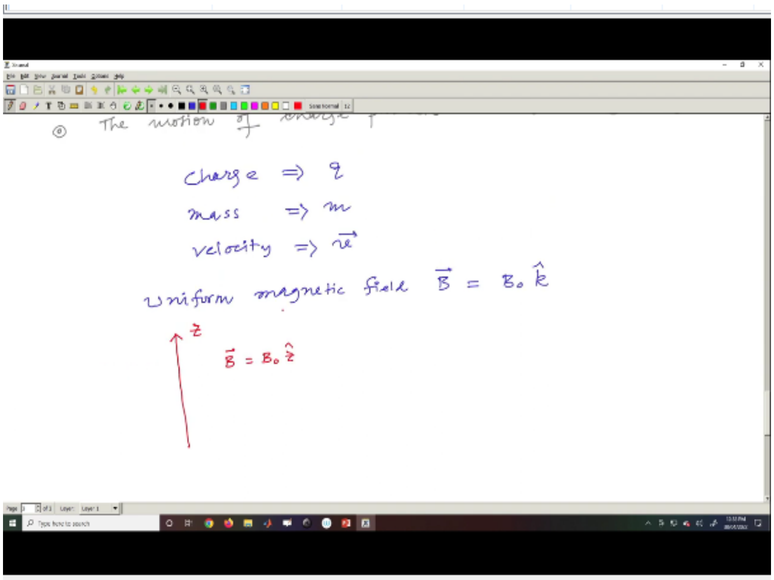

For a closed loop what we get is $\vec{F}$ = I since it is a closed loop. So, I need to have a close line integral and cross $\vec{B}$. Since I am having a close line integral then it should be simply 0 for constant magnetic field, because as this quantity has to be 0. So, for closed loop the amount of force has to be 0 that is the information we should know here. So, we will discuss this again when we are doing the Ampere's law and all this.

So, next thing is we want to understand the motion of charge particle in uniform magnetic field. We have already mentioned that just before but this thing was qualitative that how, what should be the direction etcetera. I am talking about this one. So, this is just a qualitative discussion we had. Now we will be going to calculate quantitatively that what should be the form of the motion I mean what should be the equation of motion etcetera a charged particle.

So, let us start with the charge of the particle is q single charge, mass is say m and this m travels with a velocity say v into the region of uniform magnetic field. Suppose it is a uniform magnetic field and these particles just entered there. So, what should be the equation of motion? So, we have uniform magnetic field, uniform say magnetic field $\vec{B}$. So, we will say let us assume that $\vec{B}$ is along z direction.

So, I specify the direction of $\vec{B}$ as well. So, this is the uniform magnetic field along z direction, the value is $\vec{B} = B_0\ \hat{z}$. So, uniform and now the particle is enter in this magnetic field with a velocity v having mass m and charge q. So, what should be the equation of motion? So, let us write down the equation of motion straight way because we know that the force is equal to q $\vec{v} \times \vec{B}$.

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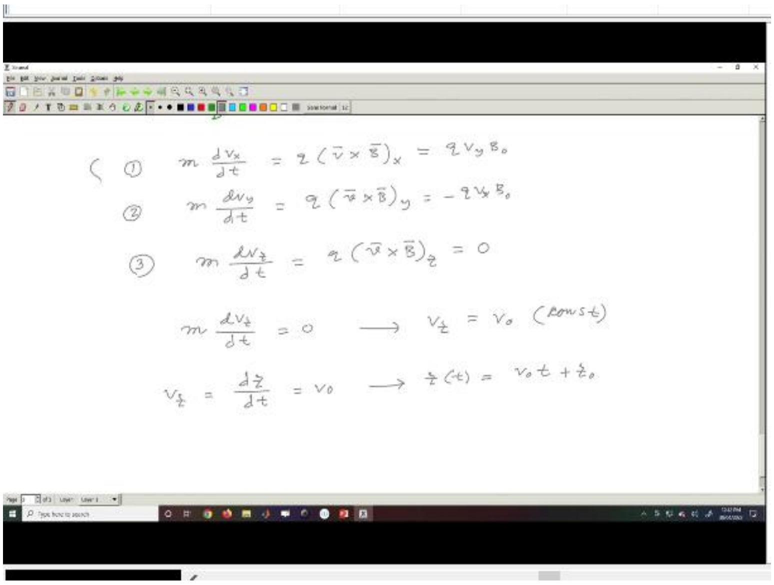

So, that means m $\frac{dv}{dt}$ that is the amount of force the particle will experience is q $\vec{v} \times \vec{B}$. Now we already assume that $\vec{B}$ = B_0 $\hat{z}$ or $\hat{k}$, both the cases it is same. So, let us write it $\hat{z}$ since we are. So, this is like this. So, then component wise this is a vector equation. So, if I write down in component form then this equation simply gives us.

So, component wise this is m $\frac{dv_x}{dt}$ it is q $\vec{v} \times \vec{B}$, the x component of this and that is simply q v_y and B_0, you should note that $\vec{A} \times \vec{B}$ its ith component it ϵ_{ijk} A_j B_k that we know. So, I just use this. So, second is m $\frac{dv_y}{dt}$ = q $\vec{v} \times \vec{B}$, y component and that should be -q v_x B_0, just use this expression and we are going to get the result and finally m $\frac{dv_z}{dt}$. The z component will be simply q $\vec{v} \times \vec{B}z$ and that value this is z component and cross.

So, it should be 0 because B does not have any other component. So, from here we have an important expression that B m $\frac{dv_z}{dt}$ is 0, which gives us v_z is a constant and say this is v_0, a constant and v_z is $\frac{dz}{dt}$. So, that is equal to v_0. That simply tells us that **z** the z coordinate of this charge particle varies like this. Now I can combine equation 1 and 2 in a in a in a unique way and that is by exploiting the complex analysis. So, that makes life simple.

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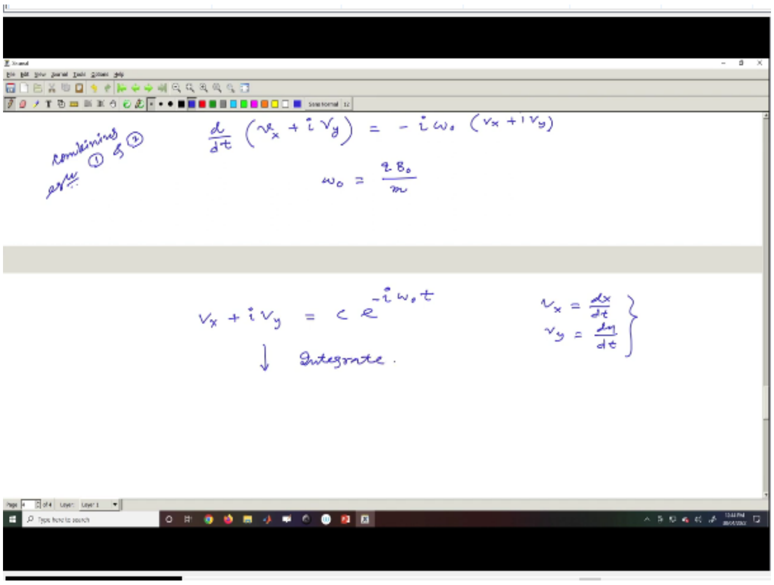

So, combining equation 1 and 2, $\frac{d}{dt}$ (v_x + i v_y), let us put one i. So, that from the right-hand side I just divided m q here and I just write it like minus of i and ω_0 and v_x + i v_y where ω_0 is q $\frac{B_0}{m}$. So, $\frac{dv_z}{dt}$ where is this equation $\frac{dv_z}{dt}$ I just divide this here m divided q, $\frac{B_0}{m}$ both the cases here is a minus sign and in order to absorb this minus and i multiply with the i.

So, that should be my expression. So, I write this equation so that I can have a ready-made solution and this solution is v_x + i v_y this quantity is simply some constant c $e^{-i\omega_0 t}$ because I have a minus here. So, that is the simple solution. Now if I integrate this because v_x is $\frac{dx}{dt}$ and v_y is $\frac{dy}{dt}$. So, again we have a differential equation here but the solution is now known.

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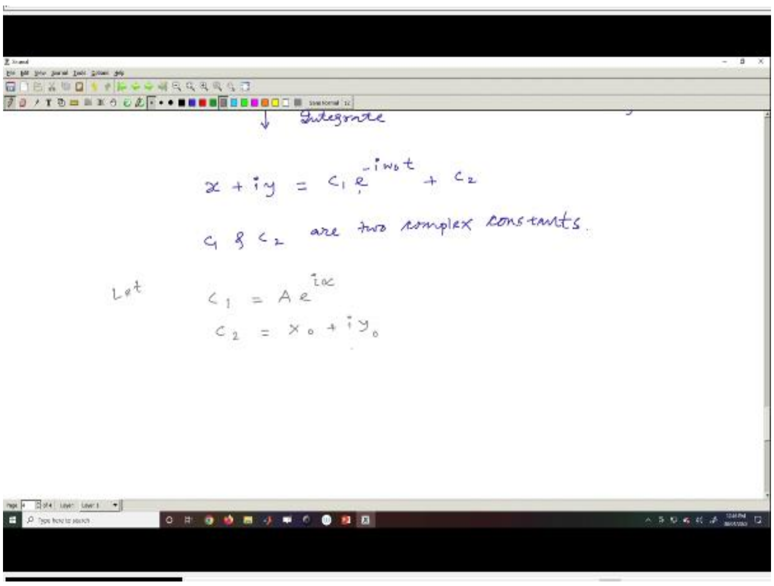

So, if I integrate then what I get is $x + iy = c_1\ e^{-i\omega_0 t} + c_2$, after integration I should have one $i\omega$ out and this $i\omega_0$ I can insert inside the c and I can make another constant where c_1 and c_2 are two complex constant. This is constant but the value can be complex constants. So, since it is a complex constant and so, I can arrange these constants suitably and c_1 let is say equal to A $e^{i\alpha}$, where I write this c_1 constant another two constant A α and c_2 I write $x_0 + iy_0$. What is the benefit of writing such thing?

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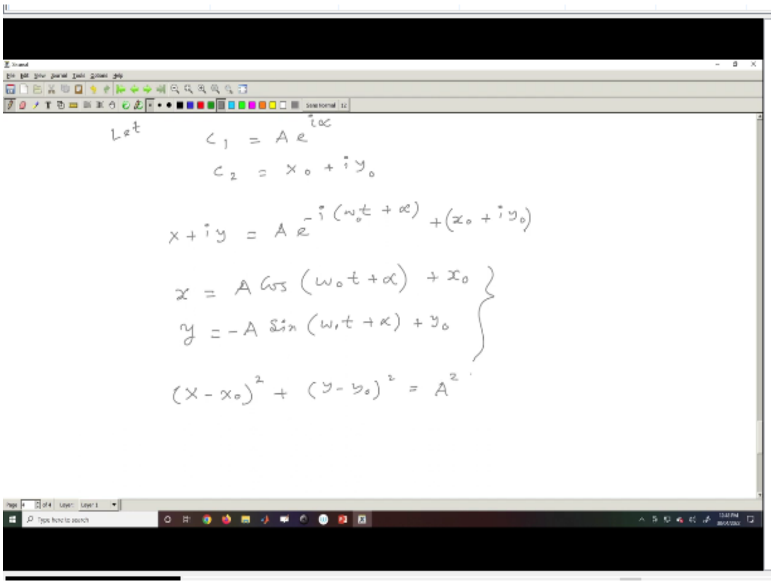

The benefit is $x + iy$ is A $e^{-i(\omega_0 t + \alpha)}$ + $(x_0 + iy_0)$. So, left-hand side we have a complex term, right-hand side we have a complex term. So, I can equate the real and imaginary part, because we know that when the two complex term are equal. So, what happened that I can equate the left-hand side and right-hand side component I mean.

So, real part will be equal to real part and imaginary part will be equal to imaginary part. In doing so, I have $x = A \cos(\omega_0\ t + \alpha)$ and x_0 and $y = -A \sin(\omega_0\ t + \alpha) + y_0$, I can write it in a form like $(x - x_0)^2 + (y - y_0)^2$ is A^2. So, I have an equation of the trajectory of the particle and that trajectory becomes a circle. That is important thing.

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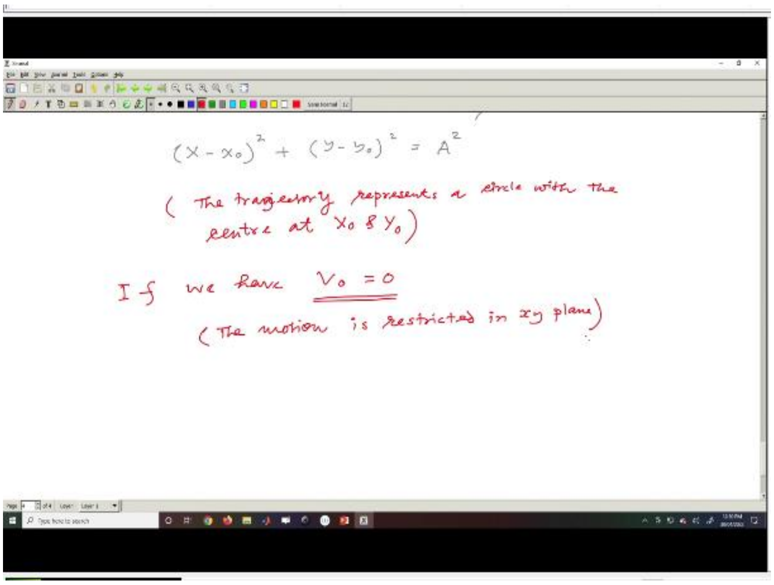

So, the trajectory represents a circle with the center at x_0 and y_0 and with a radius of A. So, now we already have v_0. So, if we have v_0 to be 0 that means the z component of the velocity is 0 then that means the motion of the charge particle is restricted in xy plane and the particle described a circular path.

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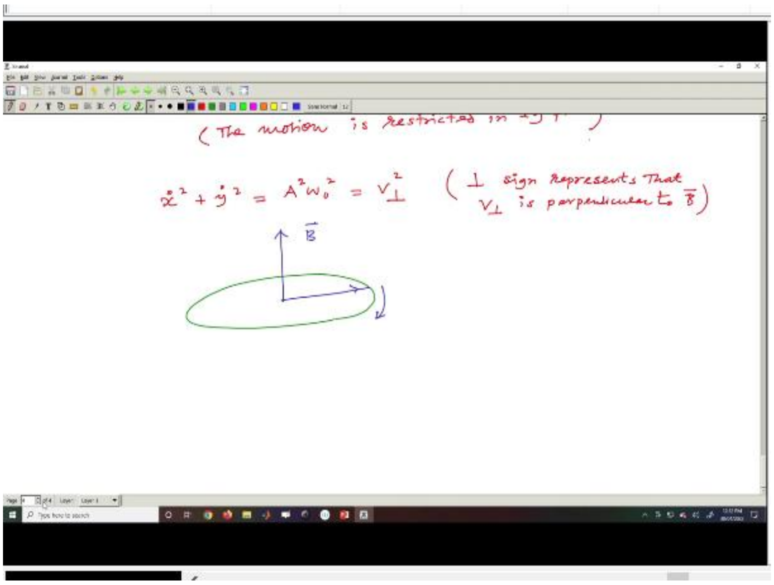

And it simply $\dot{x}^2 + \dot{y}^2$ is equal to because xy I had here. So, if I make a derivative and then make A^2 then it should be $A^2 \omega_0^2$ and that I write in this way $v_\perp^2$ why I write this, because this perpendicular sign represents that $v_\perp$ is perpendicular to the applied electric field $\vec{B}$. So, the velocity is in a plane and the corresponding magnetic field under which these things are moving is perpendicular. So, if this is a circular motion we are having then the amount of this magnetic field some this magnetic field should be perpendicular to the motion whatever the motion we are having.

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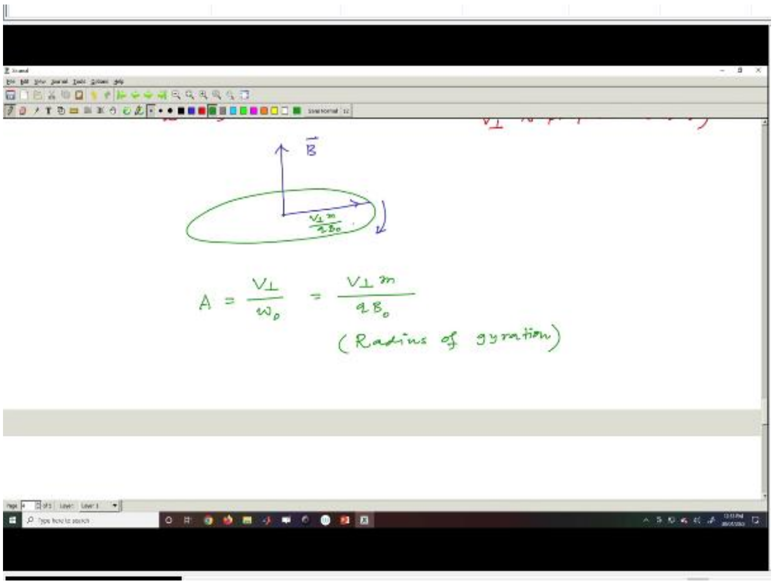

So, A here A is $\frac{v_\perp}{\omega_0}$ and this ω_0 we know. This ω_0 is $\frac{qB}{m}$. So, it is $\frac{v_\perp m}{qB}$. So, here we already had the value of the ω_0 and that is the value of the ω_0 we are writing here $\frac{qB_0}{m}$. So, here B_0 and B I just write uniform magnetic field. So, let us put this B_0 and this A is the radius of the circular path.

Because this is the expression, in the expression this A was the radius. So, this is radius of this and this is called radius of gyration. So, eventually this is my radius and this radius is $\frac{v_\perp m}{qB_0}$. Now you can see the B_0 is staying in the denominator. So, if the B_0 is high then we have the particle moving in a lower radius.

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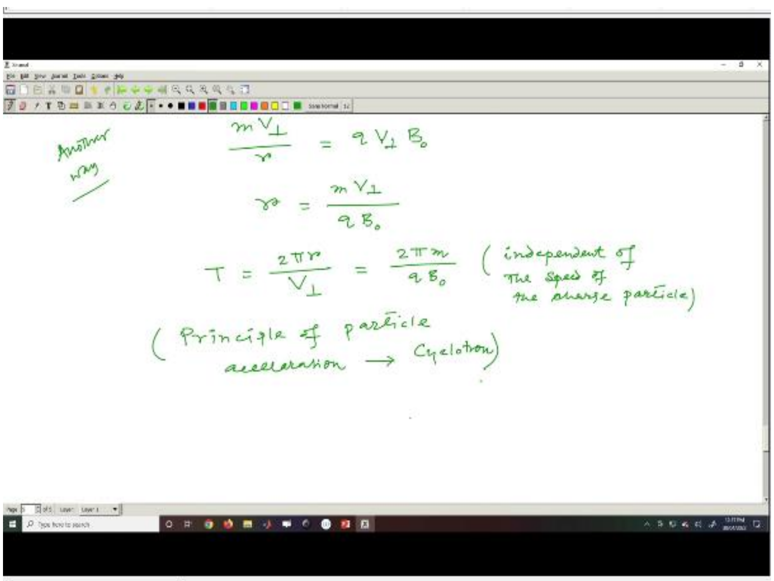

So, another way we can understand this motion and that is $\frac{mv_{\perp}^{2}}{r}$ is simply q $v_{\perp}$ B, q v B is the amount of the force and that should be counter balance this $\frac{mv^2}{r}$ and from that I can calculate my r to be $\frac{m\,v_{\perp}}{qB}$. Here I should write B_0 because this is uniform. So, let us write B_0. Now the time period because it is revolving the time period T I can also calculate.

This is $\frac{2\pi r}{v_{\perp}}$ and that is $\frac{2\pi m}{qB_0}$ because here r is my radius and I simply have this and importantly you can see that this is independent of the speed of the charge particle. So, whatever the speed of the charge particle that does not influence the time period under which the particle will go to circulate and that is the principle of particle acceleration.

So, this is the principle of particle acceleration. So, I can decrease the time period by just increase the value of B_0 that is the point. So, this is the principle of particle acceleration, which is we call cyclotron, which is related to the cyclotron. So, before going to finish also. So, this is basically the way when the charge particle is put inside a uniform magnetic field.

So, this is the way the charge particle will be going to move. So, this is the principle of the cyclotron I explain. Now quickly try to understand a very important concept and that is magnetic monopole does not exist like q, we can have a single charge like q. So, if you make a divergence of these things. So, you will be going to get a non-zero value, but for magnetic field it is not there.

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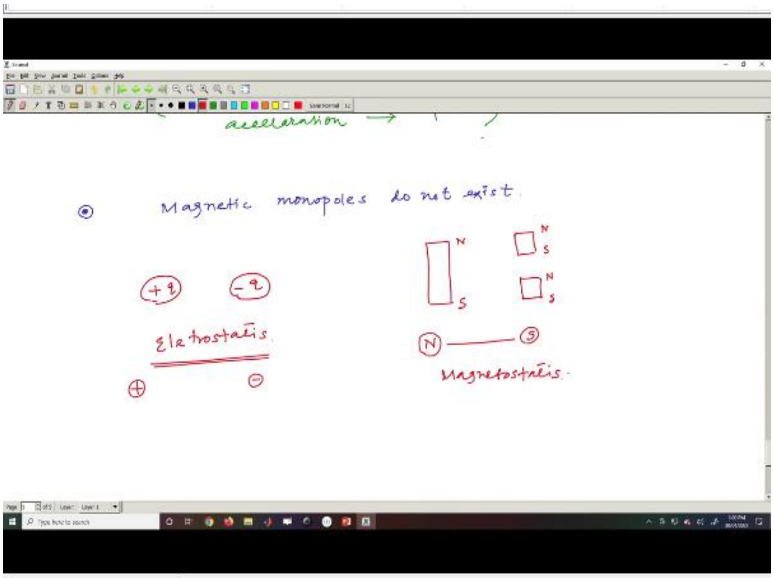

So, first you need to understand that the magnetic monopole do not exist let us first understand qualitatively then. So, I can have +q charge and -q charge. This is electrostatic, there is a basic difference between the electric charge and this electrostatic and magnetostatic. In electrostatics we have independent charge I can have plus charge or minus charge without any problem.

But for magnetostatic suppose let us consider a bar magnet here having a north pole and south pole and if I divide this tiny bar magnet then I should have a north pole south pole here and again a north pole south pole here. So, I never have an individual north pole or an individual south pole if I understand very crudely. So, that means this is the monopole and this does not exist for magnetic cases.

So, I always have a relation. So, if I have a north pole the south pole has to be there, if we have a south pole the north pole has to be there. So, that is the difference. But this is for magnetostatics, but for electrostatics I can have a plus charge or minus charge. For example if I put a plus charge so, what happened here?

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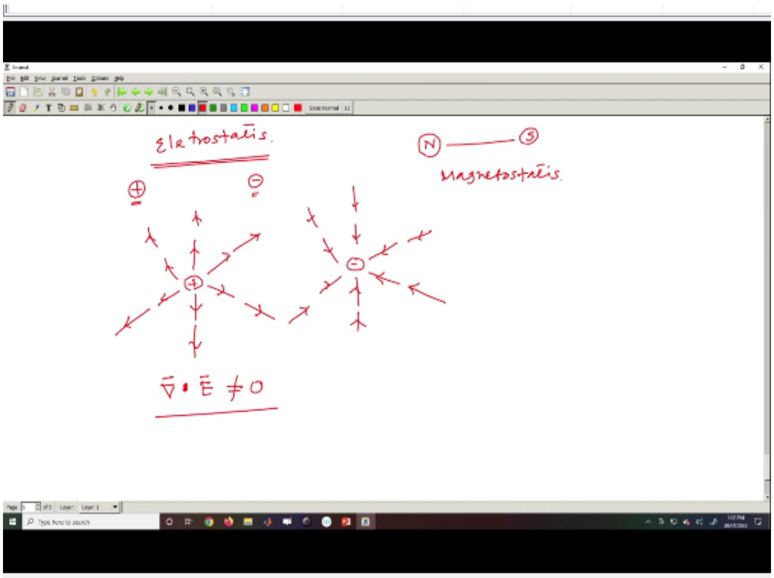

I have the field line that is moving away. So, I can have a source of a field line or a sink of a field line if I have a negative charge. So, these are the field lines, we discuss it several times in electrostatic problems. So, these are the field line and we have a source here like the field line is having a source and field line is moving this direction. On the other hand, if I have a single minus charge here I have a sink here. So, field line will move towards this.

So, the mathematical representation of such phenomena is the calculation of the divergence. So, here the electric field should have a divergence and that is non-zero, this value is non-zero. On the other hand for magnetic field this is different completely.

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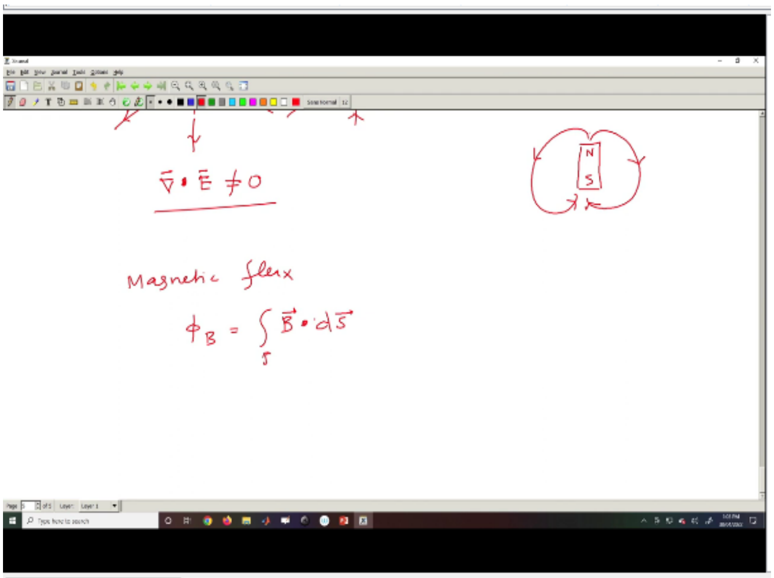

So, the magnetic flux let us calculate the magnetic flux first, we write it ϕ_B as the surface integral $\vec{B} \cdot d\vec{s}$, this is the magnetic flux. Now for magnetic field if I try to understand here qualitatively this is my given magnetic field having a north and south pole both. Now if I want to draw this magnetic field line here my pen is not working somehow. So, the field line will come here and move here like this. So, there is no such source or sink individually there. So, that is the basic problem basic difference we are having with magnetic flux and electric flux. So, let me write it here like this.

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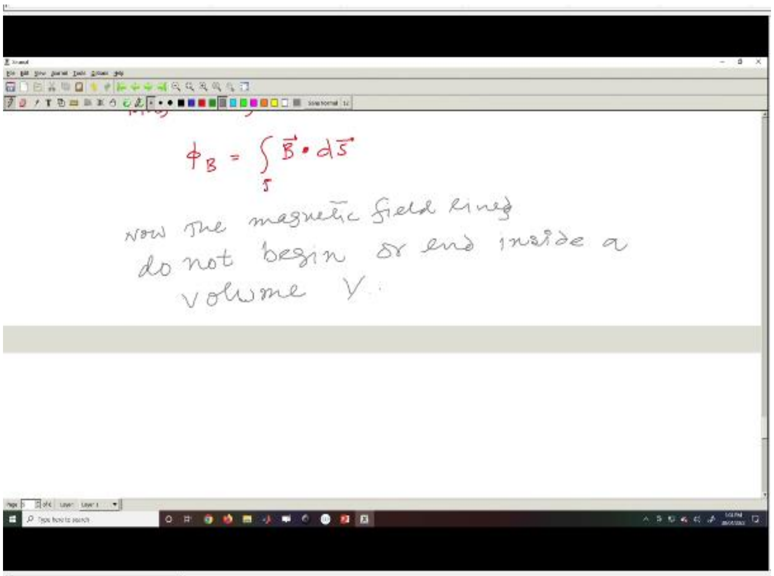

Now the magnetic field line do not begin or end inside a volume V because of this fact. So, the magnetic field lines do not begin or end inside a volume V.

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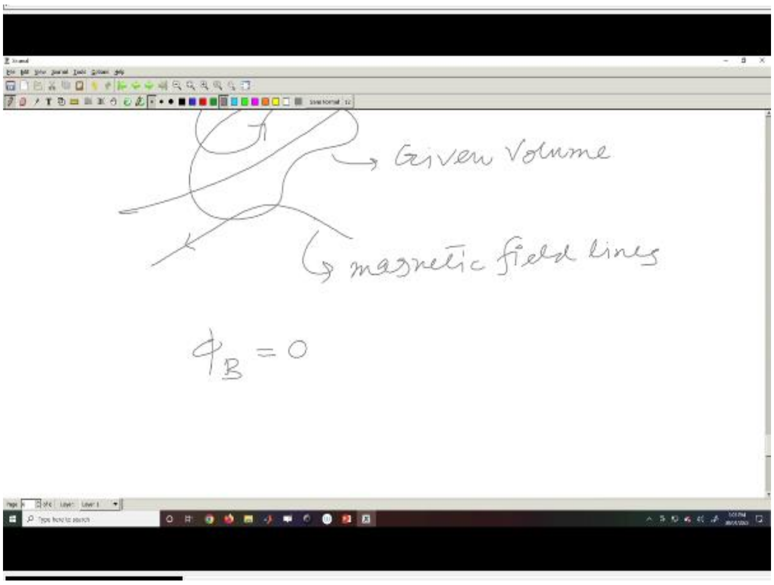

So, that means if I have a volume here like this. So, the field lines can be this enter and passing away and moving and passing away. So, every time. So, there is no source and sinks. This is the given volume and that is the magnetic field line. So, that means my effective flux is 0 over the closed surface. Whenever the surface is close it is 0.

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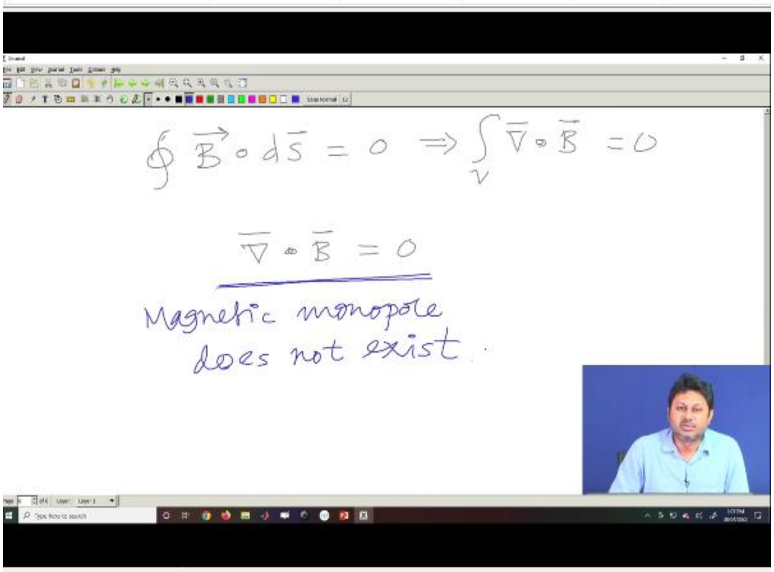

Now that means I have $\vec{B} \cdot d\vec{s}$ over this closed surface is always 0 and that eventually leads to that the volume integral divergence of the quantity $\vec{B}$ is 0 and it is true for all the surfaces. So, that gives me a very important relation that divergence of $\vec{B}$ is 0. The physical argument of this mathematical expression is the magnetic monopole does not exist. We will discuss this maybe in the future class also in detail.

So, today I do not have much time to discuss more about that. So, with this note I like to conclude here. So, hope we will do more problem in the next class, we will be going to do more problem on the magnetic field its property etcetera are going to understand more thing in details. So, with that note I like to conclude here, thank you very much and see you in the next class.