Foundation of Classical Electrodynamics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology - Kharagpur

Lecture - 44
Displacement Vector

Hello students to the foundation of classical electrodynamics course. So, under module 2, we have lecture 44 today and today we will go to discuss about the displacement vector.

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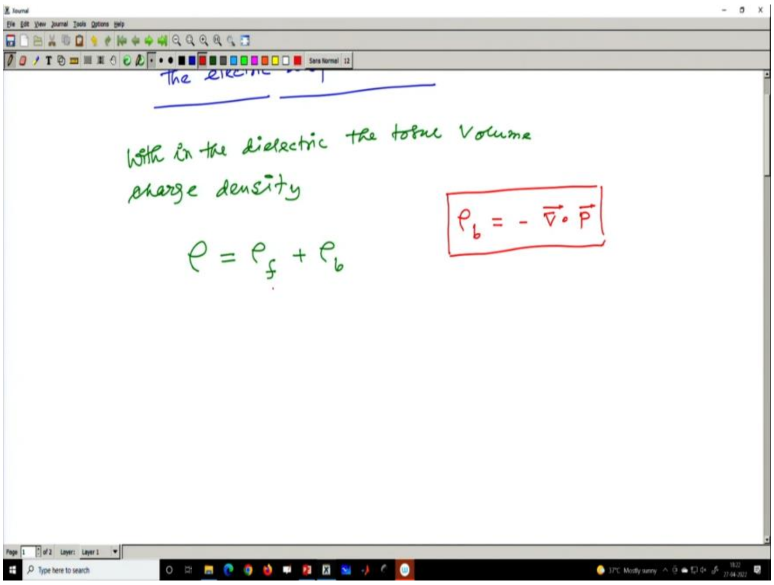

We have class number 44. And today we are going to discuss about the electric displacement or the displacement vector. So, we find that within the dielectric the total volume charge density is the contribution of the 2 kinds of charge one is we call free charge and another is the bound charge you may remember that the bound charge already we find that it is the minus of divergence of the polarisation when we have non uniform polarisation.

In that case if I make the divergence of that polarisation vector then that leads to the negative value of that leads to the amount of bound volume charge density. So, in general the volume charge density ρ can be divided into 2 parts one is free volume charge density another is the bound volume charge density.

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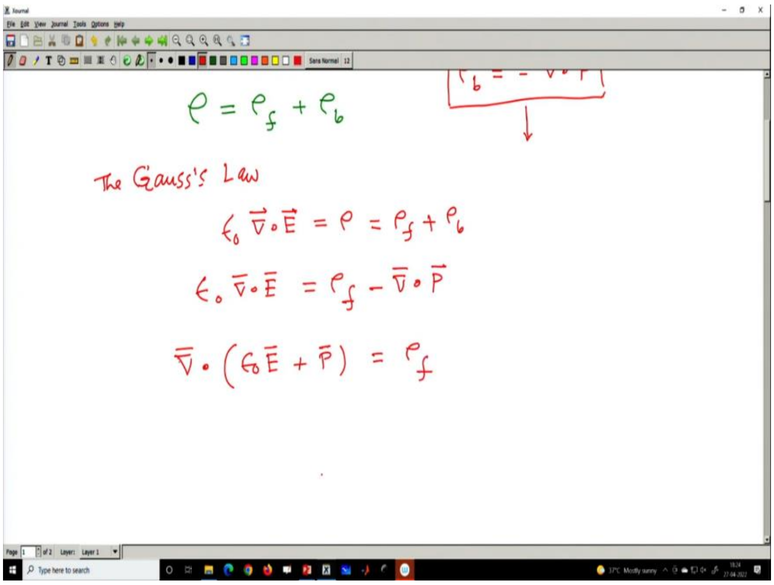

Now, in the Gauss's law what I find? The Gauss's law says that ϵ_0 and then $\vec{\nabla} \cdot \vec{E}$ = ρ, $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ I just multiply this ϵ_0 here and left side ρ. So, this ρ is now $\rho_{free} + \rho_{bound}$ already I write that ρ_b is $-\vec{\nabla} \cdot \vec{P}$. So, I can write that ϵ_0 $\vec{\nabla} \cdot \vec{E} = \rho_f - \vec{\nabla} \cdot \vec{P}$. And that quantity if I put this in this side and put this ϵ_0, which is a constant inside the divergence operator then I should have $\vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f$.

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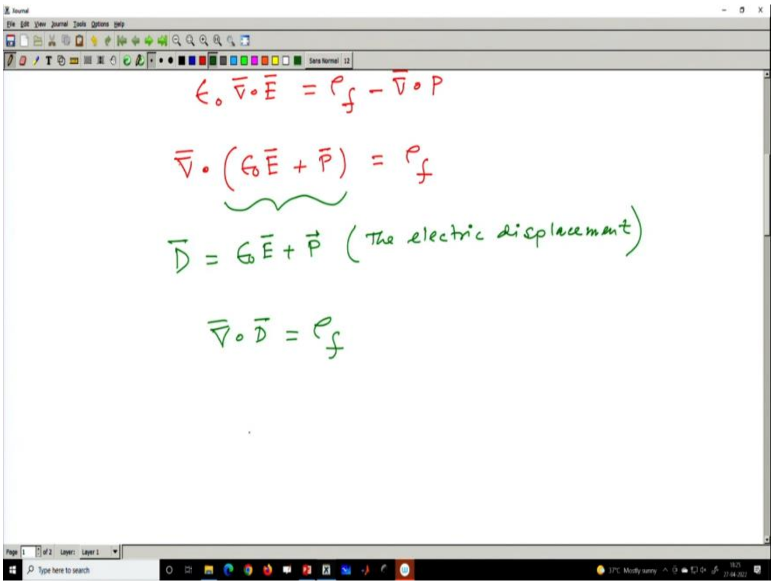

So, this new quantity we call this new quantity whatever we are getting here we call the electric displacement or displacement vector we defined it as $\vec{D}$, which is equal to $(\epsilon_0 \vec{E} + \vec{P})$ and that we call the electric displacement or displacement vector. So, the formula the Gauss's law is modified for $\vec{D}$ and now we are having the $\vec{\nabla} \cdot \vec{D}$ is equal to free charge density ρ.

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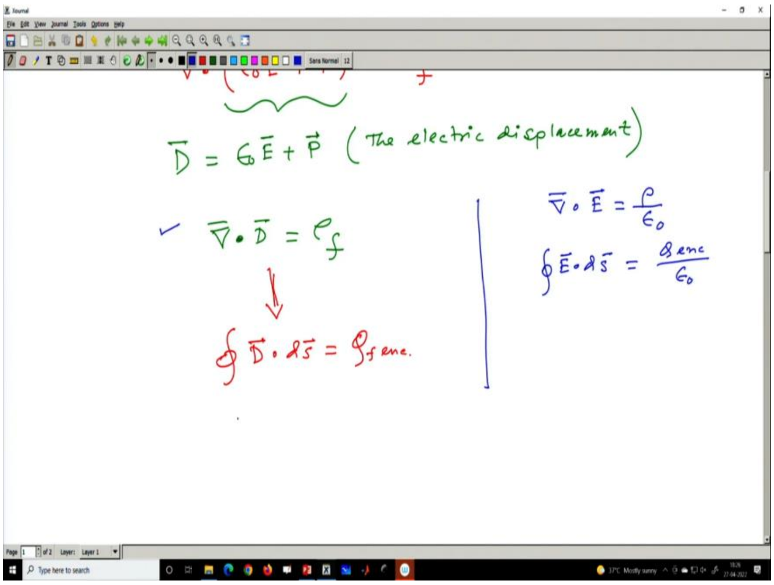

So, if I now write this expression to the integral form then the integral form leads to integral form give us that the close surface integral $\vec{D} \cdot d\vec{s}$ should be $Q_{f\ enc}$ mind it we have side by side. So, for electric field we have this expression when there is no dielectric material, so, I have $\frac{\rho}{\epsilon_0}$ and also we had the flux of the closed surface is simply $\frac{Q_{enc}}{\epsilon_0}$.

So, now for the $\vec{D}$ the expression is changed ϵ_0 is absorbed inside $\vec{D}$ and we have an expression that the differential form of the modified Maxwell's equation is this and this is the corresponding integral form.

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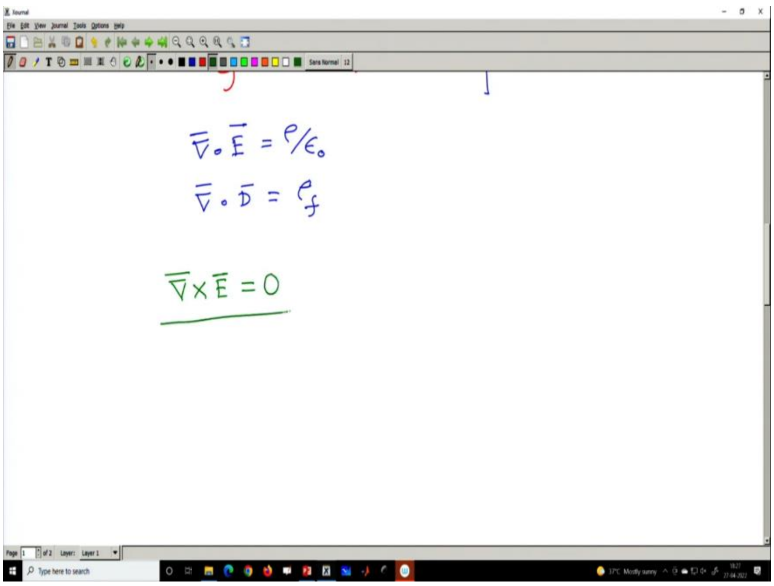

Now, as I mentioned that $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ and $\vec{\nabla} \cdot \vec{D} = \rho_f$ now, if I look carefully the $\vec{\nabla} \times \vec{E} = 0$.

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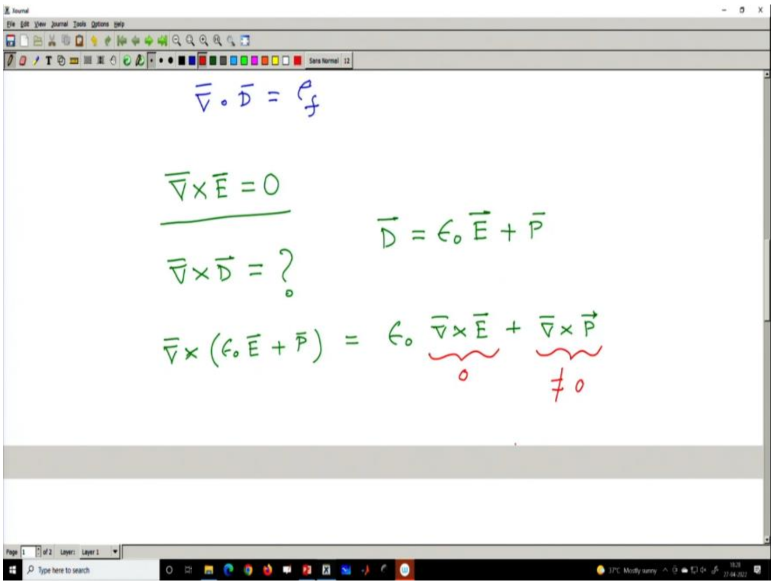

Now, the question is should I write the $\vec{\nabla} \cdot \vec{D}$ to be 0? So, what is this value should I write it 0? The answer is no, because I should have a quantity like $\vec{\nabla} \times \vec{D} = \vec{\nabla} \times (\epsilon_0 \vec{E} + \vec{P})$ because $\vec{D}$ here is $(\epsilon_0 \vec{E} + \vec{P})$ by definition. So, this curl gives us 2 quantity one is $\epsilon_0 \ \vec{\nabla} \times \vec{E}$, which is definitely $0 + \vec{\nabla} \times \vec{P}$. Now, even though this quantity 0 this quantity is not equal to 0, it is not necessarily that $\vec{\nabla} \times \vec{P}$ should be 0.

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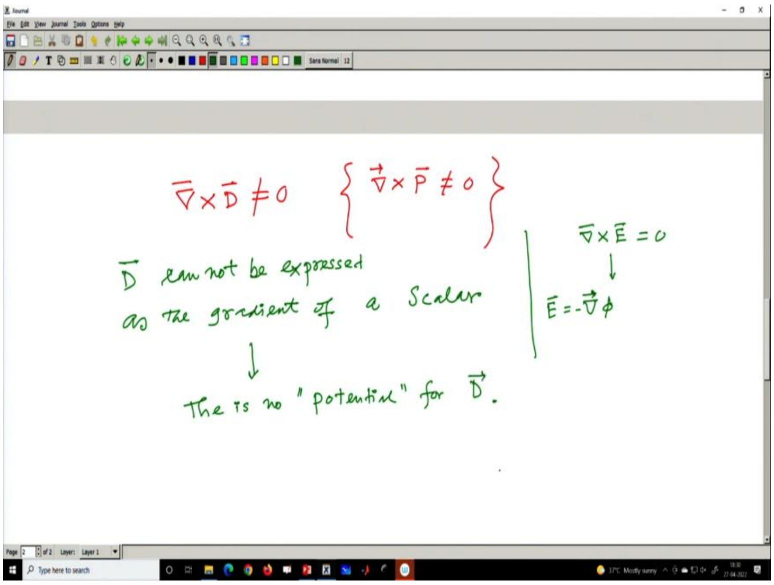

So, in general I should write that $\vec{\nabla} \times \vec{D}$ is not equal to 0 mainly because of the fact that curl of polarisation, which is arises due to the dielectric property of the material is not equal to 0 in general and that is why $\vec{\nabla} \times \vec{D}$ is not equal to 0. Since $\vec{\nabla} \times \vec{D}$ is not equal to 0 then we can also argue that $\vec{D}$ cannot be expressed as the gradient of a scalar function or scalar field, which we can have for $\vec{E}$. So, for $\vec{E}$ what happened, the $\vec{\nabla} \times \vec{E} = 0$.

And that leads to the condition that $\vec{E}$ can be represented the gradient of something and that we call the potential this is the consequence of the fact that $\vec{\nabla} \times \vec{E} = 0$ that means, $\vec{E}$ can be always represented in terms of gradient of something, but in case of $\vec{D}$ it is not possible. So, that means, there is no such thing called potential for the vector $\vec{D}$. There is no potential for $\vec{D}$. So, we have a very brief understanding of the displacement vector $\vec{D}$ and now, we will know based on that we will continue few things.

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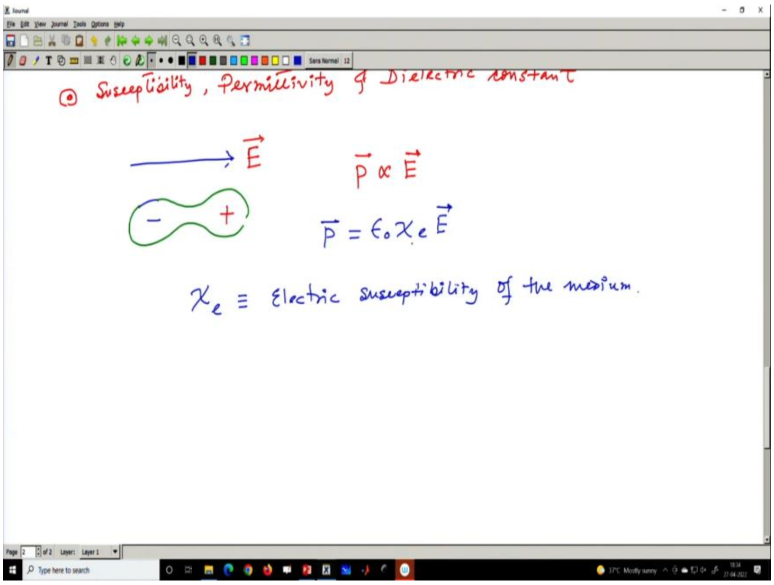

So, that is we want to understand once again the susceptibility and then the permittivity of the system and the dielectric constant these are few well-known terminology. So, we need to understand once again because we have already discussed about the polarizability and polarizability tensor etc. in last class. So and also understand just now, what is the displacement current and displacement vector.

So, with the base of that, let us understand this. So, suppose that we have an electric field here and for the dielectric system what happened a dipole is induced like this, this is the electric field and we have plus here and minus here so, some induced dipole is there and let us consider the $\vec{P}$ the polarisation is proportional to $\vec{E}$ and that is when $\vec{E}$ is not too strong obviously, if $\vec{E}$ is too strong then I can write it in this way.

And in the scalar form I can write that $\vec{P} = \epsilon_0 \chi_e \vec{E}$ this is the way we can represent earlier we represent in terms of α, but now, I write in a different way because that leads to this terminology susceptibility, permittivity and dielectric constant. So, this is the way one can write in general. So, here this χ_e is called the electric susceptibility of the medium. So, for linear medium, where this quantity is no longer a tensor quantity, so, we should have like the polarizability tensor. So, here we should have the susceptibility tensor in general form, but let us consider it the medium to be linear.

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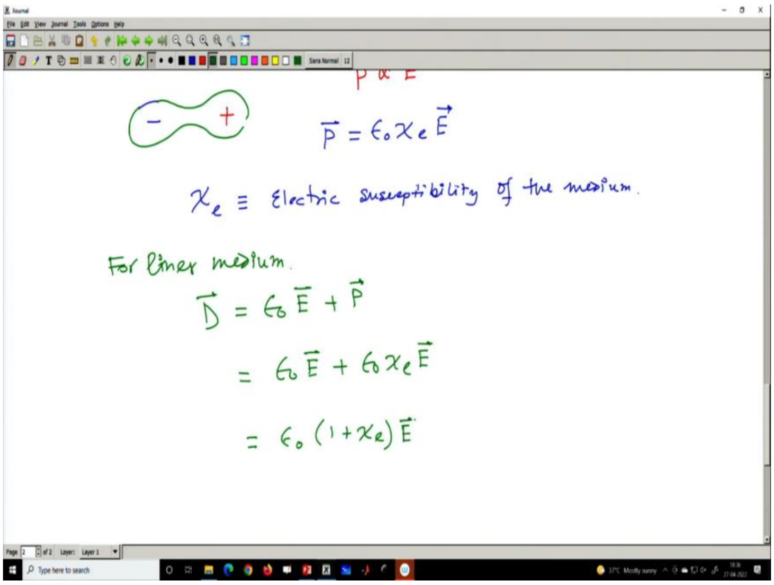

So, we will say that this is simply a scalar quantity in that case my for say linear homogeneous kind of medium we have a straightforward relation that $\vec{D}$ should be equal to $\epsilon_0 \vec{E} + \vec{P}$ by definition and my $\epsilon_0 \vec{E}$ I can write it and then $\vec{P}$ I write in this form so, it becomes ϵ_0 χ_e $\vec{E}$. So, if I take ϵ_0 common then we have $(1 + \chi_e \vec{E})$.

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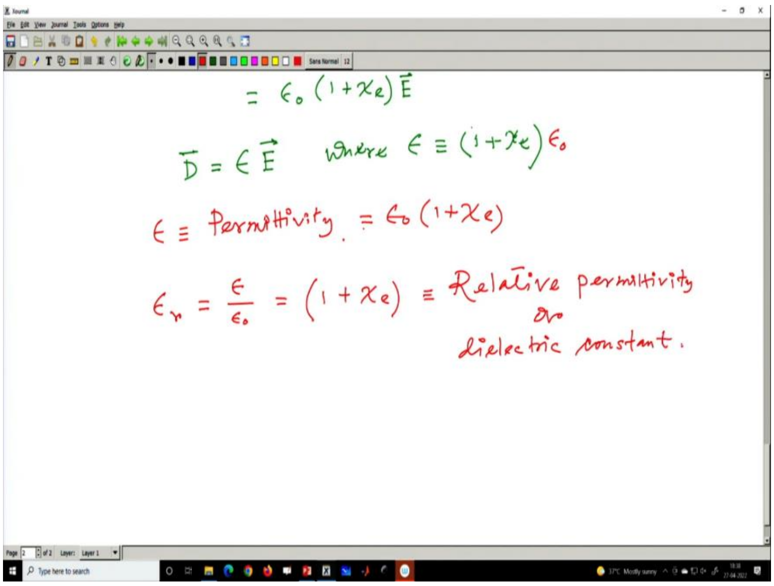

And $\vec{D}$ and $\vec{E}$ this relation now, we can write in this form it is ϵ_0 $\vec{E}$ where ϵ_0 is equal to $1 + \chi_e$ now, this ϵ_0 whatever is called the permittivity of the system. The ratio normally we have ϵ_0 is equal to which is equal to ϵ_0 here I should write an ϵ_0 multiplied by $(1 + \chi_e)$. So, the ratio be called the relative permittivity or the dielectric constant and that is represented by $\frac{\epsilon}{\epsilon_0}$ and that is simply $1 + \chi_e$, which is a number because this is the ratio of 2 same quantity.

So, this is called the relative permittivity or dielectric constant. So, now in the light of these definitions, so, we can understand few more things and that is when the dielectric is placed in a medium already we discuss this issue.

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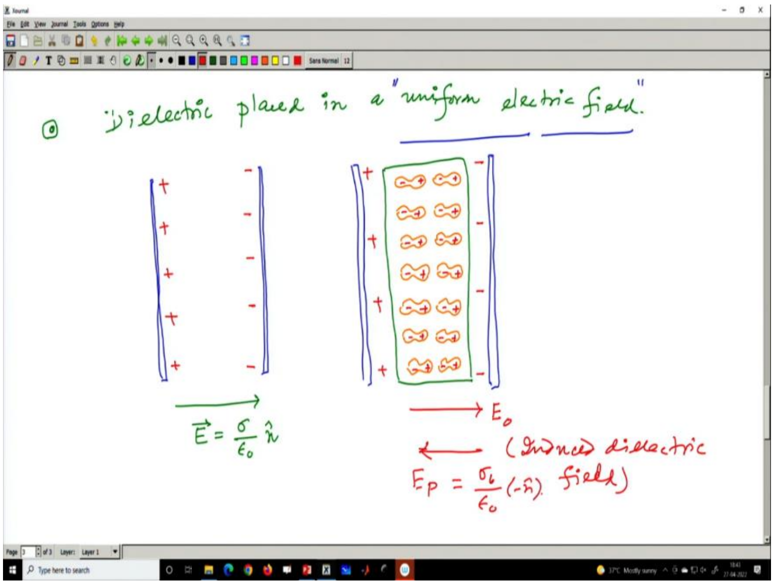

But let us consider this once again. So, say dielectric placed in a uniform electric field so, what happened so, suppose I have uniform electric field how do I get this uniform electric field that we need to understand that how to get this uniform electric field. One simple way to get this electric field is suppose I have a 2 capacitor plate like this so, this is the capacitor plate and we have a plus charge here and say minus charge here.

So, in between I can have my electric field, which is uniform and this electric field the value of this electric field is simply $\frac{\sigma}{\epsilon_0}$ along the direction $\hat{n}$ if this is my direction $\hat{n}$. Now, in this uniform electric field now, I will place the dielectric. So, what happened that this figure I have already drawn in a different way. So, this is the plate and the plus distribution of this and a dielectric material is placed.

So, this is my dielectric material that I placed in between these so, that it will be going to experience a uniform electric field so, tiny dipoles are going to you know one can find these kinds of tiny dipoles and we have like plus minus plus minus plus minus plus minus plus minus and so on. So, tiny dipole will be induced under this uniform electric field and the figure should be like something like that. So, this is the applied electric field, which is uniform and if I write this is $\vec{E}_0$ because this is a constant.

So, induced dipole field will be in opposite direction under equilibrium. So, this is say $\vec{E}_P$ that is my induced this is you know the induced dielectric field. This induced dielectric field will be simply the bound charge whatever it is having and with this.

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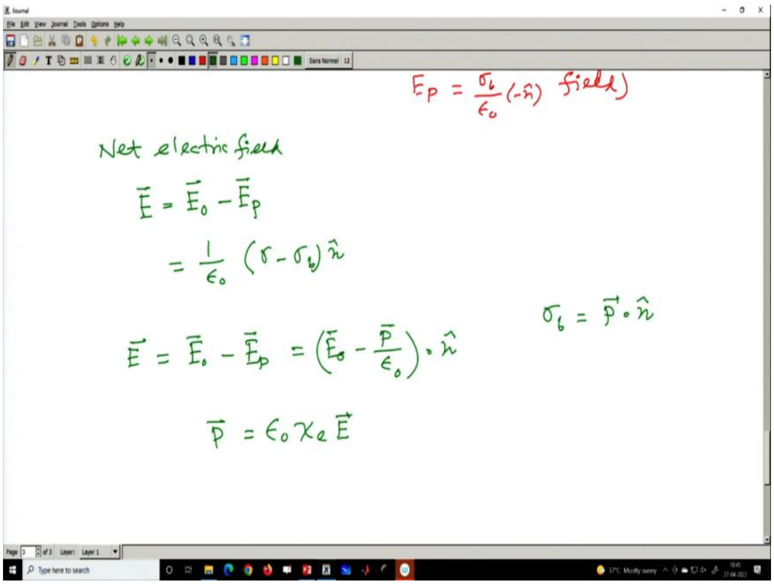

Now the net electric field should be $\vec{E} = \vec{E}_0 - \vec{E}_P$ and that is $\frac{1}{\epsilon_0}$ ($\sigma - \sigma_b$) with the direction of $\hat{n}$ and again $\vec{E} = \vec{E}_0 - \vec{E}_P$ and that is $\vec{E}_0$ - $\frac{\vec{P}}{\epsilon_0}$ because this bound charge is like $\vec{P} \cdot \hat{n}$ and $\vec{E}_P$ is this divided by ϵ_0. So, it will lead to $\frac{\vec{P}}{\epsilon_0} \cdot \hat{n}$. Now, $\vec{P}$ is again ϵ_0 χ_e $\vec{E}$ the external electric field. That so, in this case here it should be the $\vec{E}_0$ is my external electric fields it should be $\vec{E}_0$ here.

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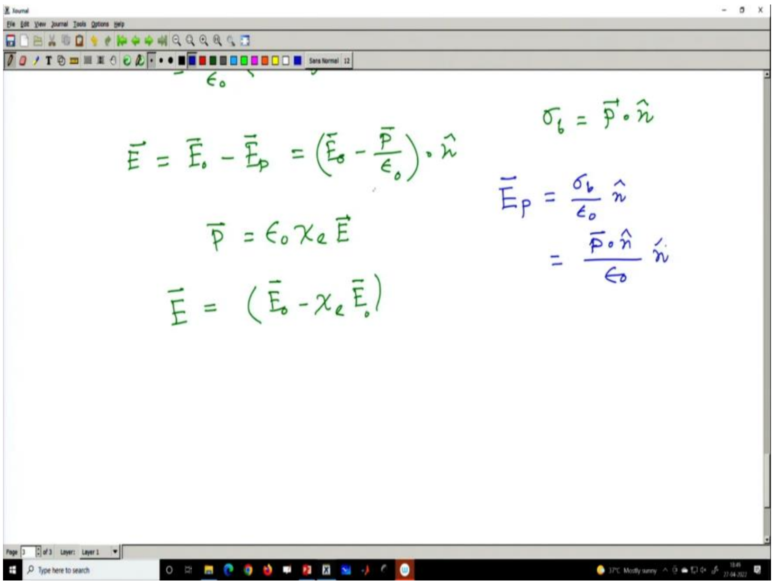

So, $\vec{E}$ the resultant electric field is $\vec{E}_0$ - χ_e $\vec{E}$ no this is not $\vec{E}_0$ because in general the polarization will happen because of this electric field whatever the electric field we are having here the net electric field and eventually we have $\vec{E}$ as $\vec{E}_0$ $\vec{E}$ has it was not so, because the field is produced by $\vec{E}_0$ here I noted I put it here $\vec{E}_0$ not $\vec{E}$. $\vec{E}$ I represent as because I should this is the uniform electric field and I write the uniform electric field in so, this is the net electric field.

So, I can write the uniform electric field should be like something this with that is the confusion I am having here. So, now I can have this expression where I should have the relationship between the electric field and that leads to you know let me erase this. So, I am now having let me go back and check it because this nomenclature is something we need to be careful. So, this is $\vec{E}$ and this is the $\vec{E}_0$ I am applying this is the uniform electric field that I am applying and this is the induced dielectric, which I write in terms of $\vec{E}_P$.

And this induced dielectric field should be $\frac{\sigma_b}{\epsilon_0}$ and it is in the opposite direction so minus now try to find out the net electric field. So, the net electric field is the addition of these 2 since it is in opposite direction, so, that is why I have a negative sign and I have ϵ_0 $(\sigma - \sigma_b)$. Now, $\vec{E}$ = $\vec{E}_0$ – $\vec{E}_P$ and $\vec{E}_P$ I now write in terms of polarisation, because $\vec{E}_P$ is σ_b here I am making a mistake I guess.

So, $\vec{E}_P = \frac{\sigma_b}{\epsilon_0}$ and then $\hat{n}$ and $\vec{E}_P$ is $\frac{\vec{P} \bullet \hat{n}}{\epsilon_0}$ and $\hat{n}$ and now, here, the polarisation that I should, because this polarisation should be based on this total electric field that we are having because this $\vec{P}$ is generated due to the total electric field that we are having here. So, that is why this should be $\vec{E}$ here. And now, I have the $\hat{n}$ this is the if I write the vector then I do not need to use this $\hat{n}$.

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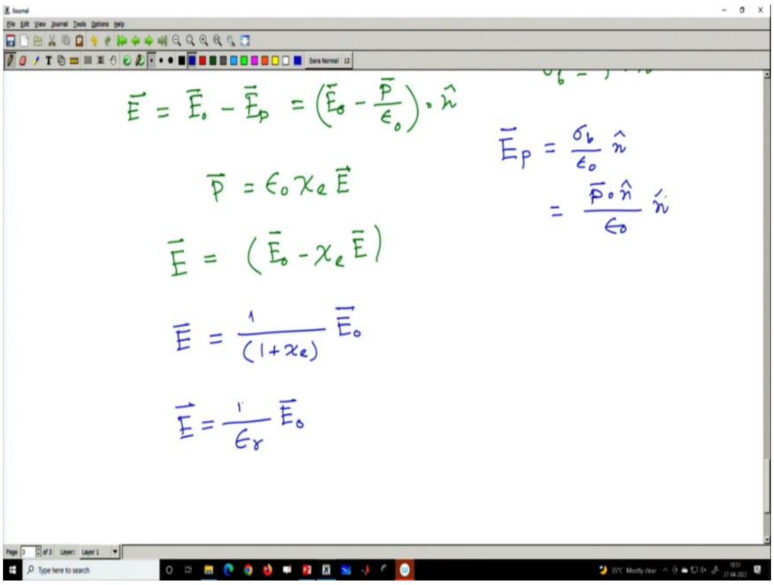

Rather I just simply write my $\vec{E}$ net electric field is equal to 1 divided by 1 plus this quantity and $\vec{E}_0$ where $\vec{E}_0$ is the amount the constant electric field under which these things is placed. So, from here I can write that $\vec{E}$ is simply $\frac{1}{\epsilon_r}$ $\vec{E}_0$. So, this is just to show that if I place this dielectric in a uniform electric field, so, what should be I mean how the dielectric will behave and what is the relationship between the net electric field and the uniform electric field that we are getting.

So, the ratio is showing that the net electric field will be reduced by this $\frac{1}{\epsilon_r}$ time of the uniform electric field that is having and this reduction is due to the fact that we are having here a dielectric that is placed and this induced dielectric is now in opposite direction and that is the reason why we have a ratio and that ratio will be reduced. Because of the place of the dielectric if it is in air, then this value should be 1, but if I placed a dielectric in between then I should have something like $\frac{1}{\epsilon_r}$.

So, this ratio should appear because of the placement of the dielectric. So, we will come this again when we discuss about when you know the 2 dielectric when we place the dielectric in between the capacitor to increase the capacitance. So, during that time again this discussion I would like to make few problems we will do and then it will be discussed once again. Now, I will do one thing that is the final task today and that is the boundary condition.

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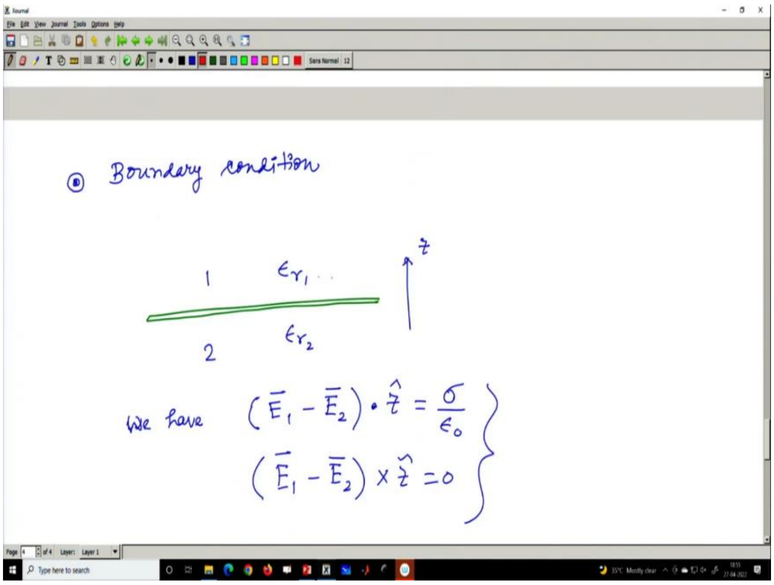

The boundary condition for the dielectric is important because I have a say boundary here say I have here dielectric 1 and dielectric 2 having dielectric constant say D_{r1} D_{r2} and this is suppose my z direction for the electric field we already have the boundary condition or matching condition we have and that condition is $\vec{E}$ in this figure $(\vec{E}_1 - \vec{E}_2) \cdot \hat{z} = \frac{\sigma}{\epsilon_0}$ and another was $(\vec{E}_1 - \vec{E}_2) \times \hat{z} = 0$.

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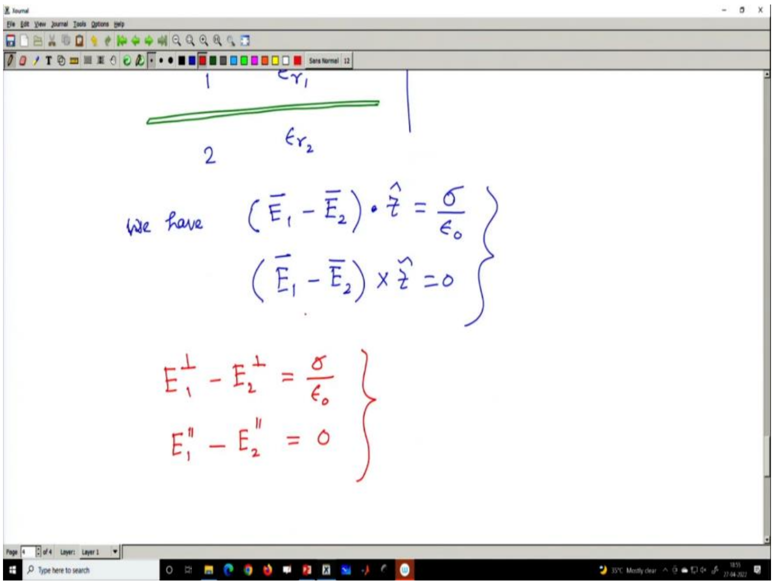

In another word the perpendicular component of $(\vec{E}_1 - \vec{E}_2)$ that is $\frac{\sigma}{\epsilon_0}$ and the parallel component of the 2 section is 0 that means, the parallel component is continuous whereas, the perpendicular component is not. Now, the question is if I know the relationship of the boundary condition of this $\vec{E}$ so what should be the condition for $\vec{D}$ because for dielectric system $\vec{D}$ is rather meaningful than $\vec{E}$.

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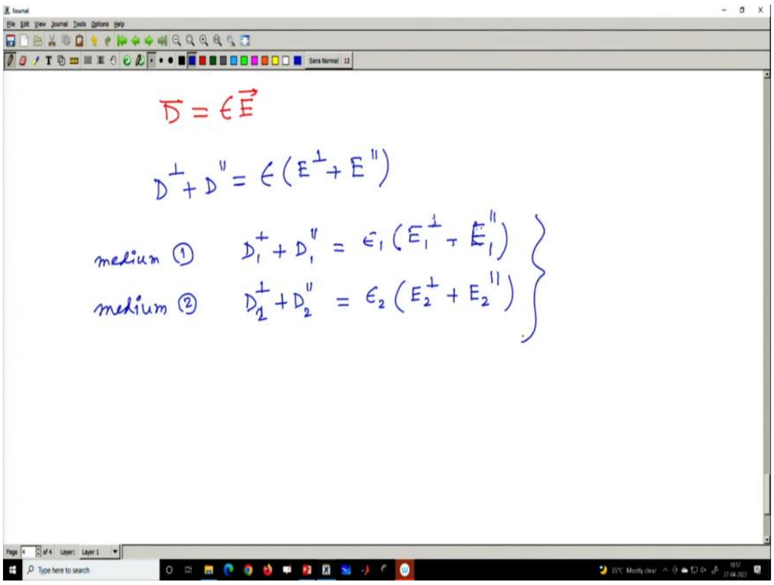

So, I know the relationship with $\vec{D}$ and $\vec{E}$ and that is simply $\vec{D} = \epsilon\, \vec{E}$. So, one thing I have here first that $D^{\perp} + D^{\parallel}$ should be equal to you know ϵ and then $E^{\perp} + E^{\parallel}$ so, for medium 1 and medium 2 for medium 1 we have $D_1^{\perp} + D_1^{\parallel}$ that is equal to ϵ_1 $E_1^{\perp} + E_1^{\parallel}$ I just divide the entire vector to perpendicular and parallel component that is all.

For medium 2 I simply have $D_2^{\perp} + D_2^{\parallel} = \epsilon_2$ and then $E_2^{\perp} + E_2^{\parallel}$. So, now we can have I mean these when we write these vector in component wise then this component wise they should be equal and that means.

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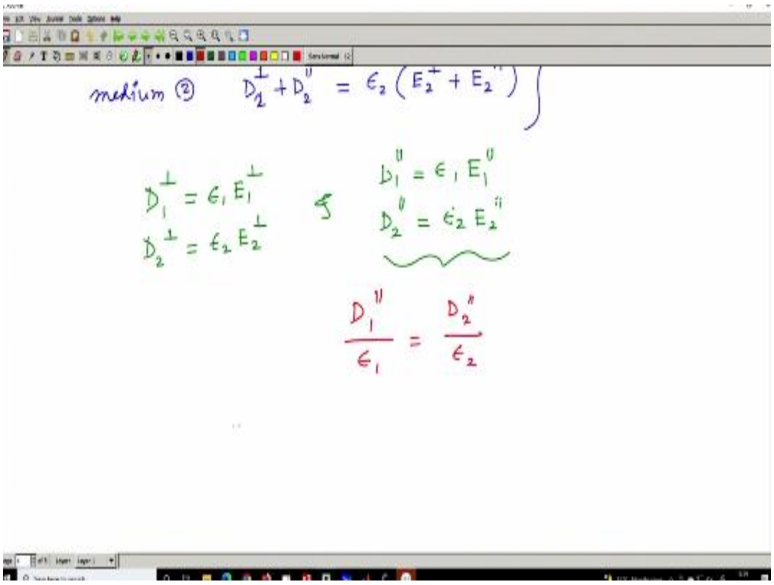

Now I am like we can write that $D_1^{\perp}$ is simply equal to $\epsilon_1\ E_1^{\perp}$ and $D_2^{\perp}$ is simply equal to $\epsilon_2\ E_2^{\perp}$ and also $D_1^{\parallel} = \epsilon_1\ E_1^{\parallel}$ and $D_2^{\parallel} = \epsilon_2\ E_2^{\parallel}$. So, from here from this expression we know that $E_1^{\parallel} = E_2^{\parallel}$. So, that means from here exploiting this equation I can simply have that very important equation. That $\frac{D_1^{\parallel}}{\epsilon_1} = \frac{D_2^{\parallel}}{\epsilon_2}$.

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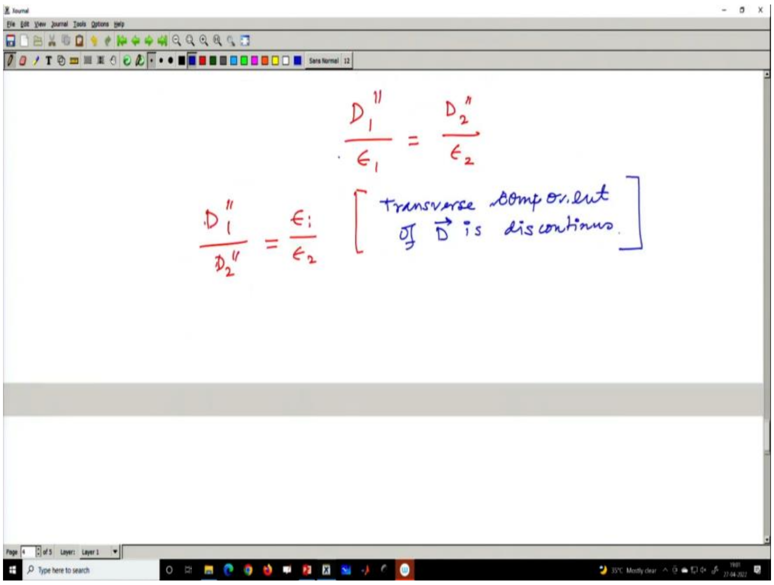

Or in other words, I have $\frac{D_1^{\parallel}}{D_2^{\parallel}}$ ratio of the parallel component of the 2 these things is $\frac{\epsilon_1}{\epsilon_2}$. That means, previously if you see that $\frac{E_1^{\parallel}}{E_2^{\parallel}}$ was 1 that means, they are continuous, but here we can see that the transverse component so, we can conclude here the transverse component of vector $\vec{D}$ is discontinuous.

So, the transverse component of the $\vec{D}$ is discontinuous, which is in contrast with the electric field $\vec{E}$ electric field the transverse component was continuous, but for dielectric system the displacement vector is rather important and we see that the parallel component is discontinuous. What about the perpendicular component because from here we cannot you know have any so, we can find it in a different way.

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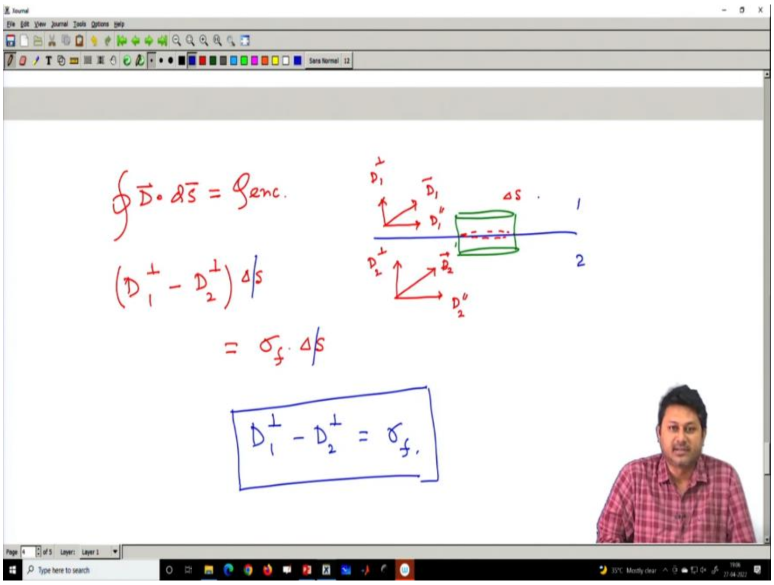

So, we know this expression the closed surface integral of $\vec{D} \cdot d\vec{s}$ is equal to the total charge enclose. So, that means, here this is mind it this is the whatever the free charge we are having here we have the surface like this and in order to execute this integral we can find a Gaussian surface a Gaussian cylindrical surface like this. So, here so, if I now divide my D say this is the D_1 this is what was my one, one is upward.

So, this is my 1 and this is 2. So, D_1 suppose, it is in this direction, I can divide into 2 parts one is this component and another is this component. So, this is my D_1 vector and this is $D_1^{\perp}$ and this is $D_1^{\parallel}$ in the similar way in the downward I can have a component in this and this. So, again this is $D_2^{\perp}$ and this is $D_2^{\parallel}$ and this is the D_2 vector and this area is Δs.

And now, if I execute that, then we find that for upper and for lower the integration is meaningful and this portion we can make these things infinitely small and we can neglect these integrals. So, in that case, we have these integral simply give gives us $D_1^{\perp}$ - $D_2^{\perp}$ because minus because of this direction and this direction the direction of the surface is different and opposite $\Delta s = Q_{enc}$. Now Q_{enc} means the total enclosed charge.

And this total enclosed charge should be the free surface charge density dot the surface area, whatever the surface charge is accumulated in this surface. So, Δs Δs will going to cancel out and we have the relationship like $D_1^{\perp}$ - $D_2^{\perp}$ is simply σ_f so, that is another relation we are having here also we find the perpendicular component of the $\vec{D}$ is not continuous in this boundary.

So, both the component perpendicular and parallel component is found to be not continuous, if the 2 dielectric if we have a boundary for 2 dielectric system. So, here we have 1 dielectric and another upper we have another dielectric and we find that for 2 dielectric medium, the boundary condition is saying that the perpendicular component and parallel component of the $\vec{D}$ both are discontinuous and the amount of discontinuity you can also calculate from this treatment.

So, well today I already finish this discussion. So, in the next class, I will start the calculation of the Laplace and Poisson's equation in solution and do some problem as a tutorial and then after that we will go to start the next module 3. So, thank you for your attention and see you in the next class.