Foundation of Classical Electrodynamics
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Lecture – 43
Dielectric (Contd.,)

Hello student to the course of foundation of classical electrodynamics. So, today we have module 2 and under module 2, today, we are going to study the dielectric that we started in the last class, we are going to continue more topic on dielectric.

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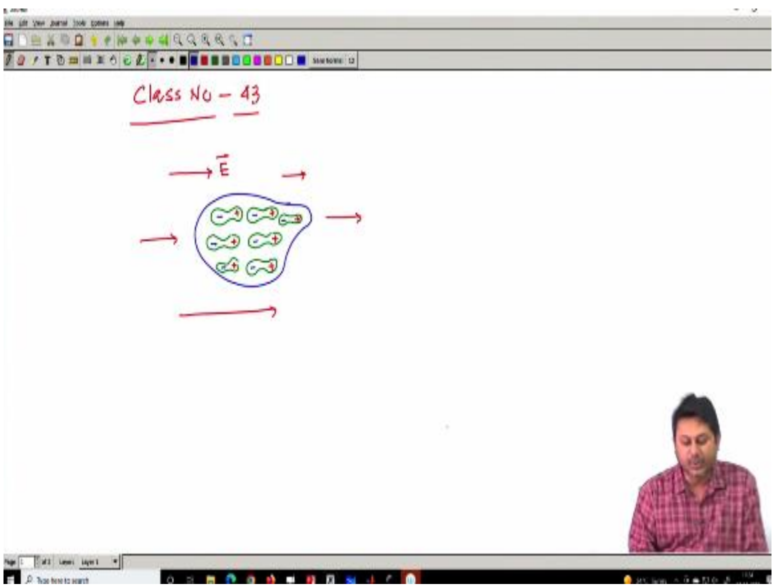

So, we have class number 43 today and we will continue our discussion on dielectric. So, let us find so, if I have a chunk like this, which is a dielectric material and if I apply some external electric field $\vec{E}$ like this. So, under this external electric field I placed the dielectric and as a result what happened that tiny dipoles will go to form if I try to present this pictorially. So, there will be charge separation like this and we will have a dielectric system here.

Now, we know that this quantity should have a property called polarization, which is the dipole moment per unit volume.

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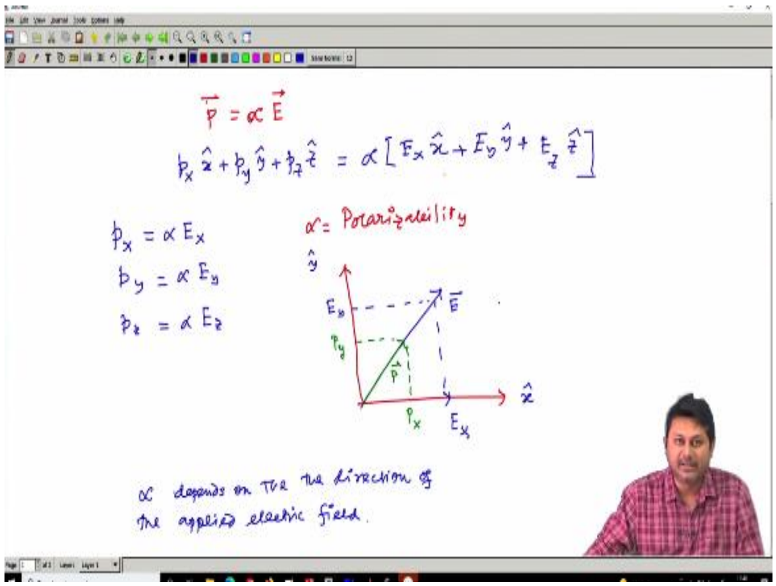

So, that polarization $\vec{P}$ is proportional to $\vec{E}$. So, I can write this proportionality constant as α and the expression the relationship with the electric field and polarization I can write in this way. Now, here what I mentioned is, so, $\vec{P}$ is proportional to the strength of the applied electric field that is and not only that, it is the same I mean if the 2 vectors are if I write in like $\vec{P}$ as $P_x\,\hat{x} + P_y\,\hat{y} + P_z\,\hat{z}$. So, that is in vector form and then I can have α then $E_x\,\hat{x} + E_y\,\hat{y} + E_z\,\hat{z}$.

So, that eventually means that $P_x = \alpha E_x$ and $P_y = \alpha E_y$ and $P_z = \alpha E_z$ component wise they are same that means the x component of the polarization vector depends on the x component of the applied electric field, y component of the polarization vector will depend on the y component of the electric field and z component will depend on the z component of the polarization will be going to depend on the z component of the electric field and all the cases that this is α.

So, here α is the polarizability of the system. Now, for this case what happened if I have a coordinate system like this and if I apply the electric field say having 45 angle and then the 2 components of the electric field say I can write like this is my E_x, this is along unit vector, this is along $\hat{y}$ and this is my E_y and whatever the P I plot here is in the same direction and I should also have P_x here and P_y here this is my $\vec{P}$ and this is my $\vec{E}$. So, this is the most simplest case.

But here I mean if α depends on the direction of the applied electric field. So, now α depends on the direction of the applied electric field that means.

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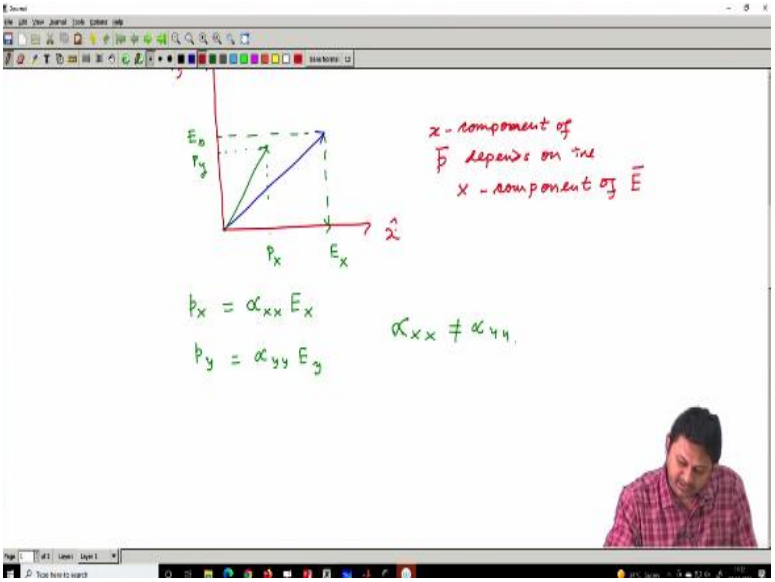

So, let me draw another case like this is my coordinate system x and y let us consider only 2 dimension and this is the direction along which my electric field is applied, but my polarization now is no longer in the same direction of the electric field here but the x component of the polarization is still depends on the x component of you know the electric field. So, this is my electric field. So, electric field should have E_x along the x direction this is E_x and this is E_y and the polarization P_x is still depends on E_x and P_y still depends on E_y.

But like before the ratio is not same. That means, now, P_x depends on still depends on E_x say but I should write here this α_{xx} E_x and for P_y it is α_{yy} E_y when α_{xx} is not equal to α_{yy} this is one condition we can think of. So, that means the α previously it was a scalar quantity, but now we find that it should have different components. And that is why it no longer becomes a scalar quantity rather I should have a tensor form of α.

Because if now here what happened the x component of P depends on the x component of E still we have these things, but more general situation can be considered and that is the x component of the more general case is so, I can pictorially I can show that.

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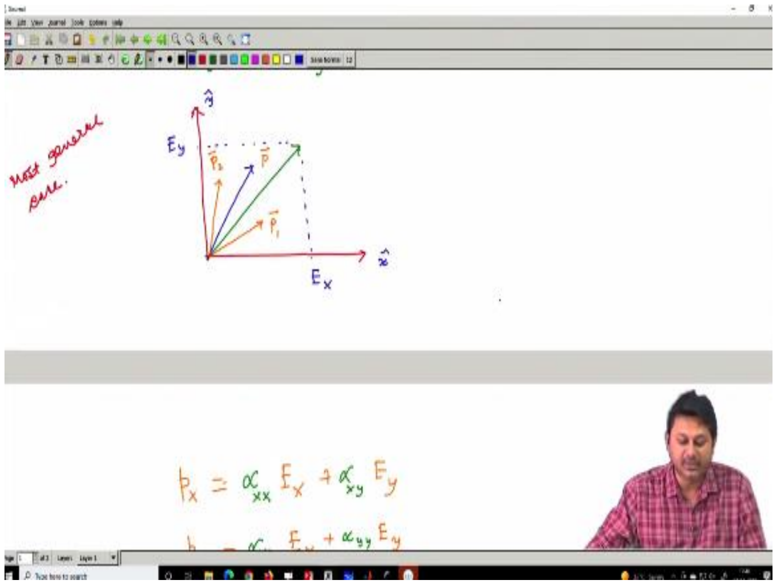

And the most general case is when E suppose, I have this coordinate system and my E is along this direction some direction P is not parallel to E so, it should have some other direction not only that, so, this is my E_x suppose, I divide this is my say E_x and this is my E_y I can separate coordinate the component in x and y direction, but whatever the P I am having its x component does not depend on only E_x it can also depends on E_y. So, maybe I can draw in this way so, P_1 I can say its another component here is P_1.

Maybe in different colors I can divide it into 2 components like P_1 and say P_2 say this is my full P component and I can have these as a combination of P_1 and P_2 and here you can see that it is not parallel, but also I mean if I divide this stuff, then the x over x component of P overall x component of P now, not only depends on the x component of E but the y component as well. And now, I can have these here the constant E_{xx} and E_{xy} because here the x component of P does not depend on the x component of E it also depends on the y component of E.

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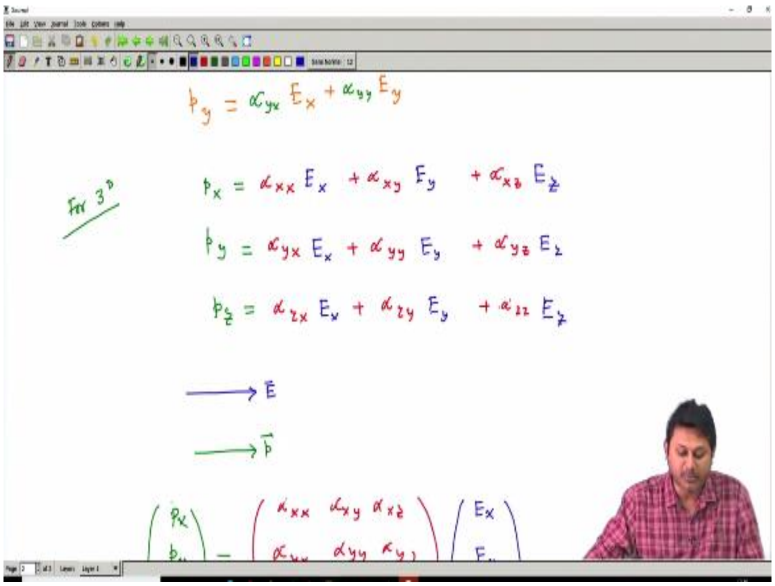

In the similar way, the y component of the P can also depend on the x component of the E plus the y component in 2 dimension y component of the E. And in that case, I should write here like yx and α_{yy} to distinguish these things. So, in general for 3 dimensional this is for 2D in general for 3 dimensional case. So, P_x component can depend on E_x, E_y and E_z and now I can write it like α_{xx} + α_{xy} + α_{xz} and my E component will be like E_x E_y and E_z in a similar way I can write P_y and P_z.

Explicitly if I write, then I should write here α_{yx} + α_{yy} + α_{yz}, α_{zx} + α_{zy} + α_{zz} and the components are E_x here I should write E_x here I should write E_y E_y E_z E_z so, my electric field is E and my polarization is P, but the x component of these things depends on the x y z component of the E and so on.

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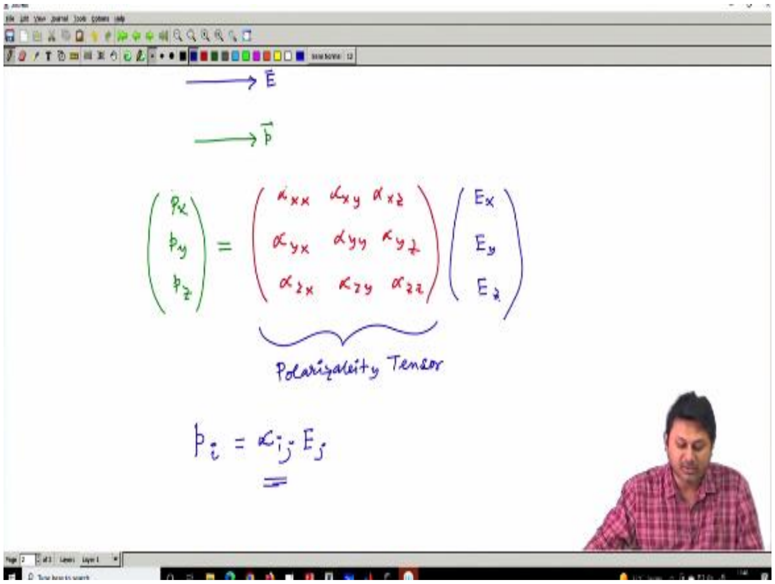

So, I can write this entire thing in matrix form and if I do I simply have this P_x P_y P_z is equal to α_{xx} α_{xy} and α_{xz} α_{yy} α_{yx} α_{yz} α_{zx} α_{zy} α_{zz} bracket close and the electric field, which is the component-wise E_x E_y and E_z this is so, α is simply the polarizability tensor and I can write it in this convenient form it is called component form P_i is simply α_{ij} E_j in Einstein notation the summation is over repeated index. So, I simply have this.

So, this is the one important expression for this polarizability where we can show that the polarizability is in fact a tensor quantity and there are different systems for which if you apply the electric field the polarization is no longer along the applied electric field there will be a different direction and not only that, its x component, y component and z component no longer depend on the x component, y component and z component of the electric field alone, the x component of the polarizability can be depended on the x, y, z components of the applied electric field.

And in that case, we are having these generalized form. So, from here you can simply find that if it is only P_x only depends on E_x and P_y only depends E_y and P_z it is only depended on E_z and the dependency is same then we should have a diagonal matrix where α_{xx} α_{yy} and α_{zz} both are all 3 are same and we just simply write α then it simply becomes a scalar equation, which we wrote in the beginning this equation, but in general α is not a scalar quantity, it is a tensor quantity and how it becomes a tensor we try to explain briefly here.

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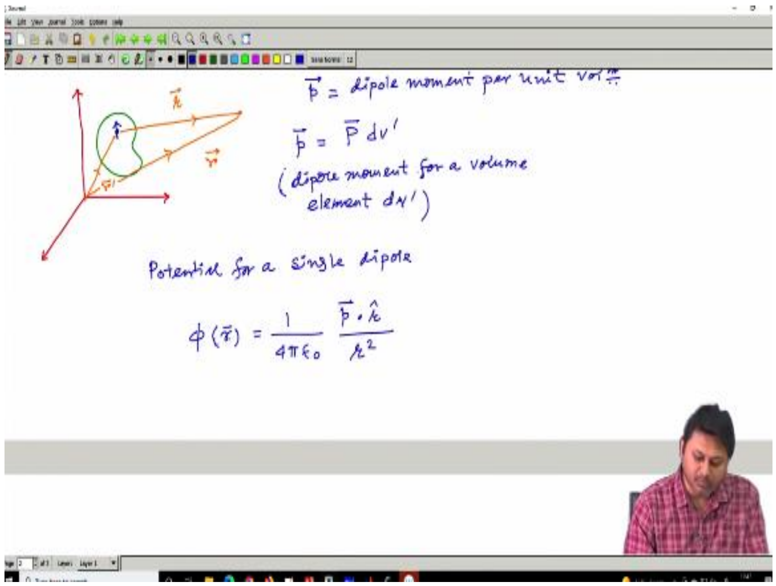

Now, we discuss about an important thing and that is the field associated with a polarized object. So, next topic is the field of a polarized object so, what does it means? So, suppose the polarized object is there and there should be a charge separation in between these polarized object and these polarized object should give us some kind of polarization and that means, some kind of dipoles are there and that dipoles give rise to some dipole moment.

So, because of that, we should have a field right for a dipole we calculate a field and here is something like happening something similar kinds of things will happen. So, we can find a corresponding field or corresponding potential due to the polarized object. So, let us try to find out what how to get it. So, suppose I have a coordinate system here like this and a polarized object is sitting here and it is characterized by a dipole moment and this is the dipole moment is the way we define the dipole.

And polarization is eventually dipole moment per unit volume. So, now, what should be the field due to this we can calculate and the location wise I need to so, the location here this coordinate is my r_1 see, say r'. So, from here to here this is a field where we point where we try to find out the field Л our standard notation and from origin to this value this is my r this is the system. So, polarization P as I mentioned polarization P is dipole moment per unit volume so, p is P dv dipole moment for a volume element is dv'.

So, if I want to find out the dipole moment per unit volume then I need to divide that. So, dipole moment for a so, this is a p is dipole moment for a volume element dv' small volume element this is the dipole moment I have. So, now for the potential for a single dipole that we know so, the potential for a single dipole is simply $\phi(\vec{r})$ we calculated this is equal to $\frac{1}{4\pi\epsilon_0}$ and dipole moment dot $\hat{Л}$ divided by $Л^2$ because the potential for dipole goes like $\frac{1}{Л^2}$. So, this is the form.

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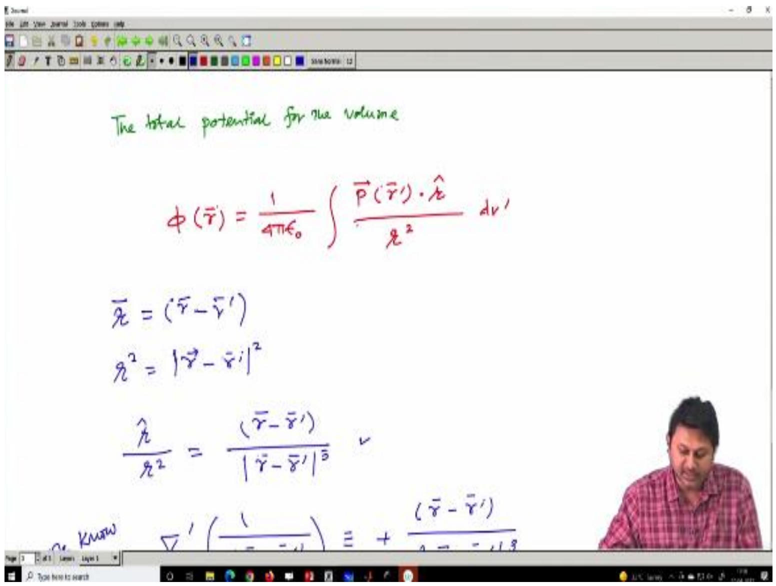

Now, for total potential for the volume so, the total potential for the entire volume that should be the integration I need to put integration because, So, $\phi(\vec{r})$ for total should be equal to $\frac{1}{4\pi\epsilon_0}$ and integration $\frac{\vec{P}(\vec{r}') \bullet \hat{Л}}{Л^2}$ and then I integrate over the entire volume, so, I have dv' so, for dv I have for single dipole I have this and for the entire volume I need to integrate it over this.

So, $\vec{Л}$ is simply ($\vec{r}$ - $\vec{r}$') and $Л^2$ simply $|\vec{r} - \vec{r}'|^2$ so, $\frac{\hat{Л}}{Л^2}$ that term, which is there inside the integral I can write it as $\frac{(\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$.

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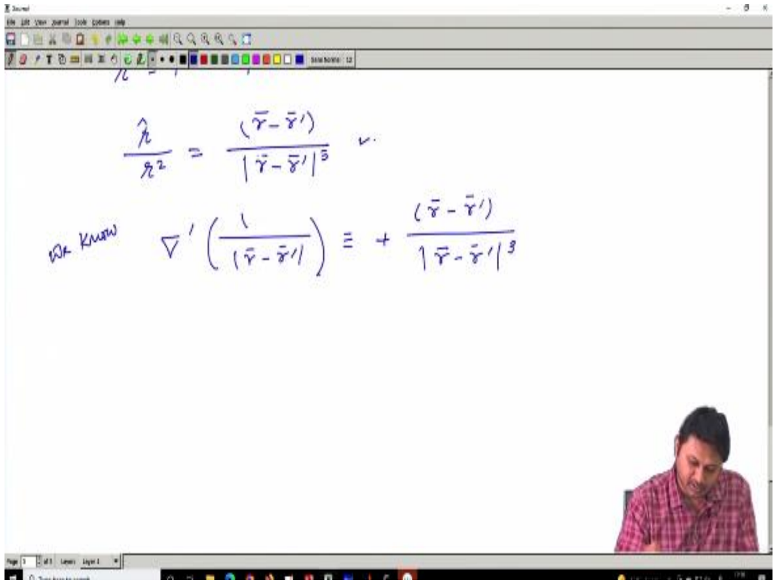

Now, we can have this so, we know this is a well-known thing that if I want to make a gradient of this vector this quantity $\frac{1}{(\vec{r}-\vec{r}')}$ in many places we have to use this then the result is + $\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ that quantity so, the positive sign because I am making the derivative with respect to prime and we have a negative sign already here.

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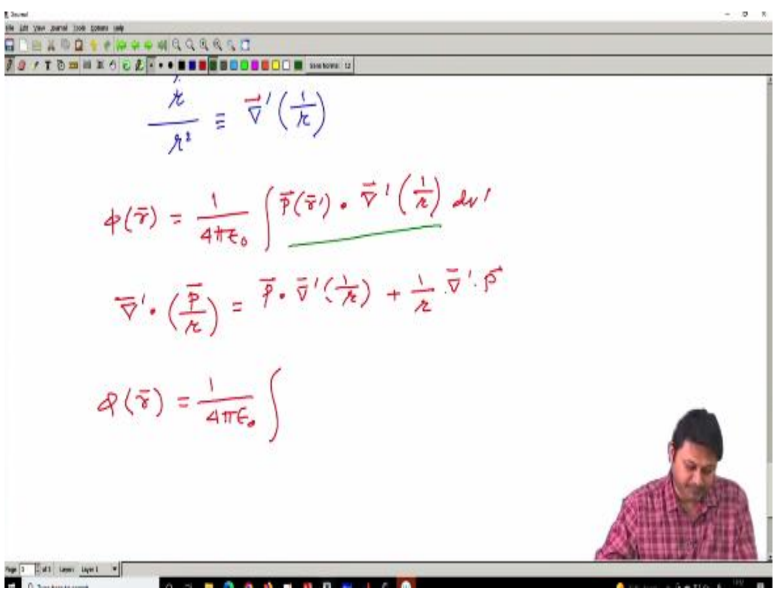

So, eventually I can write $\frac{\hat{\mathcal{r}}}{\mathcal{r}^2}$ is equivalent to grad the quantity $\frac{1}{\mathcal{r}}$, so, now I am going to make use here in this equation and I can write it as my $\phi(\vec{r})$ the potential is $\frac{1}{4\pi\epsilon_0}$ and then integration $\frac{\vec{P}(\vec{r}')\bullet\hat{\mathcal{r}}}{\mathcal{r}^2}$ I just replace this quantity prime and then $\frac{1}{\mathcal{r}}$, which we get here and then dv'. Now we are going to use another vector identity that prime dot $\frac{\vec{P}}{\text{Л}}$ the vector identities saying that it is $\vec{P} \bullet \vec{\nabla}'(\frac{1}{\text{Л}}) + \frac{1}{\text{Л}}$ and then $\vec{\nabla}' \bullet \vec{P}$.

And that we will use here that my $\phi(\vec{r})$ will be $\frac{1}{4\pi\epsilon_0}$ and if I integrate and make it 2 part then this quantity whatever I have I will go to replace here. So, this quantity I am having here this I replaced by this minus this.

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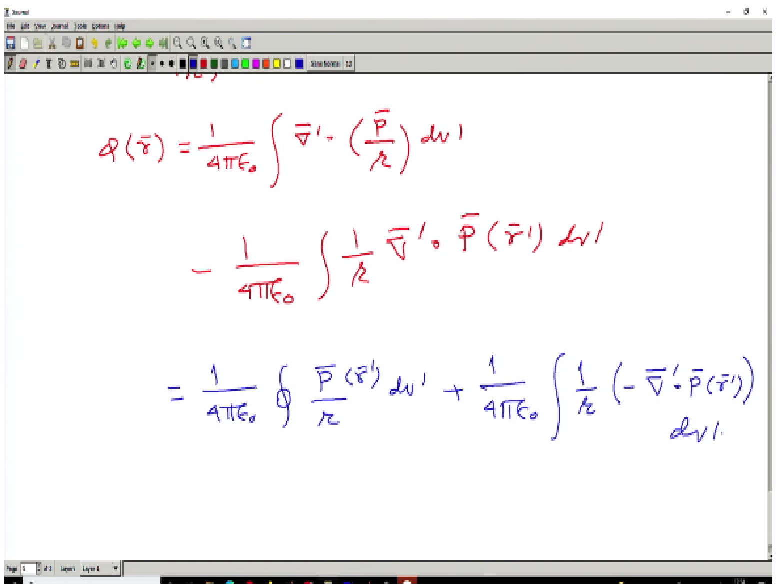

And if I do I should have say divergence my pen is not working properly $\vec{\nabla}' \bullet (\frac{\vec{P}}{\text{Л}})$ dv' and $-\frac{1}{4\pi\epsilon_0}$ and then integration $\frac{1}{\text{Л}}$ again I have prime and then $\vec{P}$ that should be $\vec{r}$' and dv'. So, now I can use this as my this integral I can replace to a closed surface integral because this is divergence of this quantity over volume. So, I can make it a surface integral like simply $\frac{\vec{P}}{\text{Л}}$ and that is over $\vec{r}$' dv' and the next integral you know here the operator is over this.

So, next integral I simply write $\frac{1}{4\pi\epsilon_0}$ and then integral $\frac{1}{\text{Л}}$ and I absorbed the negative sign to write minus of this dot $\vec{P}(\vec{r}\,')$ dv'. So, 2 term I find and these 2 terms I can now write these 2 terms in this way because one is surface integral and another is volume integral.

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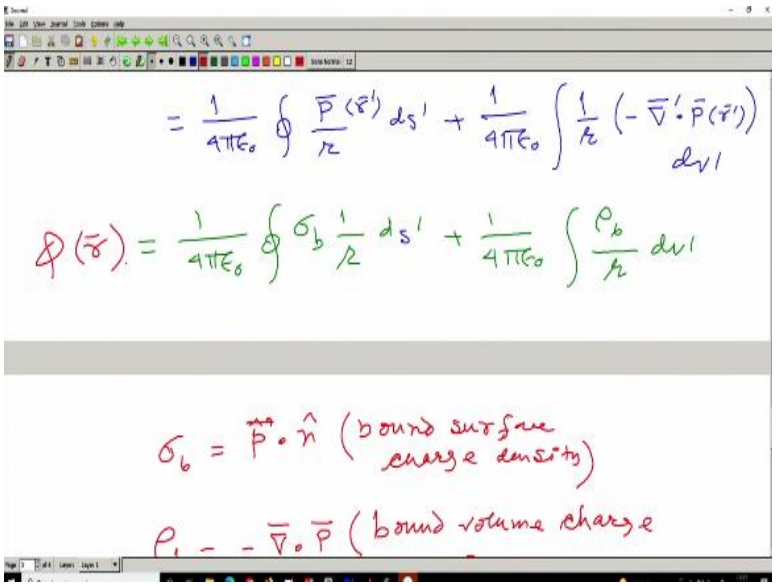

So, I can write it $\frac{1}{4\pi\epsilon_0}$ then the close surface integral and then I write the bound σ_b like a bound charge density then $\frac{1}{\text{л}}$ and then dv’ that is my sorry d here I make a mistake here because I make a surface integral so, these this should not be v this should be ds’ and here also it should be ds’ + $\frac{1}{4\pi\epsilon_0}$ and next term I can write is this write ρ_b bound volume charge density the term called and then dv’.

So, how you define that, so, I can define in this way so, σ_b is equal to $\vec{P} \cdot \hat{n}$ this is called the bound surface charge density and ρ_b, which is $-\vec{\nabla}' \cdot \vec{P}$ that quantity is called bound volume charge density from the expression of the potential you can see that this is my ϕ and if I just look carefully that this is a function of $\vec{r}$. So, it is contributing with 2 terms first here the term where we have a surface charge density and we say bound because it is the polarization polarized object.

And, because it is a polarized object it should have some bound charge and another here we have another contribution where we have a bound volume charge density. So, as if the potential is the contribution of a charge, which is bound and this charge is entirely over the surface and contribution of the another set of charges, which is again a bound charge, but this is the bound volume charge density that is why it is over the integral over volume and integrate here the integral over surface.

Now, let us try to understand the physical interpretation quickly the physical interpretation of this bound charge densities whether it is surface.

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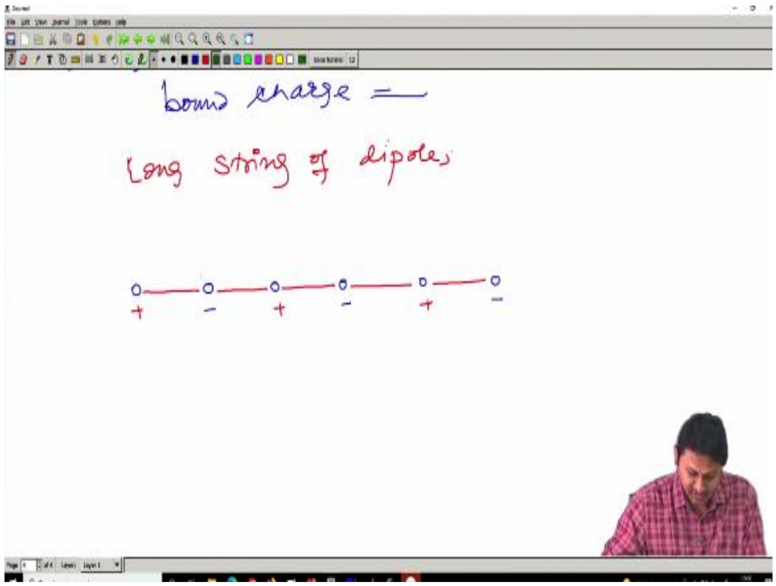

So, the next thing is the physical interpretation of bound charge so, what is the physical interpretation for the bound charge for example, you have a long string of dipoles. So, long string of dipoles means, I have here like this so, then this is if I join this, this is a chain of dipole where we have plus charge here minus charge here plus charger minus charge plus charge here minus charge here. So, these are minus charge minus charge minus charge that is simply equivalent to because these plus charge and minus charge in between they can cancel it out.

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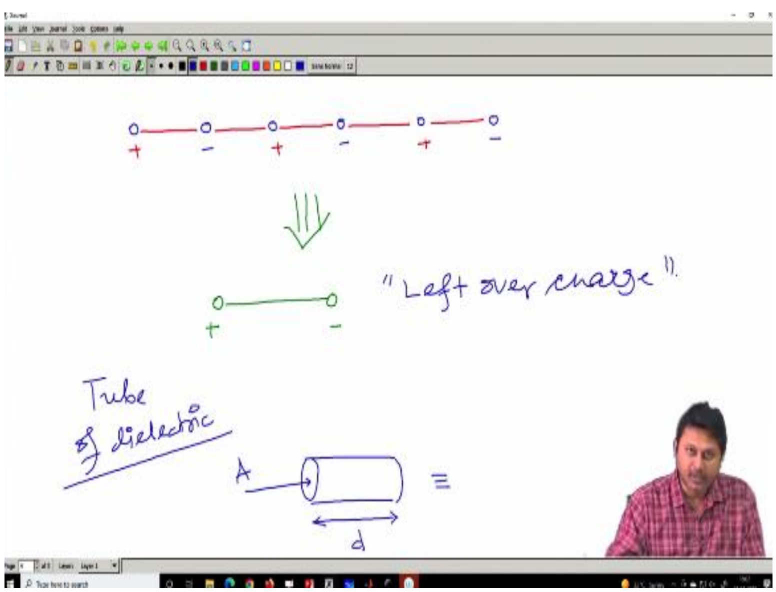

As if they are cancelling each other and eventually we can have an equivalent system where we have the only the leftover charges in the boundaries and that is plus here and say minus here. So, I have only one dipole. So, even though there is a charge the entire charge is nullified, but in the surface I can have a charge that is not being nullified. So, I should have effective charge and this charge are bound charges that is why I called the surface bound charge. So, we can understand in terms of this tube of dielectric, let us consider a tube of dielectric.

And as if I have a cylindrical shape tube made of dielectric where this is the area A and this is the length d. And it can be equivalent because the way we mentioned that only the leftover charge is here. So, this is the leftover charge sitting at the boundaries so, I have a left over charge that is sitting over this boundary.

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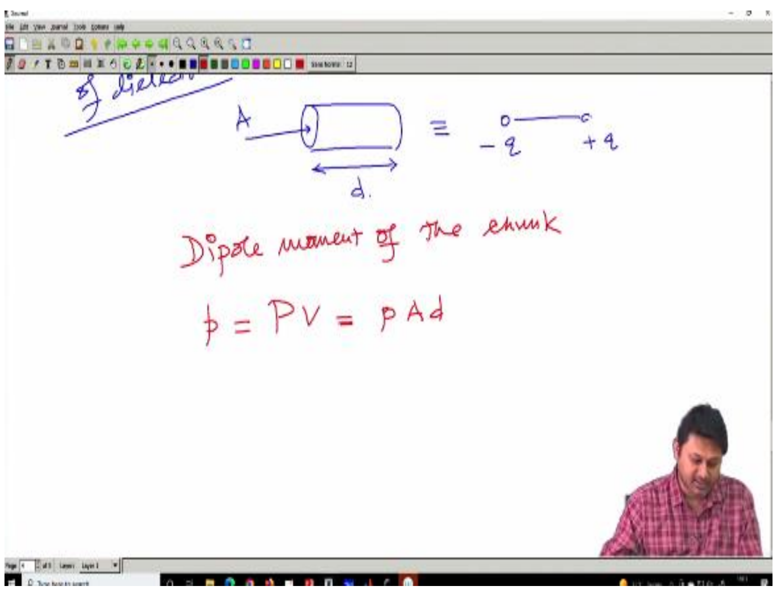

And eventually I can consider these as a dipole having the charge here say -q and +q sitting here and here. So, the dipole moment if I want to calculate this the dipole moment because I make it as equivalent dipole, because of this leftover charge whatever we get. So, dipole moment of the chunk is simply $p = PV$, V is the volume and it is a dipole moment per unit volume. So, that is equivalent to p and the volume is A multiplied by d because A is area and d.

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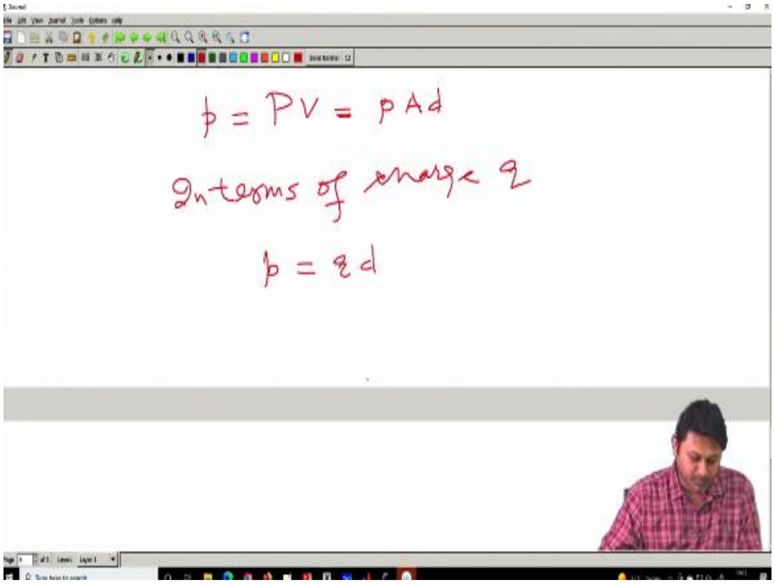

Now, the interim charges so, in terms of you know charge q what should I write the dipole moment p is simply q multiplied by d.

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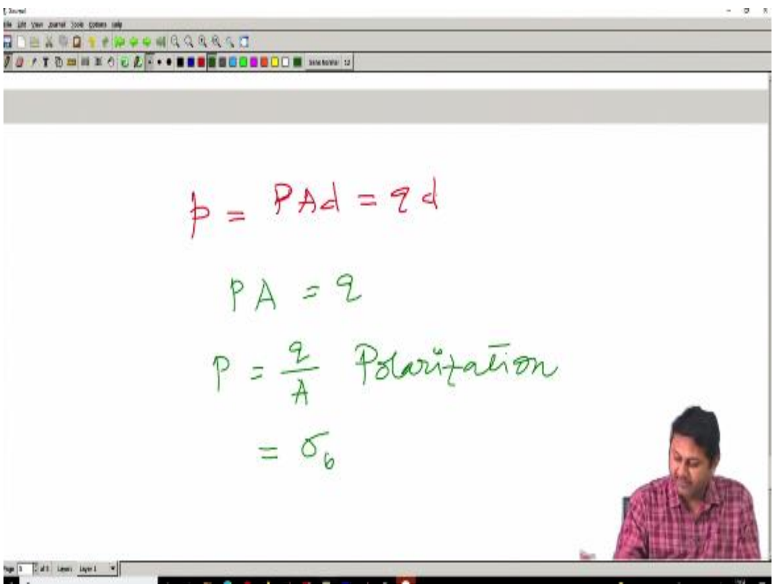

So, here this dipole moment p, which I have an expression like PAd is equal to you know q multiplied by d that is the dipole moment and now, from here we can see that P multiplied by A seems to be q and P is $\frac{q}{A}$ and that is polarization $\frac{q}{A}$ is nothing but the charge divided by surface. So, charge divided by area. So, this is the bound surface I can have the expression of bound surface charge density.

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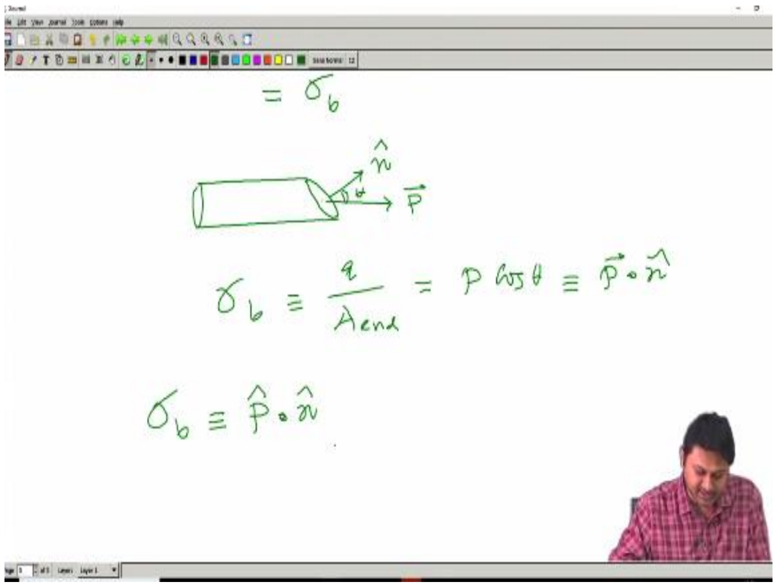

If in general if the surface is there is a surfaces like this and $\hat{n}$ is along this direction that is the direction of the surface and my polarization $\vec{P}$ is along this. So, this σ_b is simply $\frac{q}{A_{end}}$ whatever the area we are having and in order to take this if this angle is θ, so, I need to take the component cos component so it should be P cos θ, which is equivalent to $\vec{P} \bullet \hat{n}$. So, eventually what I find that my surface volume charge density is nothing but the polarization along the surface of $\vec{P} \bullet \hat{n}$.

This is the relationship between the polarization and the bound surface bound charge density. So, that is the relation we qualitatively understand. Then another quantity we need to understand qualitatively and that is the bound volume charge density.

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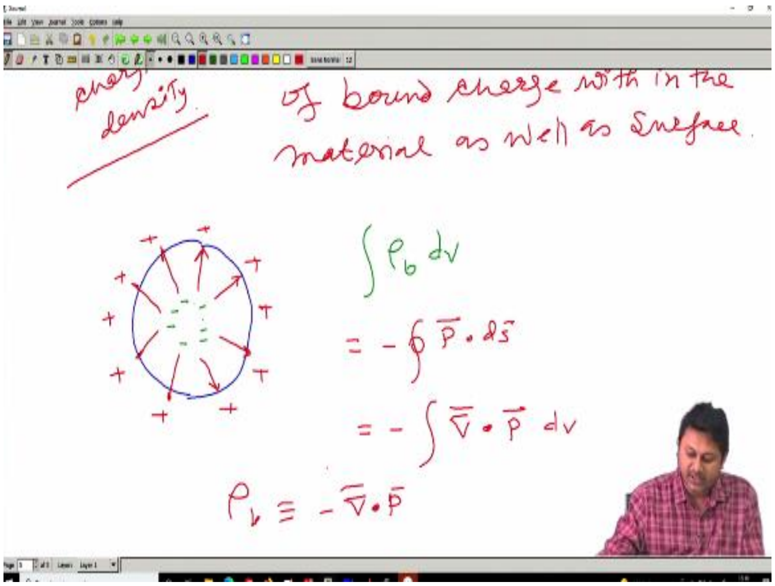

So, the bound volume charge density suppose I have a so, if say the polarization is non uniform suppose you have a polarization of the system but this is non uniform if the polarization is non uniform one can get you know the accumulation of bound charge within the material as well as surface so, what is the meaning of that suppose I have a system here dielectric, but the polarization is not uniform throughout.

So, suppose we have a bulk of negative charge sitting here and some positive charge is here over the surface. So, this is the way the charge is distributed, but this is not uniform it is not distributed uniformly now, the $\vec{\nabla} \cdot \vec{P}$ results in a you know pileup of negative charges and net bound charge whatever the net bounds charge we are having is simply I can write it as the bound charge density over dv this is because whatever the net charge and this is equal to the you know in a given volume this volume.

And it is equal and opposite to the amount of the charge that is pushed out through the surface whatever it is, so, it is equal and opposite to that thing that one can push out like this. So, I have a filling of divergence here and that filling I can write it as this is equal to minus of because it is push outward. So, I can have a $\vec{P} \cdot d\vec{s}$ because this is over the surface and that is why it is closed. So, I can have this quantity and that is minus of according to the integral law we have a close surface integral.

So, that should be equal to the volume integral Gauss's law and from that we can have that ρ v is equivalent to $-\vec{\nabla} \cdot \vec{P}$. So, what I get let me write properly otherwise.

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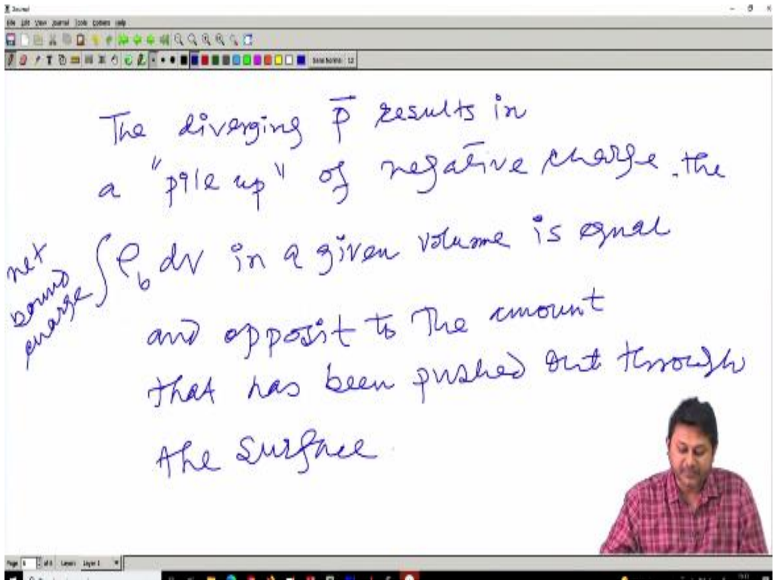

So, as if so, what we are getting here is this say the diverging that the $\vec{\nabla} \cdot \vec{P}$ results in a say pile up of negative charge as per the figure and the density of this quantity is simply this the net bound charge, which is this quantity, integral of the volume charge density where it is bound charge that is why it is written as ρ_b in a given volume is equal and opposite to the amount that has been pushed out through the surface.

So, I pictorially show that suppose I have a negative charge, so, this divergence is simply equivalent to whatever you know the volume charge density we are having. So, eventually we have if we get this very important equation that if there is a variation of the polarization, so, that variation of the polarization can be quantified with the divergence and the negative of that divergence value is eventually the density of the bound charge that is produced.

So, whenever we have a non-uniform polarization then only the concept of bound charge is there if there is no non uniformity of the polarization P then what happens if you take the divergence of that quantity, so, that will be simply 0 and you will not go to get any kind of bound charge out of that bound volume charge density out of that. So, today I do not have much time. So, I like to conclude my class here. So, in the next class, we will continue few more thing related to you know the dielectric properties of the material.

And we see some boundary conditions also that how the boundary condition of the electric field will be going to modify because in that case we should have something called displacement vector. So, how will displacement vector come and what should the boundary condition we will discuss. So, with that note I would like to conclude here thank you very much and see you in the next class.