Foundation of Classical Electrodynamics
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Lecture – 42
Energy of the Capacitor, Dielectric

Hello students to the foundation of classical electrodynamics course, under module 2, today we have lecture 42. And in today's lecture, we will be going to calculate the energy stored in a capacitor and try to start the discussion on the dielectric.

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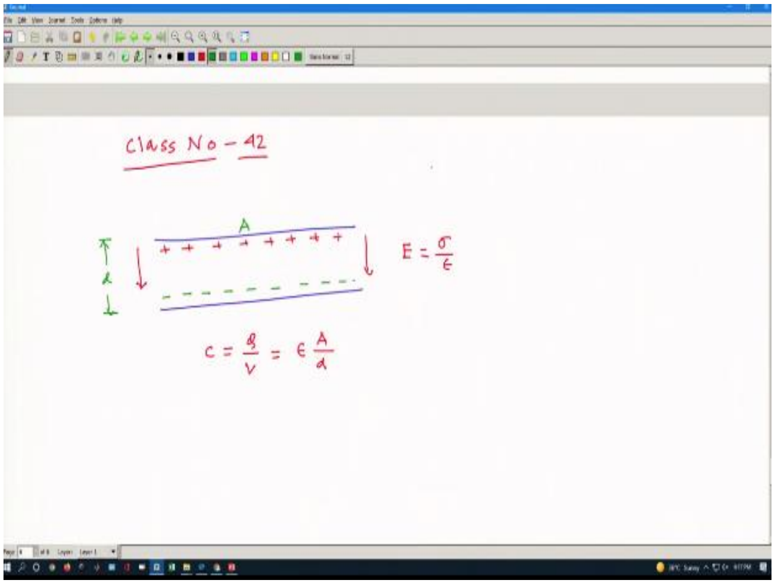

So, we have class number 42 so, in last class we started our discussion on the capacitor. So, we have 2 parallel plates where we know the charge are distributed and some negative charges here and then we have the electric fields that is in this direction $\frac{\sigma}{\epsilon}$ that was the value and then we calculate the capacitance C that is that we calculate as by definition it is $\frac{Q}{V}$ and we find that this value is essentially ϵ multiplied by $\frac{A}{d}$ where d is the separation.

This separation from here to here and A is the area of the capacitor the standard parameter for this for the capacitor is this so, now today we are going to calculate what is the amount of energy stored in this capacitor. So, the concept is like this.

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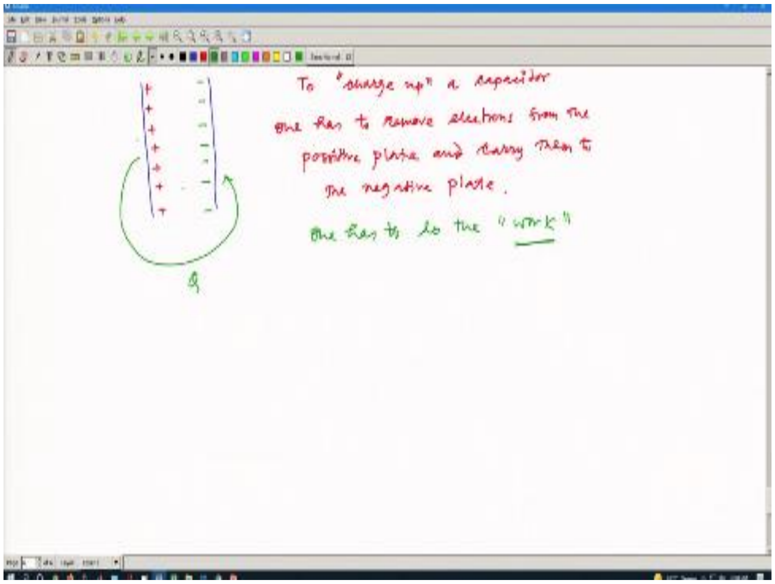

So, suppose I am having a parallel plate capacitor these are the parallel plates and where you have the charge is this negative charge like this. So, the point is to charge up a capacitor one has to remove electrons from the positive plate and carry them to the negative plate so, that means in order to form the structure as if I need to plug one electron from this side and then I put it here. This is the way suppose I am making my capacitor where I am putting some kind of charge distribution.

So, plugging one electron from the side and then put it here in another plate. So, in order to do that one has to do the work. So, that means in order to do it you know, one has to do the work to charge the capacitor up to a final amount Q because, so, this is I mean finally, Q is the amount that the charge should have on the left-hand side and right-hand side. So, the point is one has to do some work to charge up the capacitor. Now, let us some intermediate. So, that is the concept.

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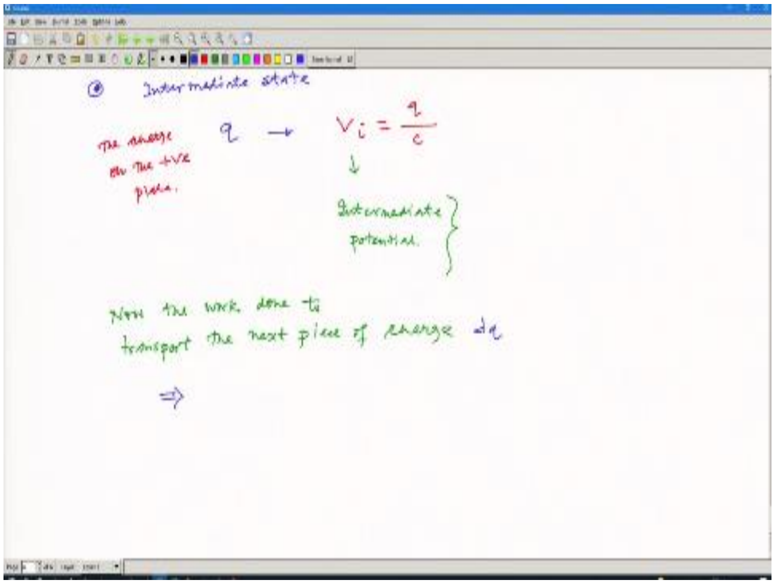

So, based on the concept we should calculate what is the amount of the energy stored here because, whatever the work we will do that is the eventually that is the energy that should store in this capacitor. So, suppose some intermediate state the charge on the positive plate is say q to some intermediate state some the charge of the positive plate is say q. So, in that case the potential difference q so, the charge at I should write the charge on the positive plate at some intermediate state it is q.

So, that leads to the potential difference V_i is $\frac{q}{C}$, V_i is again some intermediate potential. So, now, the work done required for the next piece of charge. So, now, after that so, let me write it now, the work done to transport the next piece of charge dq is because already I have a potential at intermediate state. So, now, after that if I now bring a dq amount of charge.

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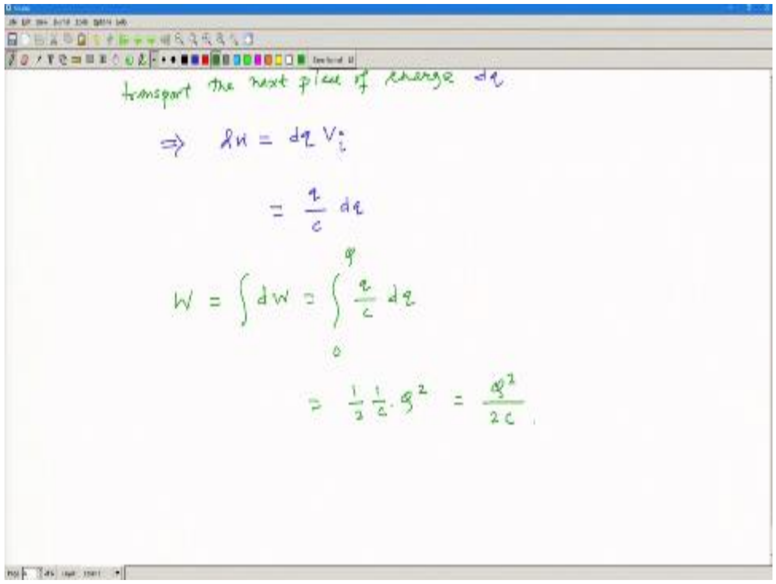

Then the amount of work done is simply dW is equal to whatever the charge I bring against the potential V_i. So, this is simply $\frac{q}{C}$ dq because $\frac{q}{C}$ is my V_i the intermediate potential where q is already there. So, now, we are in a position to calculate the total work done to bring all the charges. So, W is integration of dW, which is simply 0 to the total charge and then $\frac{q}{C}$ and dq. So, this amount is simply $\frac{1}{2}\frac{1}{C}$ then Q^2. So, simply $\frac{Q^2}{2C}$ but Q again in terms of potential.

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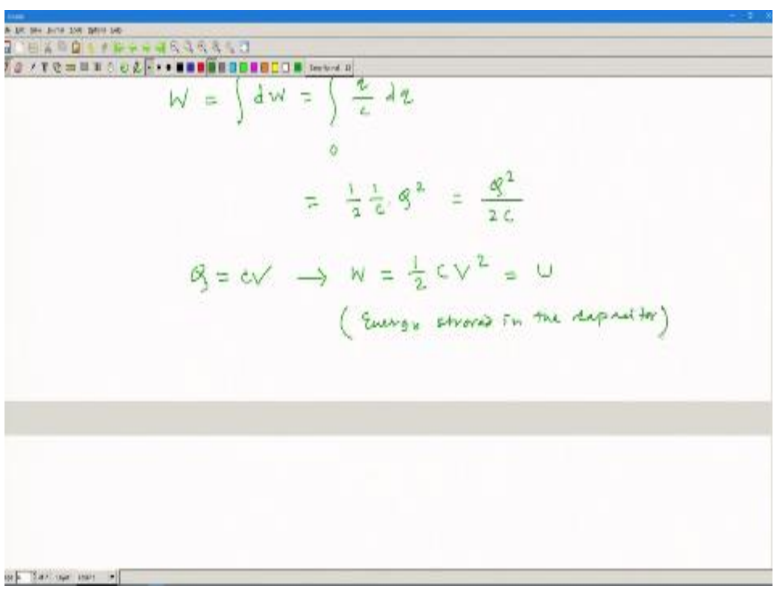

Q again for capacitor Q = CV so, that basically gives us the expression the total work done is $\frac{1}{2}$ and Q = CV one C will cancel out so $\frac{1}{2}$ CV^2 that is the amount of energy eventually the energy stored in the capacitor.

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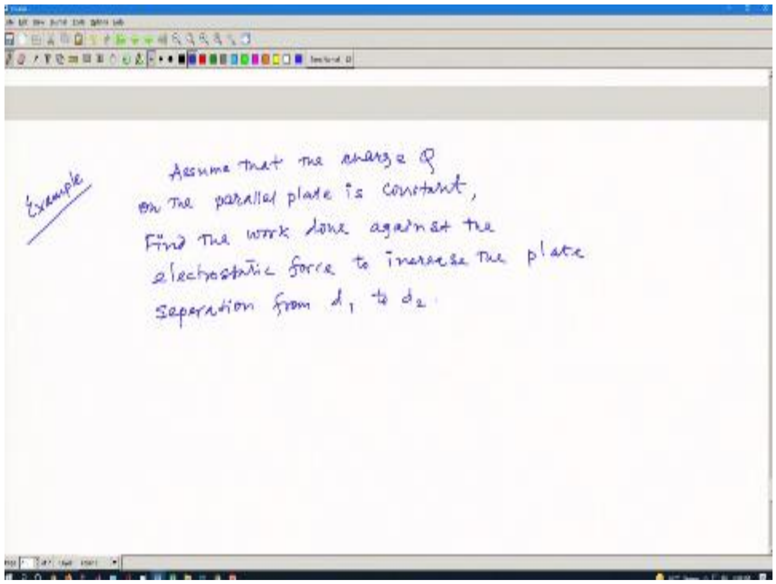

So, we can do one simple example. So, assume that the charge Q on the parallel plate is constant now find the work done against the electrostatic force to increase the plate separation from d_1 to d_2. So, the problem is straightforward.

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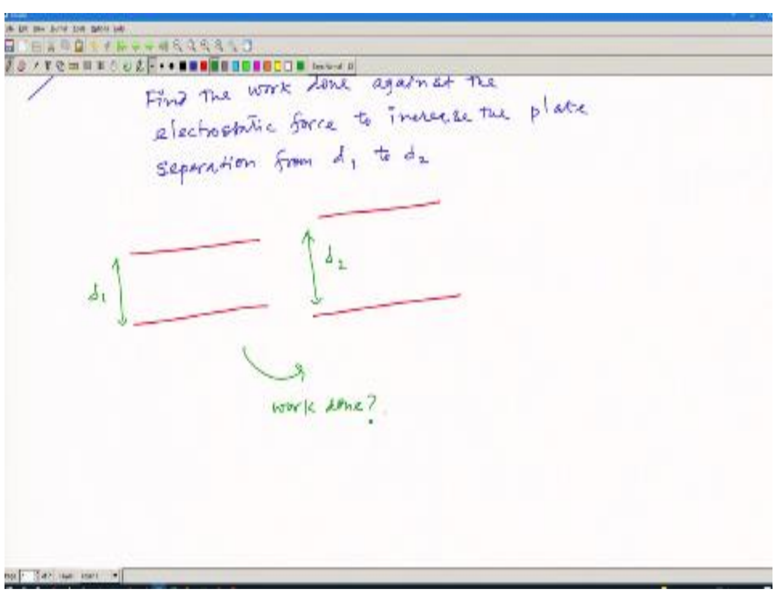

So, I have 2 parallel plate one parallel plate capacitor where this distance initially was d_1 now, I need to increase the separation of this parallel plate capacitor from d_1 to d_2 the question is what should be the work done? Now, we know the energy of the system and then from that we can simply calculate it.

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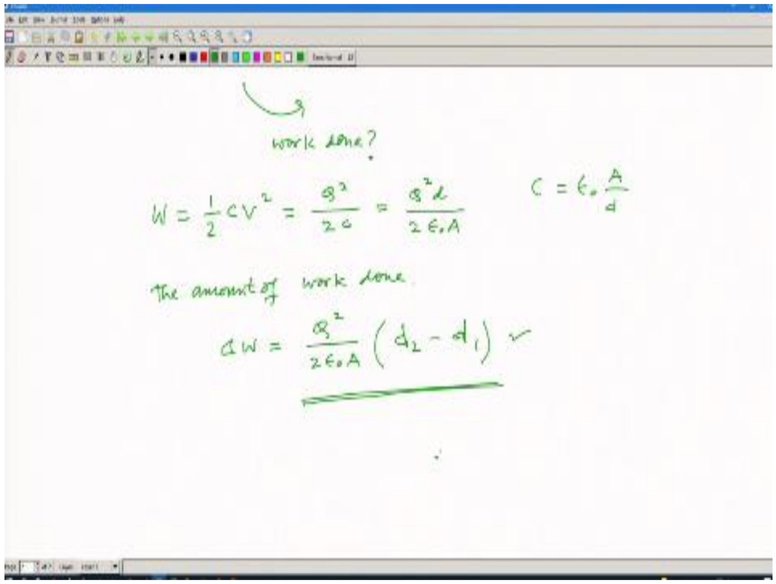

Because the energy of the system is simply $\frac{1}{2}$ CV^2 that is simply $\frac{Q^2}{2C}$ and C again because I need to change the separation so, I need to write C in terms of the separation d, $\frac{A}{d}$ that we calculate. So, from here I can write at it is simply $\frac{Q^2 d}{2\epsilon_0 A}$. Now the charge is fixed ϵ is constant, A is also fixed. So, the amount of work done $\Delta W = \frac{Q^2}{2\epsilon_0 A}(d_2 - d_1)$.

So, whatever the energy we have previously and now whatever the energy we are going to have, so, $(d_1 - d_2)$. So, that is the amount of energy now, we need to have and that is that in fact, that is the amount of work done you need to do to make the separation from d_1 to d_2. So, that is the result after that now, we like to understand what is the meaning of dielectric? That discussion, we should start now, in today's class.

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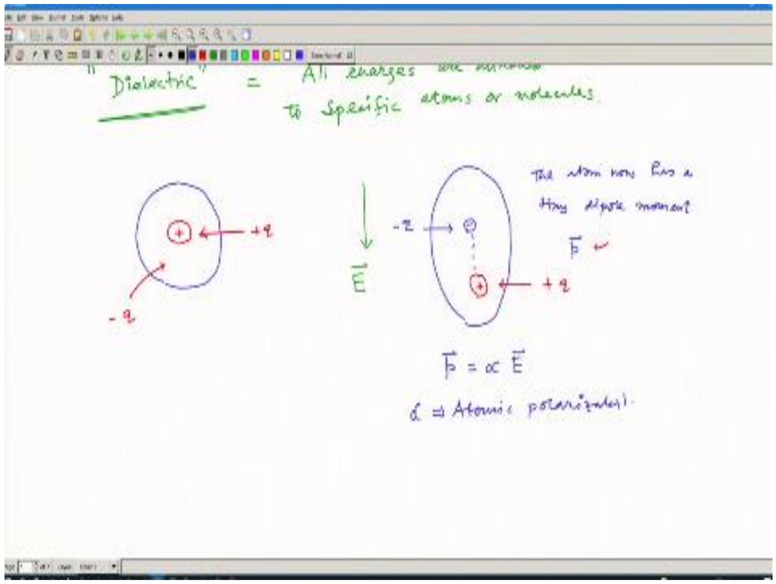

So, the next topic important topic is dielectric. So, that means so, electric field actually we are going to discuss the electric field in matter and this matter is dielectric. So, the main topic is under electric field in matter we already discuss what happened when the electric field when you have the conductor under some electric field, now, we will go to do the same thing, but now our system is different and that is called the dielectric.

So, dielectric system unlike conductor, it does not have any free electrons that is the major difference between dielectric and conductor. So, what happened here in dielectric all charges are attached to specific atoms or molecules so, there is no free electron and so, what happened that so, let me draw it here. So, suppose this is a dielectric system dielectric atoms or molecules and the plus charge and the minus charge they are super imposing each other so, this is plus and the entire.

So, this is a neutral atom and in the neutral atom what happened this is a natural atom and in this natural atom what happened the entire the electron cloud is over electron cloud we have a positive charge here +q say and this entire cloud is distributed in such a way that the charge density charge center of the charge -q and +q they are super imposing to each other that is for natural atom. Now, what happened if I put this dielectric system under some electric field suppose, I am putting some kind of electric field here for this system.

So, suppose, I have some external electric field and what happened that it induces some dipole. So, the plus charge and minus charge there will be a separation tiny separation. So, I should have a distortion here say like this cloud electron cloud and say the plus charge will be shifted to some place here and I should have some effective negative charge sitting here. So, there will be a charge separation and that leads to some tiny dipole. So, that is the concept here.

So, we have a charge separation here like this and some tiny dipole will go to form. So, here the atom now has a tiny dipole moment and I call it p. So, now my -q is sitting here and my +q is shifted under the external electric field and that is why this leads to some tiny dipole moment you know p and p is normally proportional to the applied electric field. So, I can write that p is equal to some constant and α and electric field.

So, this constant α is called in this case the atomic polarizability. Because, in an atom when you apply the electric field normally the polarization is proportional to the applied strength of the electric field and we are having an atomic polarizability.

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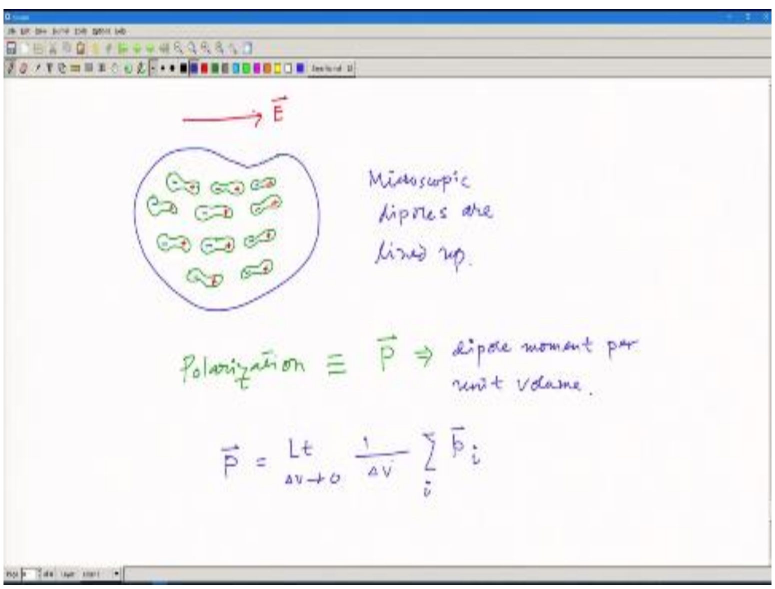

If I look at the entire picture let me draw the entire picture what happened for a bulk material. So, suppose we have a bulk material like this, this is the dielectric material. And under the external electric field suppose this is the external electric field I am having here along this direction. So, what happened the dipoles are distributed like this tiny dipoles will be formed this way there will be a charge separation and as a result of these charge separation I have tiny dipoles they lined up this dipole will go to lined up with external electric field.

So, I should have a plus sign here and minus sign here so, that is the macroscopic dipoles are lined up here no this is not microscopic this is the microscopic dipole. So, this microscopic dipoles are lined up like this. So, now we should put the concept of polarization, the polarization is dipole moment per unit volume. So, that is called a very important term called polarization, which is defined as P is a vector quantity and that is dipole moment per unit volume.

So, mathematically if I want to write this polarization that should be under the limit Δv tends to 0, $\frac{1}{\Delta v}$ whatever the dipole moment we have all the dipole moment I should have a summation over it and that should be the form now, quickly let us calculate the atomic polarizability not the bulk but the atomic polarizability.

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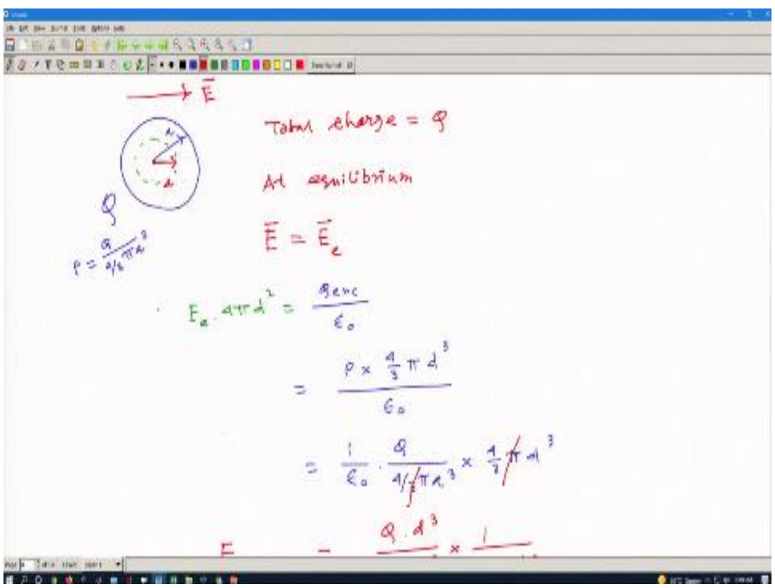

So, next thing I will calculate the atomic polarizability so, this is the structure we have and this is the external field I apply and there is due to that so, there is a separation of the charge. So, the total charge say Q under the under the external electric field $\vec{E}$ what happened that there will be a charge separation as I mentioned here and an equilibrium situation the external field and the field produced by the electron cloud at d they should be identical.

So, whatever the external field and due to the charge separation whatever the field they should counteract and at the equilibrium they should be equal. So, that is the condition. So, at equilibrium I have the external field should be equal to the field produced by the electron cloud at d because there is a separation so, this electron cloud should produce some electric field here and that should be these 2 should be equal.

So, now, we are going to use the straightforward form that whatever the field it will be going to produce here at d that I need to calculate when we have the total charge. So, it is like electrostatic problem straightforward electrostatic problem, we will go to use that concept. So, the field at a distance d how you calculate? So, I will just draw like a Gaussian curve here the Gaussian surface and our aim is to find out what is the field it is producing so, that $\vec{E}_e$.

So, $\vec{E}_e$ so, if this is d so, Gaussian surface suggests that I should write $4\pi d^2$ that should be the total charge enclosed mind it the total charge over this entire volume is Q. So, the volume charge density σ should be Q divided by if it is a radius of a then $\frac{Q}{\frac{4}{3}\pi a^3}$. So, the charge enclose divided by ϵ_0. So, now what is charge enclose? Charge enclose is the density multiplied by this volume and this volume is simply $\frac{4}{3}\pi d^3$ divided by ϵ_0.

Now, let us put the charge value of the charge density then it should be $\frac{1}{\epsilon_0}$, ρ is $\frac{Q}{\frac{4}{3}\pi a^3}$ and then multiplied by $\frac{4}{3}\pi d^3$. So, 4 by 4 by will be going to cancel out so, this will be going to cancel out.

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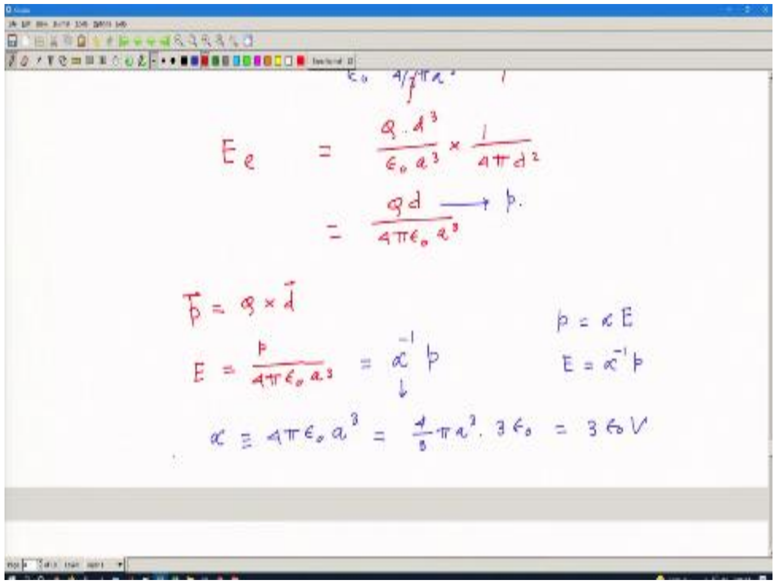

And eventually we have $\vec{E}_e$ we have $\frac{Q\,d^3}{\epsilon_0 a^3}$ multiplied by $\frac{1}{4\pi\, d^2}$, which is already here in this side. So, eventually this value becomes Q total charge $\frac{Q\,d}{4\pi\epsilon_0\, a^3}$ now the atomic polarizability we know how to calculate the dipole moment. So, p dipole moment is total charge multiplied by the distance d and electric field is equal to p and Q d sitting here, so, it should be $\frac{p}{4\pi\epsilon_0\, a^3}$.

Because this quantity is nothing but my p so, here this is equal to again $\alpha^{-1}p$ where α is my atomic polarizability. So, from that I can write that α is equivalent to $4\pi\epsilon_0$. So, p is this is α^{-1} because the formula is $p = \alpha E$ so, $E = \alpha^{-1}p$. So, what I calculate is α^{-1} is one by this.

So, α is equal to $4\pi\epsilon_0\ a^3$ and that is eventually the value. So, I can write in a different way also I just put $\frac{4}{3}\pi a^3$ and then $3\epsilon_0$ so, that gives me the volume. So, it is simply $3\epsilon_0$ and the volume of the system or volume of the atom.

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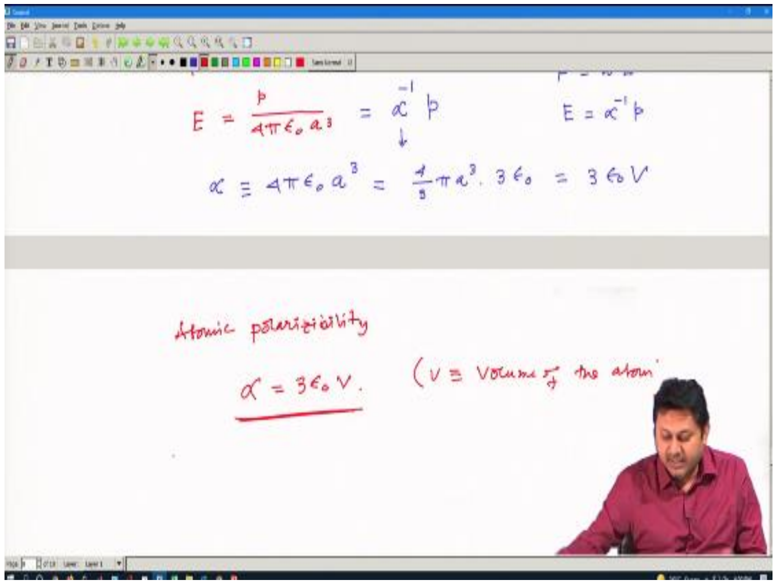

So, α is roughly the atomic polarizability is simply α is equal to $3\epsilon_0$ and then V where V is the volume of the atom so, this is the way one can calculate the atomic polarizability. This is a technique I mean just use the standard tool we have tool we use here in form of Gauss's law and then we can find out what is the value of the atomic polarizability. So, today I do not have much time. So, I would like to conclude here. So, in the next class, I think I should discuss about the susceptibility more about the susceptibility and susceptibility tensor.

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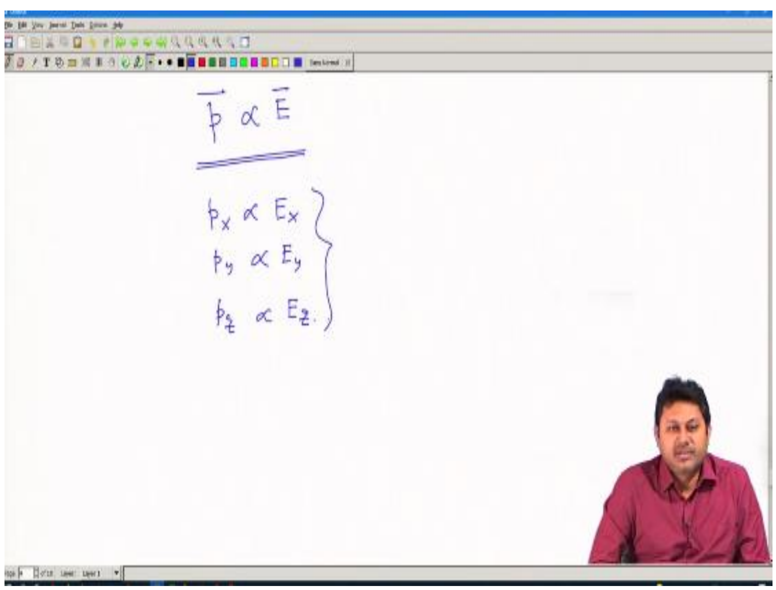

Because here we see that the p I write the polarization let me so, polarization is proportional to the electric field. So, that means, if I write this equation, that is essentially means, that the p_x component is proportional to the E_x component and p_y component is proportional to the E_y

component and p_z component of the polarization should be proportional to the E_z component. So, each component wise they are proportional.

So, then only I can have a relation like this and not only that this proportionality constant is same for all the cases for E_x E_y E_z this proportionality constant should be same, but normally this is not the case. So, we can have a system where p_x does not only depends on the E_x rather it can also depends on E_y and E_z and that gives rise to something called the susceptibility tensor or polarization tensor so, that we will discuss in the next class.

And also try to understand based on this concept of dielectric, what is called the bound charge density and bound surface, bound volume charge density and bound surface charge density in the form of polarization. So, with that note, let us concludes today's class so thank you for your attention and see you in the next class.