Foundation of Classical Electrodynamics
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Lecture - 39
Conductor (Contd.,)

Hello students to the foundation of classical electrodynamics course. So, today under module 2, we will have lecture 39 and today we are going to discuss the conductor and we will continue the discussion.

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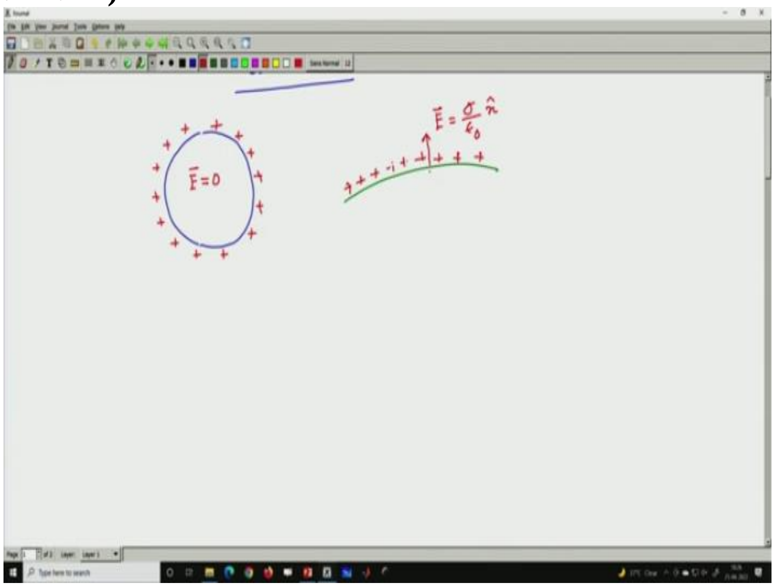

So, we have class number 39 today and we will be going to continue the discussion on conductors. So, we already mentioned in the last class that in conductor we have free electrons and suppose this is a conducting sphere and all these free electrons on the charge will say accumulative if I put some charge in the conductor say +q so, they will be residing on the surface making the total field inside of the conductor $\vec{E}$ = 0, also whatever the field that one can expect to just have the conductor.

So, this is the conductor where all the say positive charge induced charge is residing on the surface like this and just above the surface. If I want to find out what is the electric field then that value has to be calculated it is $\frac{\sigma}{\epsilon_0}$ with $\hat{n}$ where $\hat{n}$ is along this direction which is perpendicular to the surface.

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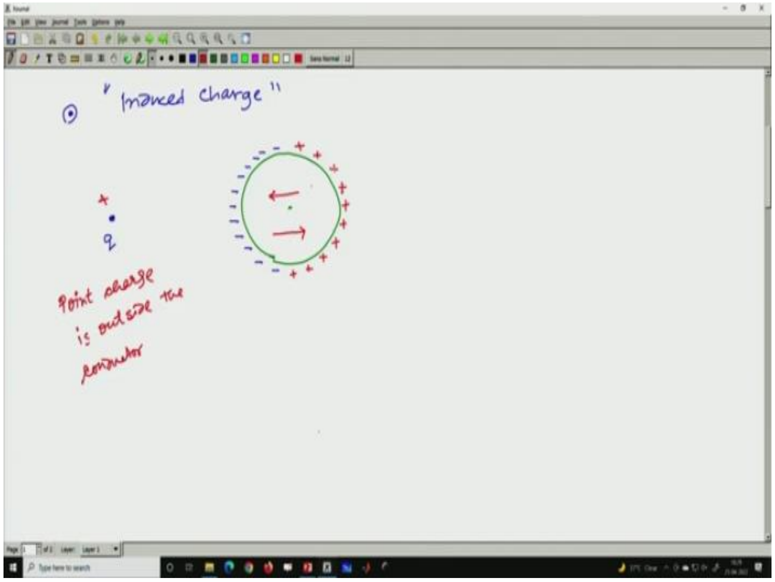

Today, we will try to understand few more interesting things regarding the conductor. So, that is the induced charge how the charge is induced I already mentioned. But today we will try to understand with different aspects. So induced charged so, suppose I have a conducting sphere here sphere or shell and I put a charged particle here outside the charged particle q is outside now, since the free charge in the conductor can move around in such a way that it cancel of the field of the q inside the conductor. So, all the charge should reside on this.

So, if say I put a plus q charge here that this is a positive charge all the negative charge will sit here this side and the positive charge will be here. So, this is the way the charge will be distributed over the conductor. So, here the point charge is outside the conductor and since all the free charge can move and they can arrange in such a way that they can cancel of the field inside so that all the charges can be so whatever the charges here we have are equal and opposite.

Whatever the field we have here equal and opposite field will be so 1 field will be here another field will be like this. So, they will cancel each other one is the external field and another is the induced field by this charge distribution.

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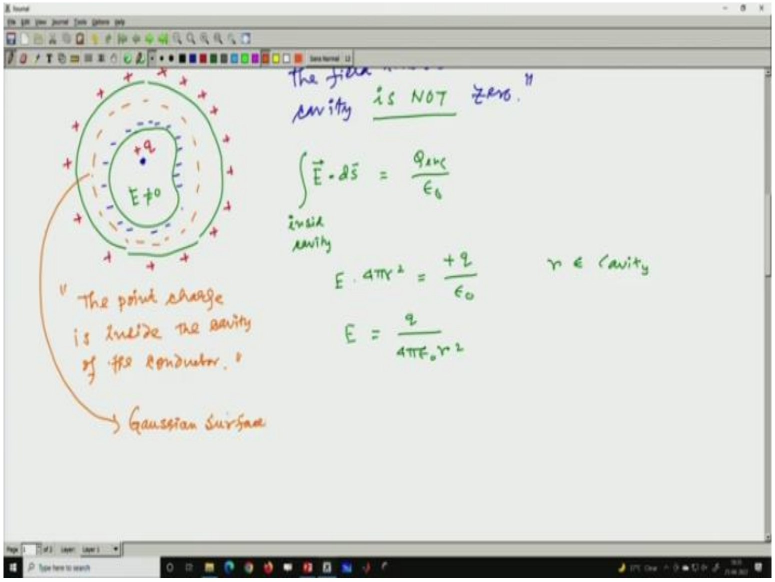

Now, the thing is let us put the point charge inside this sphere. So, how I can do that, so, suppose I am having a sphere like this and inside I am making a cavity like this, so, this is the cavity inside and I place my point charge here which is say +q so, then the question is now that the problem is the point charge is inside the cavity of the conductor that is the situation we are having we have a conductor and inside that cavity so, we are having the point charge.

Now, the field inside the cavity is not zero that is the interesting thing first we need to understand so the field due to the charge within the cavity is not zero. So, what happened that now, this is the thing we need to first understand why it is not zero and you can understand easily by just when we place a charge inside this cavity. So, there are 2 surfaces of this system this conductor 1 is the inner surface and another is outer surface. So, all the negative charge will be going to sit here over this inner surface all the negative charges will be this.

Whereas, all the positive charges are going to sit here like this. So, now, here the important thing is here my $\vec{E}$ should not be 0 and using simply the Gauss's law if I make a Gaussian surface here you will find that these things so, if I make a Gaussian surface here, then according to the Gauss's law we have $\vec{E} \bullet d\vec{s}$ inside the cavity that is total charge $Q_{enclose}$ divided by ϵ_0. So, if I have a sphere here with R, so, then $\vec{E}$ and then the surface $4\pi r^2$ the quantity let us put some this vector sign then the total charge inside is $\frac{+q}{\epsilon_0}$.

Where r belongs to the cavity so, r is inside this cavity then $\vec{E}$ should be simply $\frac{q}{4\pi\epsilon_0 r^2}$ that is the field, which is nonzero that we understand, but the point is what happened here I, mean in this region. So, in this region again I need to draw a Gaussian surface, so, this will be my Gaussian surface now, because I want to find out the electric field inside the conductor and already we know that inside the conductor there should not be any electric field.

But here the situation is slightly different because inside the conductor we have a cavity and that cavity contain a charge +q. So, this is my Gaussian surface. So, this surface is my Gaussian surface so, now again I am going to use the same law to find out what is going on there and my laws suggest that here I need to make this notation total because it is inside the cavity and the total surface I need to calculate.

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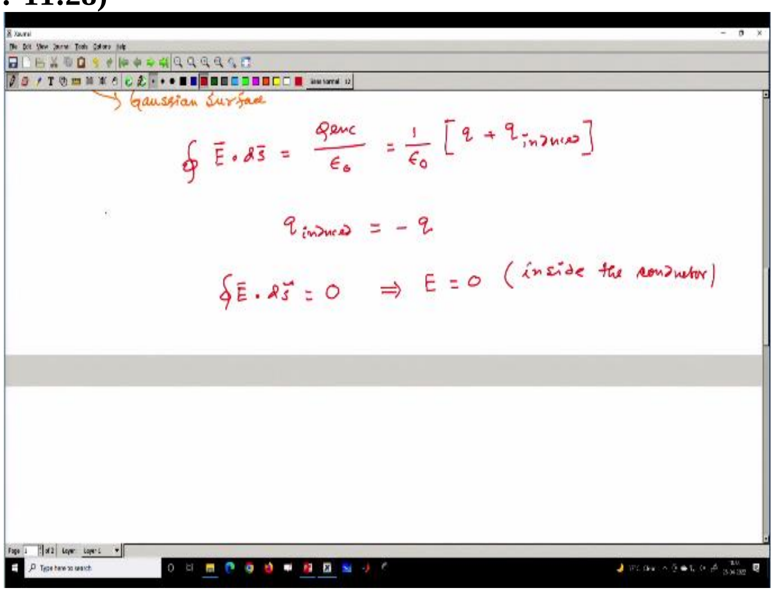

So, here I will do the same thing that the integration closed surface integration $\vec{E}$ • d$\vec{s}$ will be for this Gaussian surface $\frac{Q_{enc}}{\epsilon_0}$. Now, note here the Q_{enc} contain the +q charge and also these induced negative charges because you can see these induce negative charges and the +q both charges are there inside this Gaussian surface. So, I should write here $\frac{1}{\epsilon_0}$. There will be 2 charges, one is simply +q that is already there in the cavity plus another is the induced charge.

And now you can see that $q_{induced}$ is nothing but -q because whatever the positive charge you place, the equal amount of negative charge should be here in this over the surface. So, that makes this quantity zero so, eventually we have closed surface integral $\vec{E} \cdot d\vec{s}$ is simply equal to 0 that means that gives us that $\vec{E}$ is 0 this is where inside the conductor but not the cavity you should inside the conductor means only this annular region in this region.

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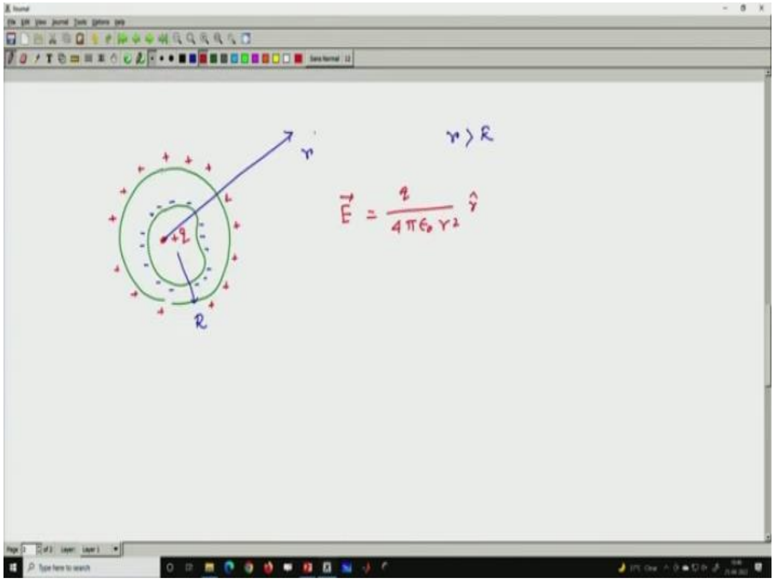

Now, a similar thing one can ask that I have a sphere here and inside the sphere I am having a cavity like this and again I have a point +q charge sitting inside this cavity. So, what should be the field at some point r, where r is greater than the radius of this sphere say a or say R, R is the radius of this sphere, but I want to find out what is the field outside and the result if I draw a Gaussian surface again the distribution if I look carefully, the distribution will be the negative charge will be sitting here.

And the positive charge will be over this equal amount can we just distribute it like this. And if I want to find out what is the electric field here, it will be like I am only having 1 charge q because at this point it will be going to ignore all the induce charge, because they are equal and opposite. So, as if only 1 charge here and you just simply ignore what is happening here and I should have a result like $4\pi\epsilon_0$ the standard result r^2 with the $\hat{r}$ if I put in vectorial form so that should be the field at the outset.

So, 3 cases I just discussed one is if there is a charge outside what happened? So, there will be induced charge like this, if I put a charge inside the cavity then what happened that inside the cavity there will be a field nonzero field, but in the annular region inside the conductor, we should have a 0 electric field and another is far away if I want to find out what is happening far away even though I put a charge inside you know the cavity of a conductor, then the result will be simply like this.

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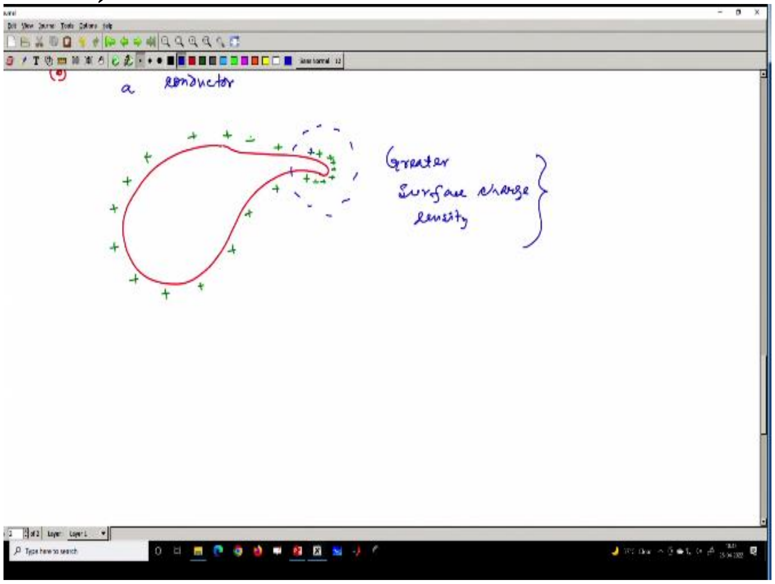

Now, next I will discuss the charge distribution so, next topic is the charge distribution on the surface of a conductor. We mentioned that the charge will be distributed over the surface for a given conductor always, but the charge distribution may not be uniform. So, that is the point I try to make suppose I have a conductor with this kind of shape this is a conductor this is not a sphere, but it shape like this I have a shape edge here.

So, in this kind of conductor, if the total charge is distributed, the charge will not going to be distributed with a uniform, you know uniform if I could give a charge here, so, it will not going to distribute uniformly. So, in this edge, the charge will be distributed like this there will be a very dense charge distribution one can expect in the sharp edges again there will be so, here in this region the greater charge density one can expect greater surface charge density can be observed in this region here we have a + sign.

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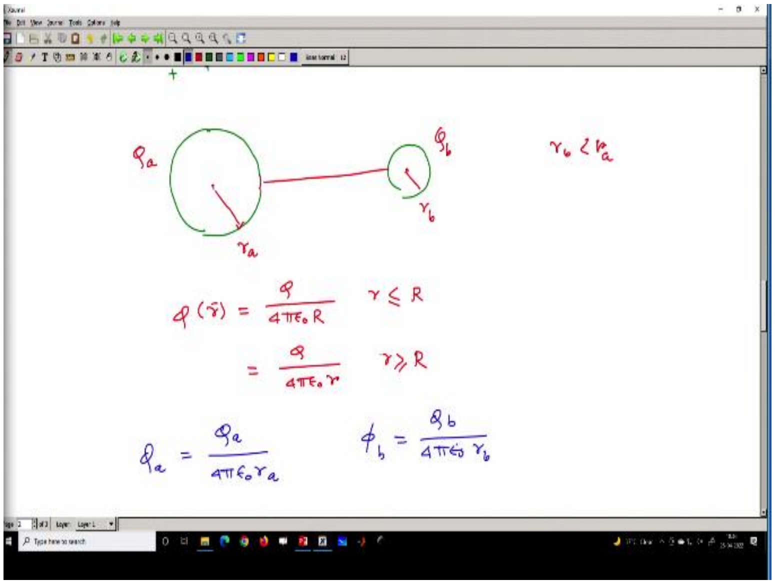

So, we can easily find it out that what happened? So, suppose I have a sphere here 2 conducting sphere 1 with radius r_a and another radius r_b when r_a is greater than r_b. So, this is 2 radius 2 conductor and this is r_a and the radius is say r_b where r_b is less than r_a that is the condition. Now, I already draw a line here. So, let me just erase this part. so, I already draw a line that means I am just before joining these 2 conductors with a wire or something, I put a charge here. So Q_a is amount of charge is given here and Q_b is a charge. I just give to this conductor.

So, now Q_a Q_b charge will be distributed uniformly and for the conducting sphere we know the potential for the conducting sphere is simply that we know that it is $\frac{Q}{4\pi\epsilon_0 R}$ when r is less than R for a sphere when R is the radius of that and equal to $\frac{Q}{4\pi\epsilon_0 r}$ when r is greater than or equal to R. So, that means, in the inner region for the conductor the potential is constant, and when we go outside the conductor, then it reduces in the form of $\frac{1}{r}$.

So, that means, in our case, we have ϕ_a is simply the total amount of charge is given $\frac{Q_a}{4\pi\epsilon_0 r_a}$ and ϕ_a for other conductor is $\frac{Q_b}{4\pi\epsilon_0 r_b}$ now, I joined these 2 conductor with a you know with wire or something in by doing that, it becomes an equipotential now, this entire conductor became a single conductor with the condition that it is having a equipotential surface.

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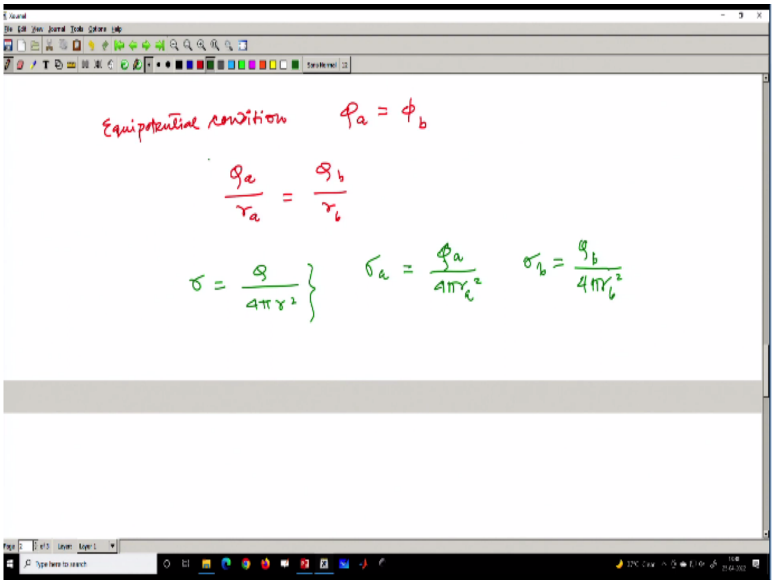

So, the equipotential condition leads to the condition leads to $\phi_a = \phi_b$ so that means, from this equation I can simply have $\frac{Q_a}{r_a} = \frac{Q_b}{r_b}$. Now, the charge is distributed, so, I should have *a* over the surface so, I should have a surface charge density σ and that is $\frac{Q}{4\pi r^2}$ in general so, σ_a is simply $\frac{Q_a}{4\pi {r_a}^2}$ and σ_b is $\frac{Q_b}{4\pi {r_b}^2}$ so, this is Q_b. So, σ_a and σ_b we know and also I have a condition here.

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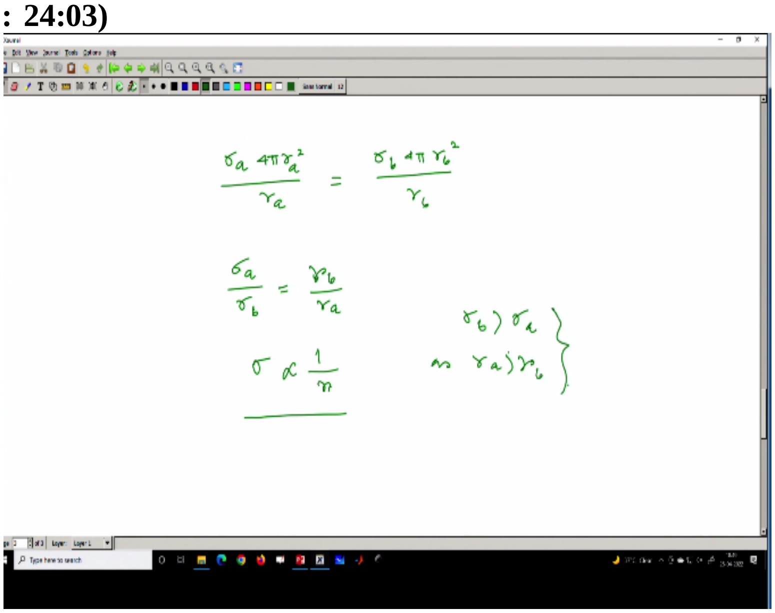

So, if I just replace this Q_a in the form of σ, so, I can simply have $\frac{\sigma_a 4\pi {r_a}^2}{r_a} = \frac{\sigma_b 4\pi {r_b}^2}{r_b}$ or in other word $\frac{\sigma_a}{\sigma_a}$ is simply $\frac{r_b}{r_a}$. So, from that we can understand that σ is inversely proportional to r. So, in this case, the charge will be distributed so that means here simply σ_b is greater than σ_a as r_a is greater than r_b because it is inversely related this is the condition I get.

So, that means, we have a very in this case you can see the radius of this curvature is smaller and since the radius of this curvature is smaller, it is expected that the charge density will be higher compared to this region, where the radius of this curvature is larger. So, if the radius is larger the charge density over this region will be smaller and if it is smaller than the charge density will be larger. Now, the corresponding electric field because in the surface it is distributed, so, I should also have an electric field associated with this these things, which is along this direction that should be the associated electric field $\vec{E}$ and that again depends on σ.

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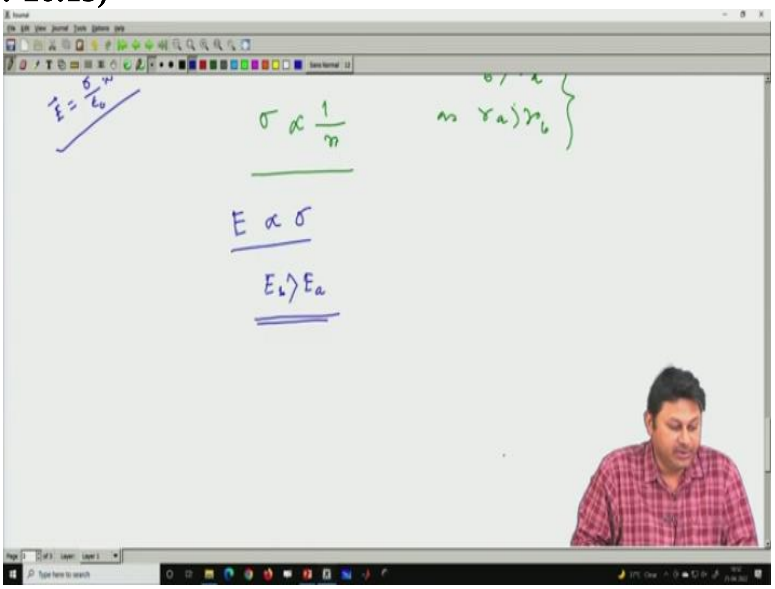

Because we know that electric field $\vec{E}$ is how much it is $\frac{\sigma}{\epsilon_0}$ with $\hat{n}$ this is the vectorial form so that means the electric field is proportional to σ, so, the electric field will be also higher. So, that means, if I want to find out the electric field for these 2 sphere one is say E_a and another is say E_b then E_a so from here we can also write that E_b is greater than E_a, this is the condition I should have.

So, this is the thing I actually wanted to cover in today is lecture. So, I just mentioned that how for a conductor, what are the properties of the conductor few important properties and how the charge is accumulated over the surface, what should be the charge density etc. This is not a very new thing, because most of the students should know, but I just recap in the context of this conductor. So, today I am going to stop here, because I do not have much time.

In the next class what we do that we try to understand some phase matching condition or the matching boundary not phase matching condition rather I should say, the boundary condition of the electric field when the electric field is going from one medium to another medium. And in the interface if we have some kind of surface charge density, then how to deal with that, what should be the boundary condition of the electric field different component of the electric field that we are going to discuss. So, with that note, let me conclude here. Thank you very much for your attention and see you in the next class.