Foundation of Classical Electrodynamics
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Lecture - 38
Conductor

So, hello student to the foundation of classical electrodynamics course, under module 2, today we have lecture number 38 and today we are going to start the topic related to conductor.

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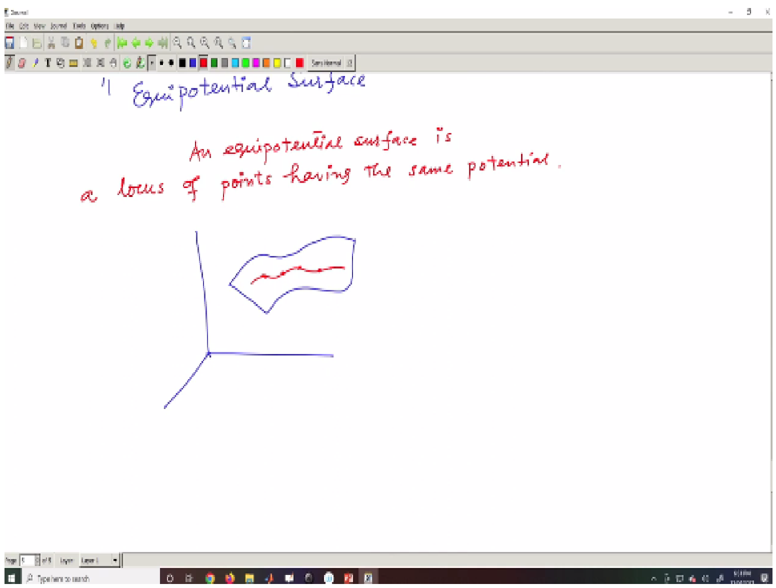

We have class number 38 and our topic here is conductor, but before understanding conductor let us quickly recap a few things that you already know and the first thing is the equipotential surface. So, the equipotential surface is a surface so, an equipotential surface is a mathematically locus of points having the same potential. So, that means, in the space I just start you know calculating the points, which are having the same potential I have a space and at different point we are having the potential suppose this is my space.

And a different point we are having the potential and I am tracking it that which potential is same for the same potential. If I am tracking and I find a plane like that and if I find a plane over this plane eventually all the points will should have the same potential.

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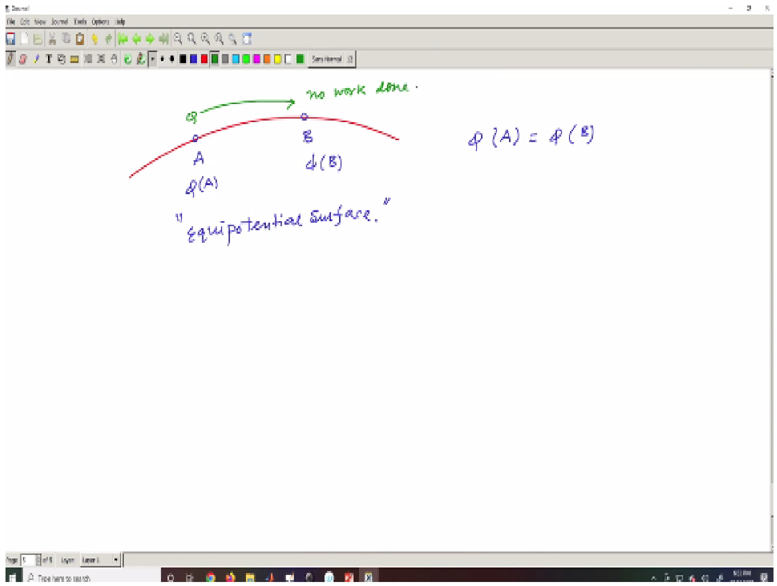

So, if I simplify this that this is a surface and over the surface I can have 2 point A and B this is A and B the potential of A and B is same. So, that means if I write the potential here at $\phi(A)$ and the potential for this point is $\phi(B)$. So, I should have $\phi(A) = \phi(B)$ potential is same not only on this point all the points over the surface. So, these surfaces are called the equipotential surface. Now, if we move the charge particle from point A to point B, since it is equipotential surface, so, we should not do any work.

So, if I move a charged particle say from here to here, say charged particle Q if I move from here to here, over this equipotential surface, so, no work will be done.

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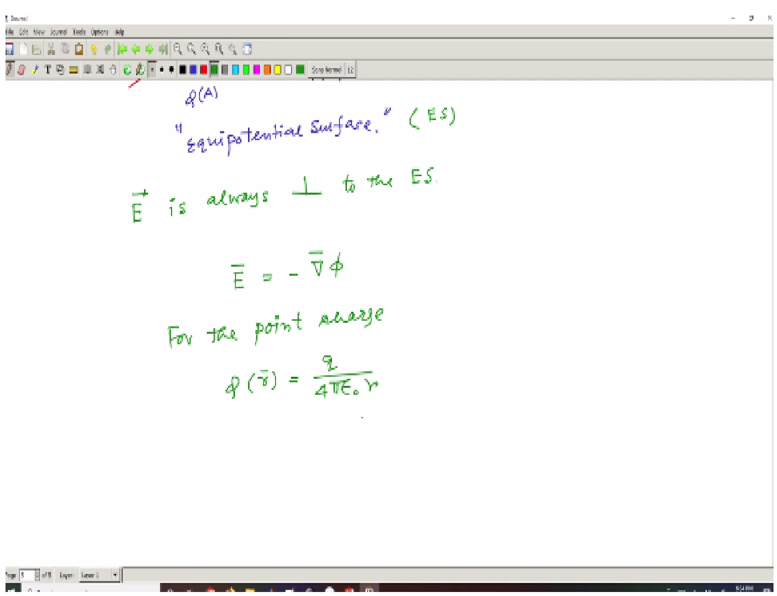

Now, $\vec{E}$ the electric field is always perpendicular to the equipotential surface. Let us write in a shorthand note equipotential surface at ES equipotential surface. So, I know that why it is that because $\vec{E}$ is equal to you know in general it is $-\vec{\nabla}\phi$. So, for the point charge what do we have $\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0 r}$.

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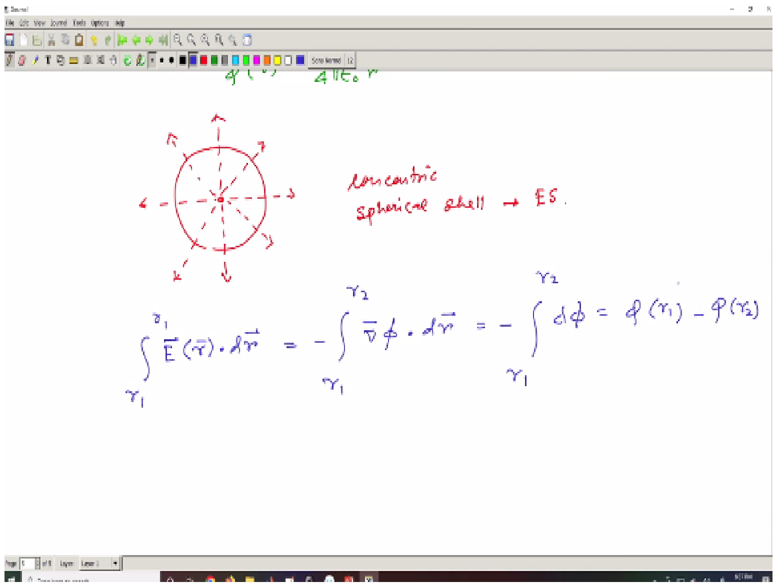

So, this is my point charge and the electric field is moving like this, this is the electric field the way because we know the field lines for point charge and it is along the direction of $\hat{r}$. So, this is the direction along which we have the electric field. So, this shell whatever the shell I am drawing here is containing the equipotential surface if you have a circular cylindrical the spherical shell over this entire region.

Then that basically this concentric all the concentric spherical shell are equipotential surface for a point charge. Now, let us go back to whatever we are doing suppose, I on this electric field $\vec{E}$ is a function of $\vec{r}$. I move from point $\vec{r}_1$ to $\vec{r}_2$ and if I want to find out what is the work done, so, that is the formula and it is $-\vec{r}_1$ to $\vec{r}_2$ then this quantity because $\vec{E}$ is $\vec{\nabla}\phi \bullet d\vec{r}$. I can write it and this is equivalent to $-\vec{r}_1$ $\vec{r}_2$ $d\phi$ and it is $\phi(r_1)$ - $\phi(r_1)$.

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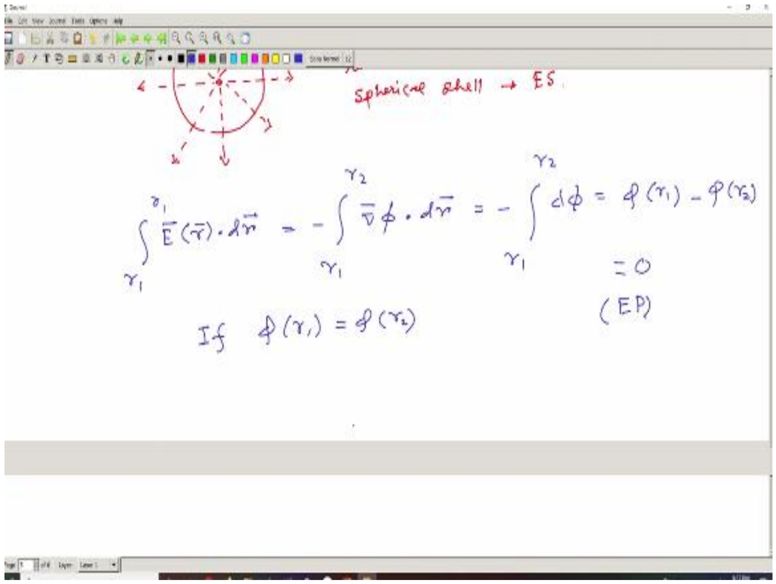

Now if $\phi(r_1)$ and $\phi(r_2)$ are same that means if I am moving over a equipotential surface, then this quantity is simply 0 under the condition that I am having a equipotential surface the potential at point $\vec{r}_1$ and potential at point $\vec{r}_2$ is same.

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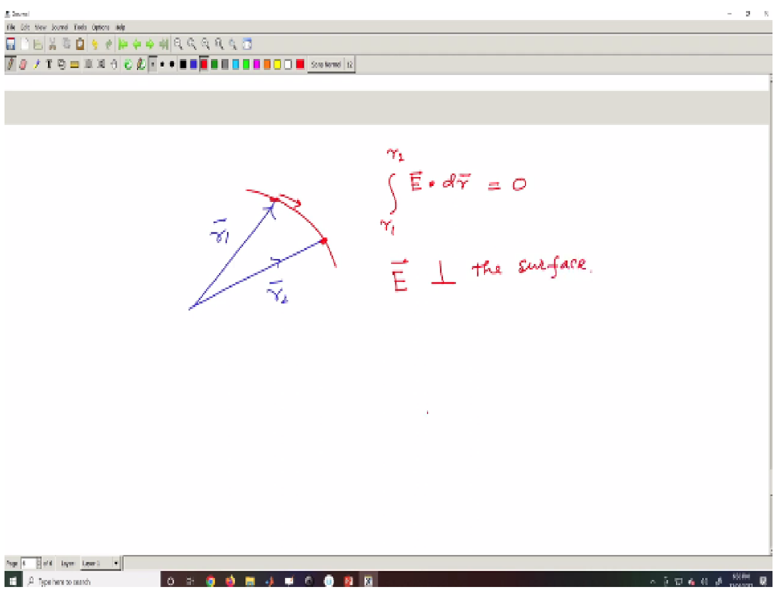

That is suppose this is my point $\vec{r}_1$ and this is the point $\vec{r}_2$. So, if I join a line here so this line is over equipotential and I am doing my work over this line. So, whatever I calculate is $\vec{E} \cdot d\vec{r}$ r_1 to r_2 is basically I am doing the work going from here to here and that this quantity is 0, this quantity is 0 means $\vec{E}$ is along this direction. So, $\vec{E}$ has to be perpendicular and $\vec{r}$ is in this direction. So, $\vec{E}$ and $\vec{r}$ should be perpendicular to each other where $\vec{r}$ is over the surface.

So, that tells me that $\vec{E}$ is perpendicular to the surface, which is equipotential by the way, so, I mean also you can understand in this way the way I draw let me elaborate that.

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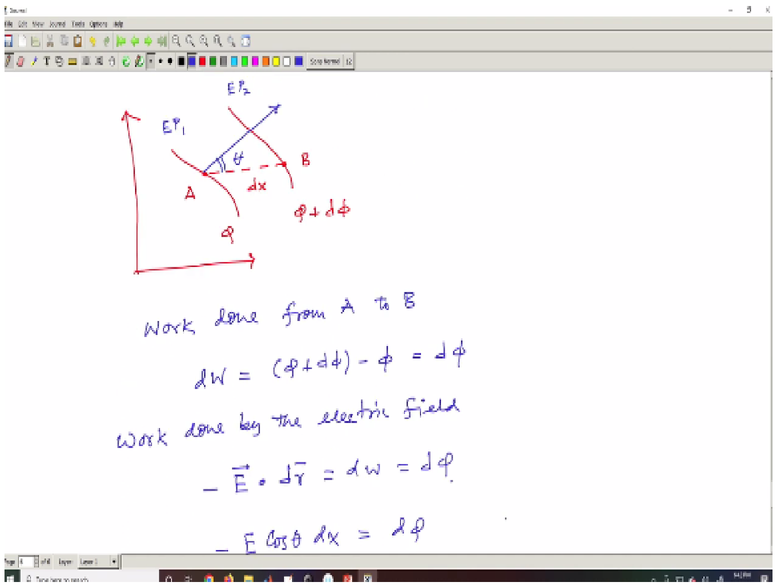

Suppose I am having a coordinate system and I am having a surface equipotential surface here ϕ and another surface here say ϕ + dϕ this is the value over the surface and I have a point here A and another point here say B with a small increment from here to here it is say δx and my $\vec{E}$ is going along this direction and having an angle θ. So, 2 equipotential this is equipotential surface 1 and this is equipotential surface 2 E is along this. So, the potential value here is ϕ and at dx distance I have a different potential.

But still it is producing equipotential surface over this line. So, now, if I want to calculate the work done from A to B then the work done I can calculate as the amount of change of the equipotential surface because it is the equipotential the change of potential, which is simply dϕ. Now, the work done by the electric field this is $-\vec{E} \cdot d\vec{r}$ and that is dw and that is dϕ. So, this quantity is simply - E cos θ and dx taking the component because this angle is θ along x axis if I take the component and this is dϕ.

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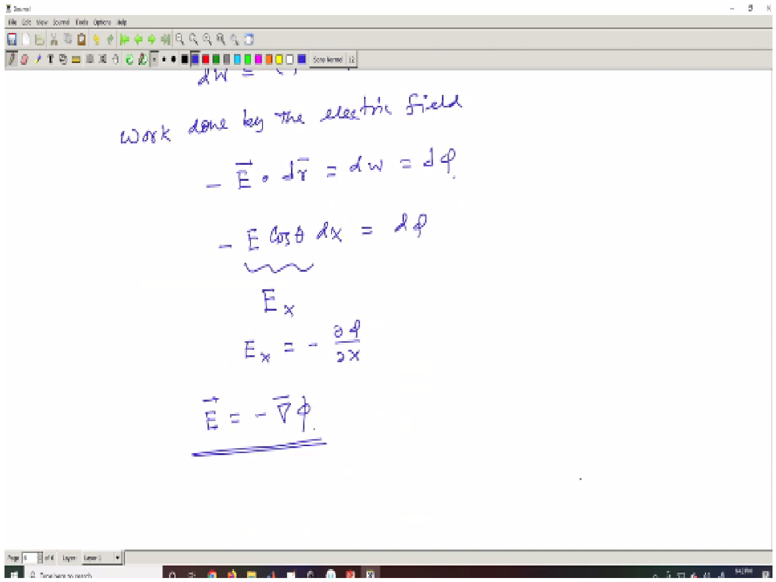

So, simply this is my E_x component so, E_x is simply minus of and in general that is the reason in general I can write as $\vec{E}$ as a $\vec{\nabla}\phi$. So, I can also calculate this very important equation with the concept of equipotential surface and this work done thing.

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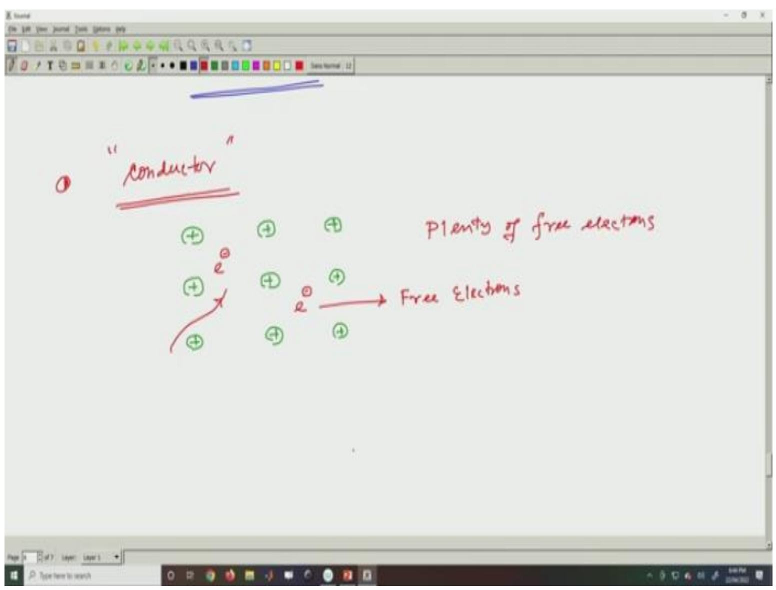

Next, let us now come to our original topic, the topic we wanted to discuss today is conductor. In general the conductor is a system is a where we have plenty of free electrons. So pictorially if I want to understand that suppose I have a material where the positive and negative charge in the atom are distributed but over that we can have a free moving plenty of free charges in the form of electron. So, these electrons are called free electron. So, plenty of free electron we have in the conductor plenty of free electrons are there which is moving

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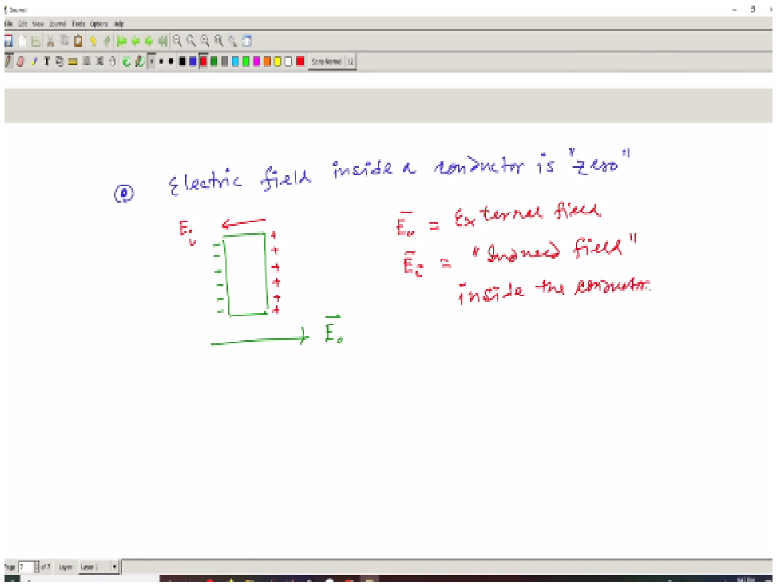

Now, a few properties once you know that what is conducted roughly not the definition and just say that if plenty of electrons are there we normally call it but there are a few properties. So, electric field the first thing is that the electric field inside a conductor is 0. So, one can understand this suppose we have a conductor like this and if we have an electric field outside say $\vec{E}_0$ then I can always since there are free electrons they can arrange inside the conductor in this way and that can produce and reverse electric fields $\vec{E}_i$.

So, $\vec{E}_0$ is external field and $\vec{E}_i$ is induced field inside the conductor so, that induced field basically nullify whatever the external field you have. So, that is why always inside the conductor you have the field 0 the charge densities also vanish everywhere.

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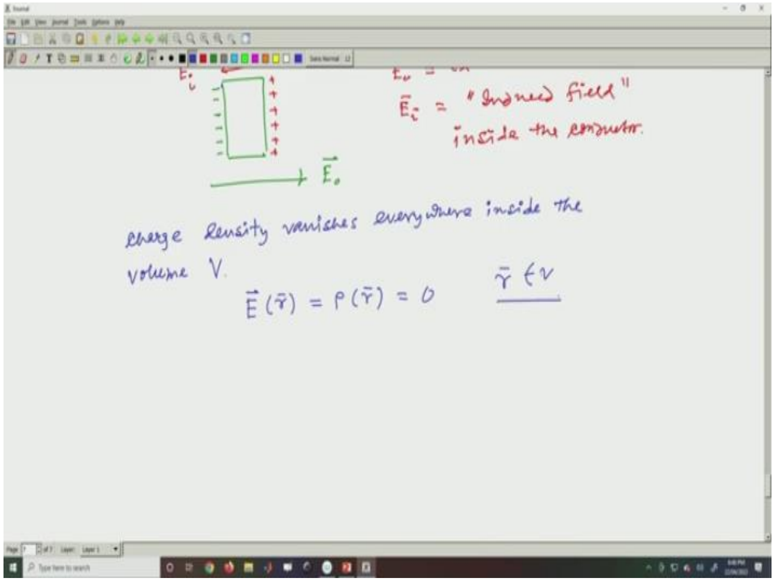

So, here we can also say that the charge density vanishes everywhere inside the volume V. because what happened that when we have a conductor all the electrons will be accumulated in the surface. So, inside the volume there should not be any charge density. So, the charge density vanishes everywhere inside the volume V. So that means E = 0 and ρ is also 0 when r is inside the volume.

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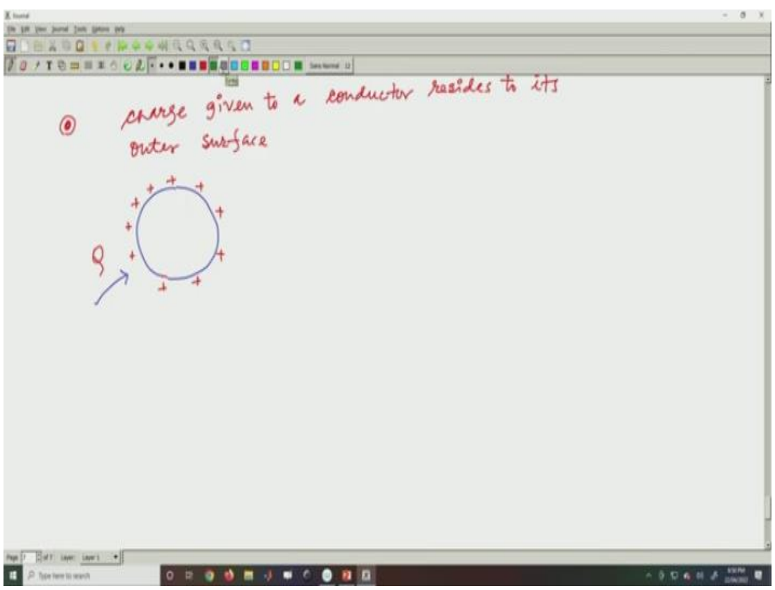

So, the next thing is the charge as I mentioned the charge given to a conductor resides on its outer surface so, whenever you put some kind of charge suppose this is a conducting sphere and from outside you put some kind of charge here so, these charge say it is a plus charge will resides over the outer surface like this it is distributed over the outer surface. So, you can see that the field inside and outside is 0 so, the total field this is fine I do not need to do anything here because this is the point I wanted to make.

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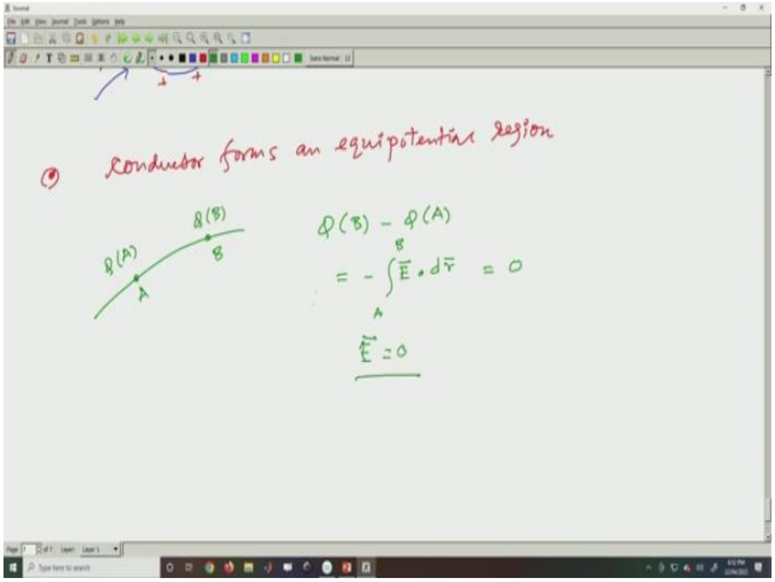

Next third point conductor forms an equipotential region whenever we have a conductor the surface of the conductor is essentially equipotential surface that is why I started with equipotential a brief outline of equipotential surface to make you understand this point. So, if I have over the conductor 2 point so, the $\phi(A)$ and $\phi(B)$ should be same. So, $\phi(B) - \phi(A)$ that quantity I can write as $\vec{E} \cdot d\vec{r}$ from A to B and that basically gives me 0 and it is true because $\vec{E}$ is 0 because we know that there is no electric field in the conductor. So, $\vec{E}$ vanishes and that makes the surface of the conductor as equipotential surface.

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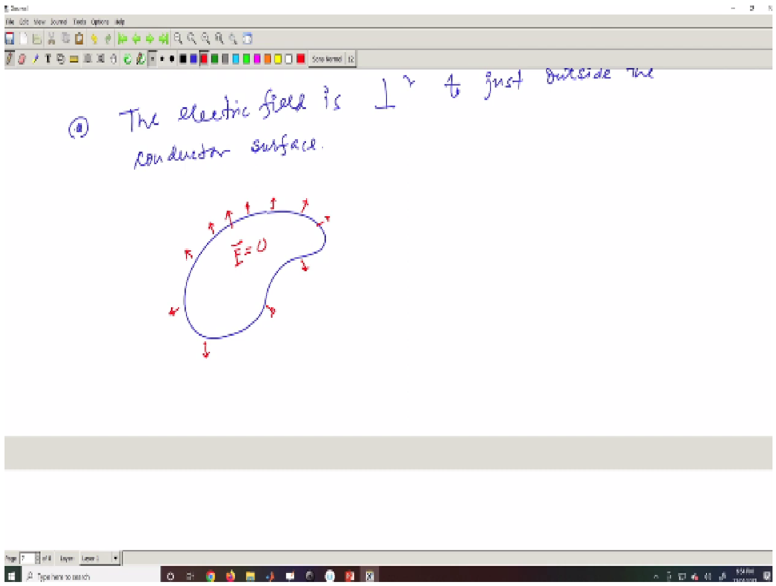

Next, these points I already mentioned the electric field is perpendicular to just outside the conductor surface. So, suppose I am having a conductor like this. So, the electric field just outside here because the charge are located to sides on the surface so, that produces some electric field. So, that electric field just above the surface is perpendicular and it has to be because we know that the electric field is perpendicular to a conduct this.

Here $\vec{E} = 0$ mind it inside that I am talking about just outside the region what is the electric field that is you know that electric field is perpendicular. Now, if I calculate the strength of the electric field.

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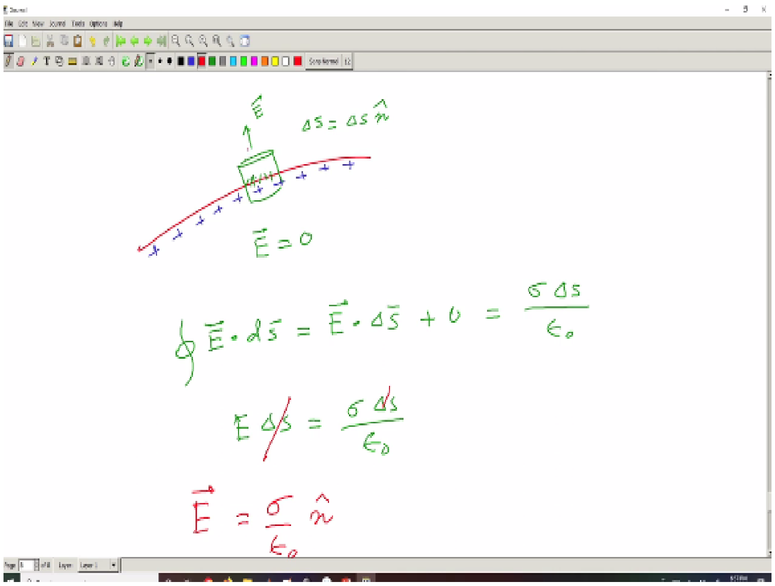

So, the strength of the electric field if I calculate so, suppose I have a conducting sphere here and the charges are here very near to the surface or exactly what the surface like this distributed and if I want to find out what is the electric field just above this region, I can use our old Gaussian surface Gaussian law. So, I can have a Gaussian surface here like this. I can prepare the Gaussian surface where electric field is along this direction.

And the surface is $\Delta s\ \hat{n}$. Here inside the conductor $\vec{E} = 0$. So, what do we have the law is closed integral $\vec{E} \bullet d\vec{s}$ should be equal to in this case is should be equal to the just electric field outside dot area $\Delta\vec{s}$ + 0 because inside there is no electric field and that quantity should be charge enclose divided by ϵ_0 that means $\sigma\ \Delta s$ that is the charge divided by ϵ_0, σ being the surface charge density.

So, whatever the charge it is enclosing is here in this region is $\frac{\sigma \Delta s}{\epsilon_0}$. So, $\vec{E}$ simply comes out to be from here from this equation $\vec{E} \bullet d\vec{s}$, $\vec{E}$ is perpendicular so $\vec{E} \bullet d\vec{s}$ is simply E $\Delta s = \frac{\sigma \Delta s}{\epsilon_0}$ Δs both the side will cancel out and eventually we have E = $\frac{\sigma}{\epsilon_0}$ magnitude wise and if I want to write the unit vector if I write in terms of vector then it should be when n is the direction of the surface n is the unit vector of the surface, which is perpendicular to the surface.

So, these are the few properties I just tried to discuss, because there are other properties also maybe in the next class we will discuss those properties. Today I do not have much time to discuss more.

So, overall in today is discussion. We try to understand the few basic properties of the conductor and in the next class I should discuss more about the conductor and how the induced charges are there.

If we have a hole inside the conductor, what should be the electric field there these kinds of problems and also the equipotential surface etcetera. So with that note I like to conclude my class here so thank you for your attention and see you in the next class.