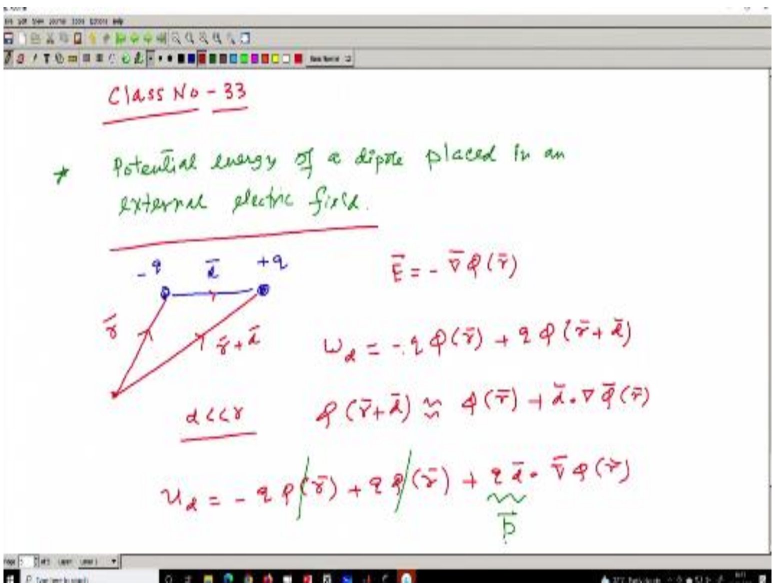
Foundation of Classical Electrodynamics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology - Kharagpur

Lecture – 33
Electrostatic Dipole (Contd.,)

Hello students to the foundation of classical electrodynamics course, under module 2 today we have lecture 33 and in today's lecture we will continue the different aspects of this electric dipole.
(Refer Slide Time: 00:32)



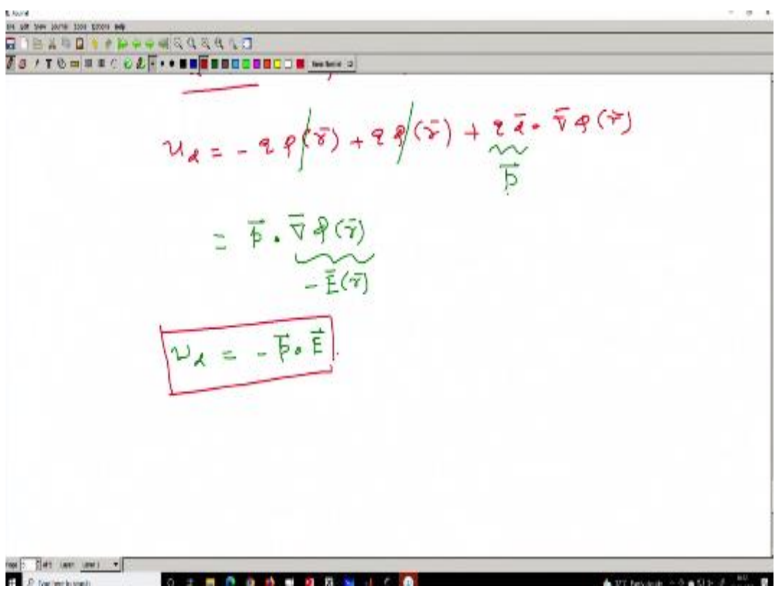
We have class number 33 now and the thing today we are going to calculate the first thing is the potential energy last time I mentioned that potential energy of a dipole placed in an external electric field so, there is an external electric field and I place a dipole and we need to find out that what should be the potential energy we discussed earlier. So, that thing we calculate. So, let us do it quickly. So, we have a dipole here say this is -q, +q and this length is say $\vec{d}$ and the location it is located with some origin like at point say $\vec{r}, \vec{r}+\vec{d}$.

So, the external electric field $\vec{E}$ described by the potential ϕ here, so, I can write this external field with that potential. So, this is this quantity so, this is the potential ϕ that we are going to experience by the dipole and if that is the case, we know what should be the potential energy and we know the potential energy is charge multiplied by the potential it will go to experience last day we calculate last to last class.

So, for the dipole it is simply whatever the charge they have and the corresponding potential ϕ at point $\vec{r}$ and then +q this is the summation of all the thing and this is ϕ and $\vec{r} + \vec{d}$ again $\vec{d}$ tends to 0 that is the condition we should have. Now, normally $\vec{d}$ is very small compared to $\vec{r}$ magnitude wise then $\phi(\vec{r} + \vec{d})$ that quantity I simplify and like before last class also I did this same technique it should be simply $\vec{d} \bullet \vec{\nabla} \phi(\vec{r})$ and I should not extend much because it should be the higher order of $\vec{d}$ and I can neglect safely.

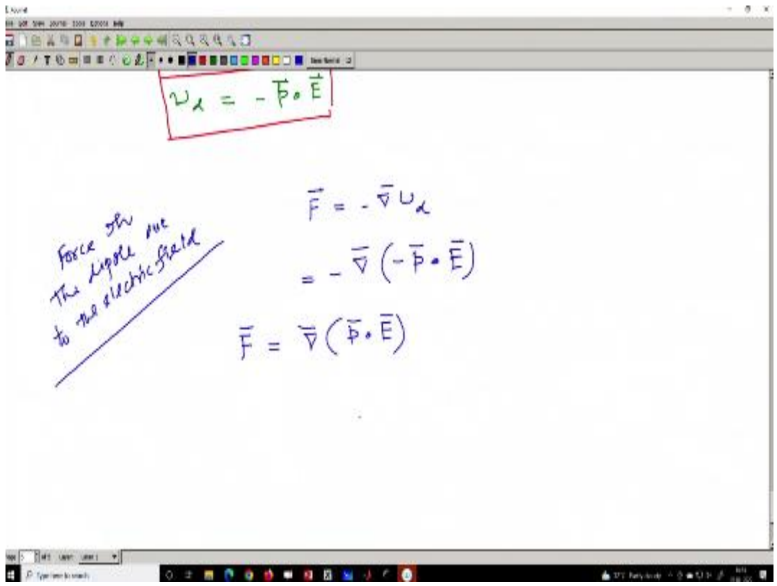
So, U_d is now -q $\phi(\vec{r})$ and then plus this quantity +q $\phi(\vec{r})$ and +q $\vec{d} \bullet \vec{\nabla} \phi(\vec{r})$. Now again I have this q $\vec{d}$ term q $\vec{d}$ that is the dipole moment $\vec{p}$ this term this term we are going to cancel out.

(Refer Slide Time: 05:27)



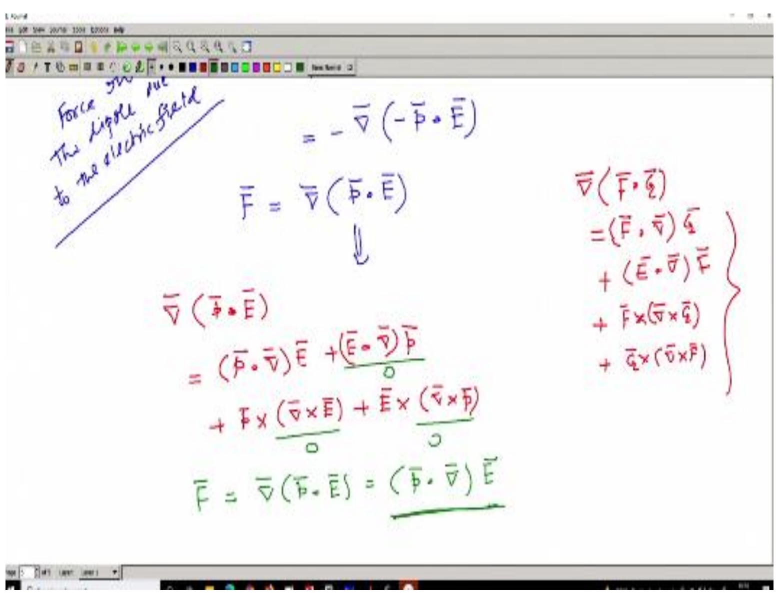
And simply we have dipole moment $\vec{p} \bullet \vec{\nabla} \phi(\vec{r})$ and this quantity is nothing but $-\vec{E}(\vec{r})$. So, my potential energy that the dipole should experience due to the presence of this external field is simply $-\vec{p} \bullet \vec{E}$.. Now, whatever it is experiencing an electrostatic potential energy then due to the external field so, some sort of force it should exert this external electric field. So, that again we can calculate.

(Refer Slide Time: 06:37)



So, the force on the dipole due to the external electric field so, what is the force that the dipole is going to experience now, I know what is the energy. So, the force should be simply minus of this quantity this potential energy and this expression I already figured out so, it should be $-\vec{\nabla}(-\vec{p} \bullet \vec{\mathrm{E}})$ and I can absorb this minus, $\vec{\mathrm{F}}$ simply becomes the gradient of the scalar quantity $\vec{p} \bullet \vec{\mathrm{E}}$ how I can use a vector identity here.

(Refer Slide Time: 08:07)

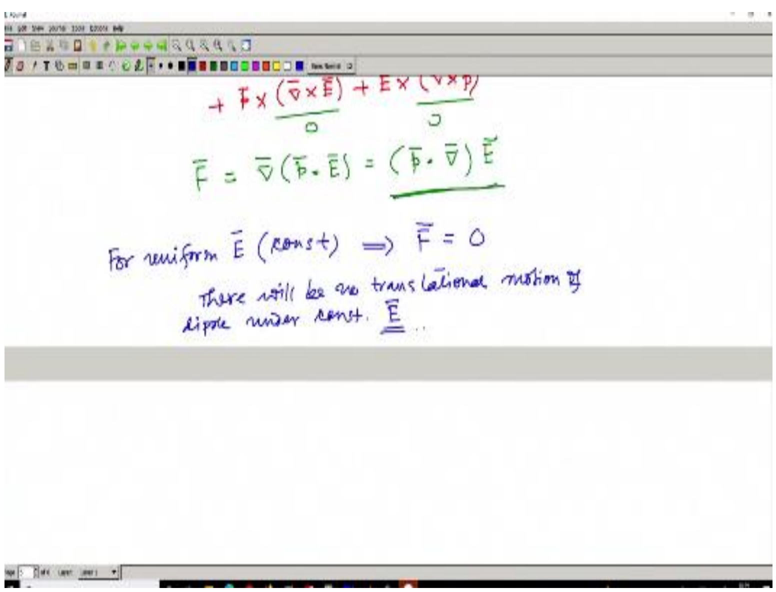


This is a very important vector identity when we have 2 vector field here then the vector identity that follows like this. So, that I can write is if I extend the vector identity, vector identity means I am saying this I have that I want to find out the divergence of 2 vector quantities and say vector field $\vec{F} \bullet \vec{G}$ this is. So, this quantity we know that this is not a very straightforward thing and it should be like here I am writing directly so, it is let me write it first here.

So, $\vec{F} \bullet \vec{G}$ there are many times be careful and then $\vec{E}$ dot this operator that will operate over $\vec{F}$ and then I have $\vec{F}$ curl that is operate over $\vec{G}$ and I have $\vec{G}$ and then $\vec{\nabla} \times \vec{F}$ this is not a very straightforward I must say and that will go to us here. So, there were my first term will be $\vec{p}$ this will operate over $\vec{E}$ and then $\vec{E}$ dot these things operate over $\vec{p}$ and now $\vec{p}$ curl of this things and $\vec{E}$ curl of this thing now, if you look carefully many terms will cancel because $\vec{p}$ is fortunately $\vec{p}$ is a constant, since $\vec{p}$ is a constant, so, this operator over $\vec{p}$ does not mean anything.

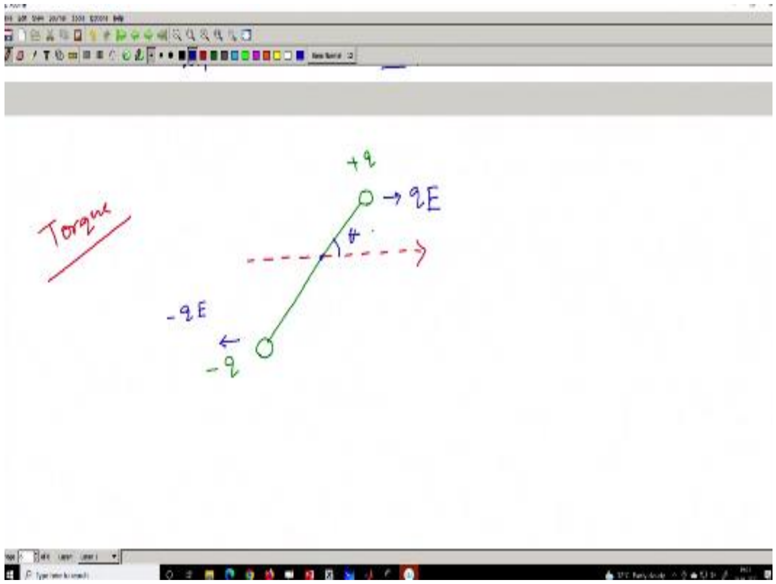
So, this term will 0 $\vec{\nabla} \times \vec{E}$ we know that it is 0 and $\vec{\nabla} \times \vec{p}$ again it goes to 0. So, simply we have $\vec{F}$ is equal to which is divergence of $\vec{p} \bullet \vec{E}$ and that quantity is $\vec{p}$ dot this operator that will be going to operate over $\vec{E}$. So, this is roughly the expression we have now, for uniform $\vec{E}$. Now, let us consider the $\vec{E}$ is uniform. So, now $\vec{E}$ is uniform.

(Refer Slide Time: 11:49)



So, for uniform $\vec{E}$ that is uniformly mean to this simply constant then simply I find that this operator over a constant so, it is simply gives 0. So, that means, I should have $\vec{F}$ to be 0. So, what is the conclusion? The conclusion is there will be no translational motion of the dipole under constant $\vec{E}$ but, what about the torque and that you find that that we will get something.

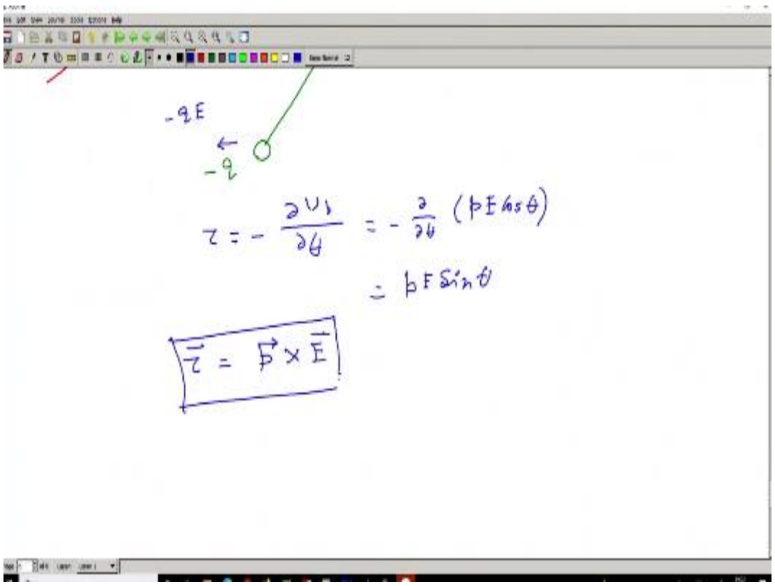
(Refer Slide Time: 13:22)



So, next we calculate the torque so, for uniform field we find that there is no translational motion, but for uniform field the force on charge +q and -q they are equal and opposite. So, that basically creates a torque so, what should be the scenario let me draw it. So, this is suppose the constant electric field moving in this direction and I placed a dipole tiny dipole magnify that and this is one charge and this is another charge.

And this charge is say +q and this charge is -q and when we have a constant electric field so, the force that it will experience is simply q $\vec{E}$, which is in this direction but because of this minus sign here it will experience a force in the opposite direction, which is minus of q $\vec{E}$ these are equal but opposite. So, obviously it should experience a torque if it is a θ then you should experience a torque and that we can calculate the torque τ.

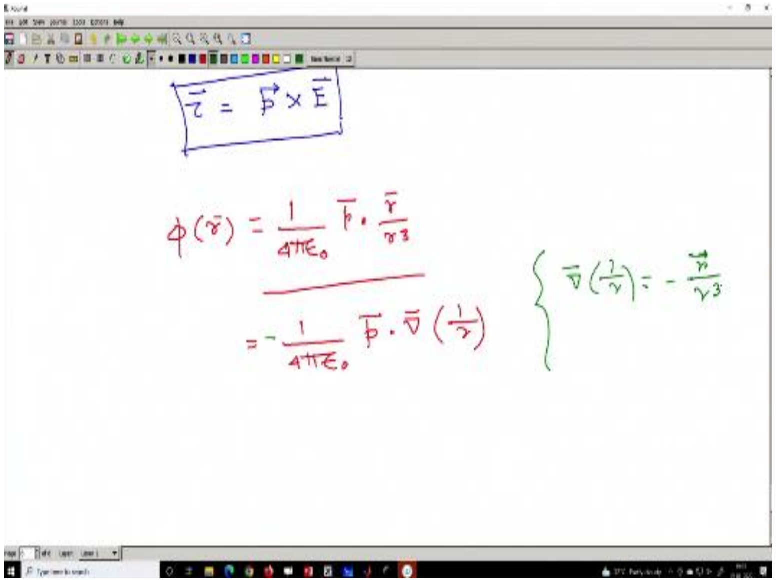
(Refer Slide Time: 14:46)



We know that if we know the potential, then the torque can be calculated. Derivative with respect to θ and that quantity is simply $-\frac{\partial}{\partial \theta}$. And U_b we know it is $\vec{p} \bullet \vec{E}$, so, it is simply $\vec{p}$ and then $\vec{E}$ and the angle between these 2, which is cos θ let me write properly cos θ so, this is simply if I make a derivative with respect to θ it is p $\vec{E}$ and sin θ in vectorial note this sin θ can be absorbed and I can write it as simply $\vec{p} \times \vec{E}$ this is the amount of torque it will be going to experience by dipole moment is $\vec{p}$.

And if it is placed in a constant external electric field $\vec{E}$. So, also we calculated the potential last class and then we tried to find out the electric field out of that that was the homework if you remember. So, in another in many books it is written in terms of polar coordinates. So, I can also calculate this we can also calculate this in polar coordinates, but in this gradient form.

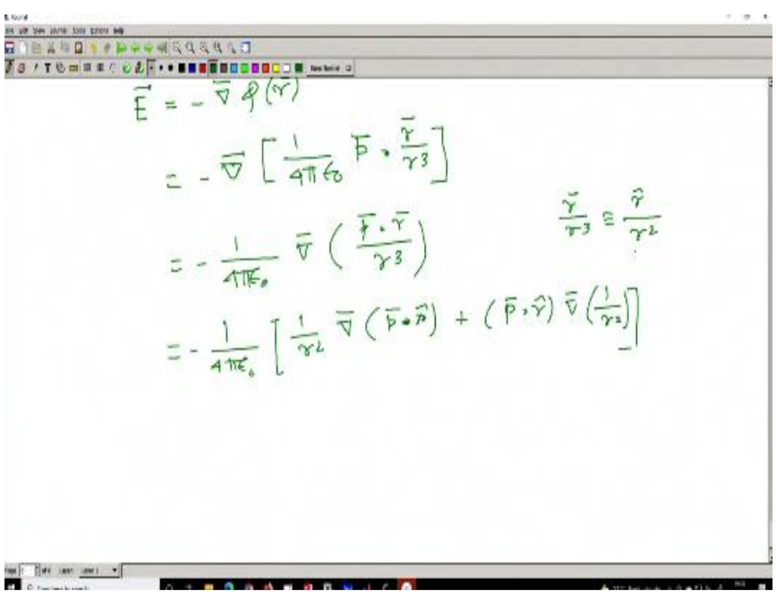
(Refer Slide Time: 16:44)



So, a dipole potential as a gradient if I want to write say $\phi(\vec{r})$ is $\frac{1}{4\pi\epsilon_0}$ and then $\vec{p} \cdot \frac{\vec{r}}{r^3}$ that was the potential we calculated and also we did it in gradient form if you remember by using this sticky method and it was $\frac{1}{4\pi\epsilon_0}$ and it should be $\vec{p} \cdot \vec{\nabla}(\frac{1}{r})$ where, this I believe you can take check by yourself that $\vec{\nabla}(\frac{1}{r})$ is simply I should put a negative sign here so, it is simply $-\frac{\vec{r}}{r^3}$ or $\frac{1}{r}$ in a vector form it is $\frac{\vec{r}}{r^3}$ or $\frac{\hat{r}}{r^2}$.

The pen is not writing properly this is say $\frac{\vec{r}}{r^3}$ so, that I use here to get the potential.

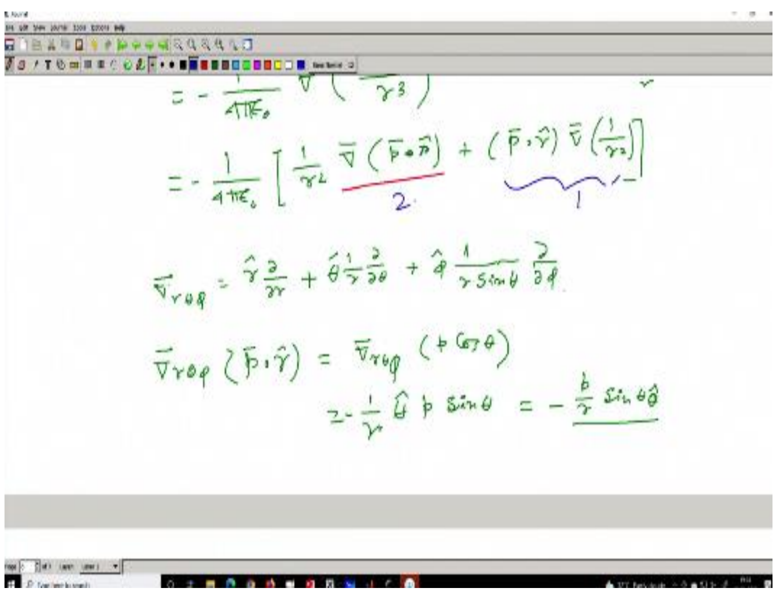
(Refer Slide Time: 18:53)



Now the $\vec{E}$ I figure out from that so, my $\vec{E}$ electric field due to dipole is $-\vec{\nabla}\,\phi(\vec{r})$ and I simply have if I simply put this one then it should be. So, let me do that I can use the whole. So, minus of this and $\phi(\vec{r})$ if I use this one it is simply $\frac{1}{4\pi\epsilon_0}\,\vec{p}\bullet\frac{\vec{r}}{r^3}$ so, that quantity I can manipulate and try to find something. So, $-\frac{1}{4\pi\epsilon_0}$ and then it should operate, the operator should operate over the quantity $\vec{p}\bullet\frac{\vec{r}}{r^3}$.

And that quantity I can expand to find $-\frac{1}{4\pi\epsilon_0}$ and I can expand this is a vectorial so, it should be $\frac{1}{r^2}$ and then $\vec{\nabla}(\vec{p}\bullet\hat{r})$ then + $(\vec{p}\bullet\hat{r})$ and $\vec{\nabla}\left(\frac{1}{r^2}\right)$ you can write this $\frac{\vec{r}}{r^3}$ is equivalent to $\frac{\hat{r}}{r^2}$. So, I use this one to get this now, this operator if I write in polar coordinate because dipole is placed in a plane so, I can write also in polar coordinates.

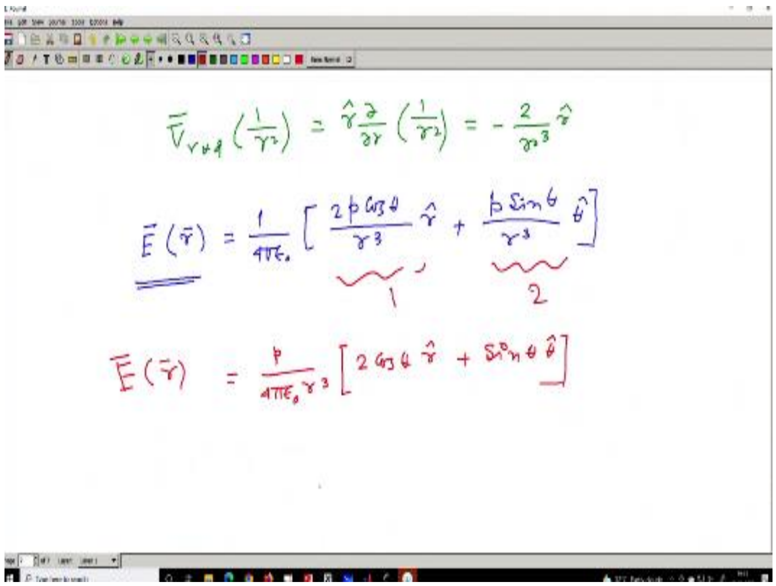
(Refer Slide Time: 21:55)



So, in polar coordinate the operator r θ ϕ it is $\hat{r}\,\frac{\partial}{\partial r}$ then $\hat{\theta}\,\frac{1}{r}\frac{\partial}{\partial\theta}$ and then $\hat{\varphi}$ I believe you remember the scaling factors in their case the scaling factor is r scale factor h_1 h_2 h_3 θ and then $\frac{\partial}{\partial\varphi}$ that was the expression. Well now I operate this r θ φ over $(\vec{p}\bullet\hat{r})$ and that eventually gives me operator r θ φ and $(\vec{p}\bullet\hat{r})$ is simply p cos θ.

Because the angle between $\hat{r}$ and $\vec{p}$ is θ so, that quantity depends on θ. So, all the other component will not be there, so, it will be simply $\frac{1}{r}$ so, it is $\frac{1}{r}$ then $\hat{\theta}$ and I have $\frac{\partial}{\partial \theta}$ so, it should be p and then sin θ the derivative will be sin θ with a negative sign so, eventually we have $-\frac{p}{r}$ and then sin θ and then $\hat{\theta}$ so, this is one component. Another component is still there, so, I figured out this component and another component is still there and that again we can execute.

(Refer Slide Time: 24:53)



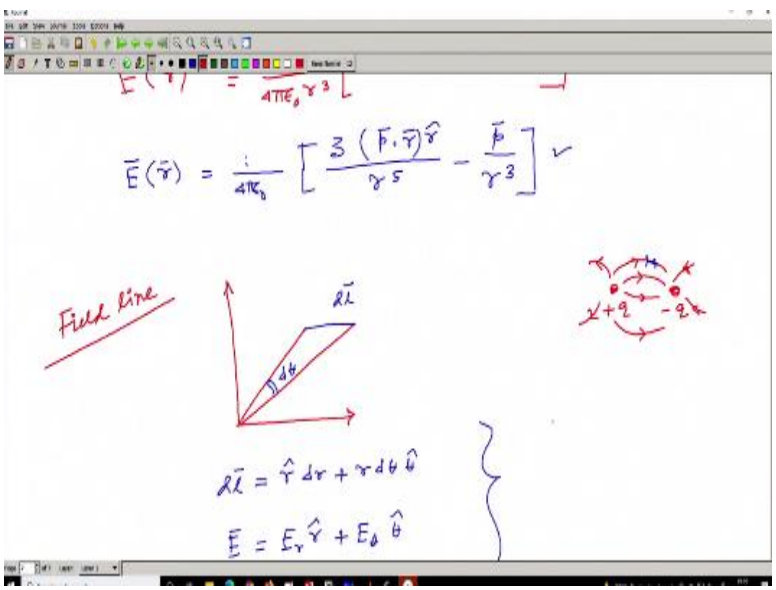
So, if I do that, then $\vec{\nabla}_{r\theta\varphi}$ $(\frac{1}{r^2})$ is simply $\hat{r}\ \frac{\partial}{\partial r}$ $(\frac{1}{r^2})$ and that quantity is $-\frac{2}{r^3}$ and $\hat{r}$. So, the overall $\vec{E}$ I can now write as a function of $\vec{r}$ and that is $\frac{1}{4\pi\,\epsilon_0}$ it is in polar form mind it and it is $\frac{2p\cos\theta}{r^3}$ and $\hat{r}$ + $\frac{p\sin\theta}{r^3}$ $\hat{\theta}$ mind it now, I write my total electric field in terms of r θ φ coordinate φ is not there, only r and θ is there because it is a 2 dimensional electric field.

But let me go back to here this is $\frac{1}{r^2}$ and this quantity so, it should be $\frac{1}{r^3}$ and this quantity $\vec{p} \bullet \vec{r}$ I calculated and that let me check. This operator $\vec{p} \bullet \vec{r}$ I calculated it should be p cos θ and then I have sin θ, $\vec{p} \bullet \vec{r}$ is cos θ and this I write the next term first. So, here $\vec{p} \bullet \vec{r}$ is p cos θ and $\frac{1}{r^2}$ is minus of 2 so, I should have a minus sign, so, this minus is absorbed here and other is minus both minus and minus is absorbed. So, I write this term here.

So, this is my one term and this is my second term and this term I write here this is my one term and this is my second term. The second term I write here like this whatever I get p by r sin θ, so, $\frac{p}{r^2}$ already $\frac{1}{r^2}$ is there, it is fine. So, now, I can simplify a bit because this r^3 is sitting here. So, it should be 1 divided by no, p is also there. So, you can take p outside $\frac{p}{4\pi\epsilon_0 r^3}$.

And then it should be 2 cos θ $\hat{r}$ + sin θ $\hat{\theta}$. So, this is the the expression of the electric field due to a dipole, but in polar form. In a similar expression you people should we calculate it in other form where we did not. So, that was the homework. So, let me write it that as well that what was the expression we got earlier and that expression let me write side by side so that you can realize.

(Refer Slide Time: 29:23)



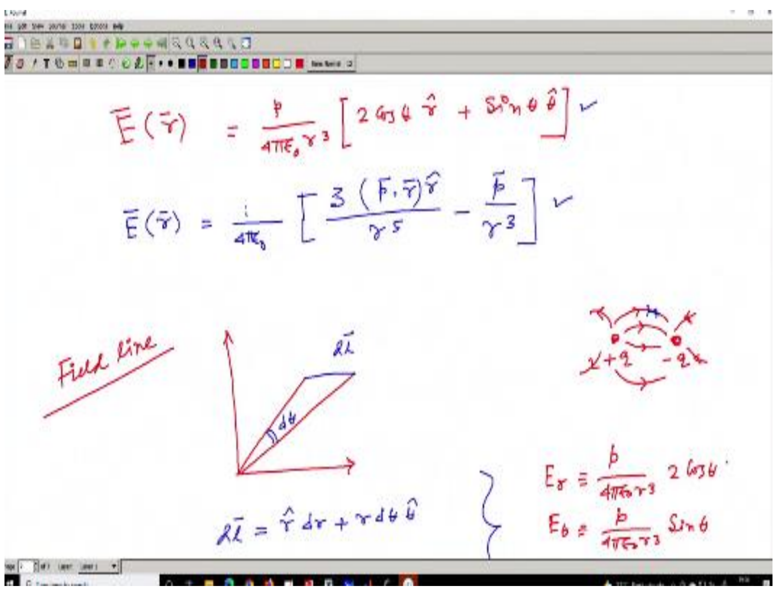
That expression you are supposed to calculate from the potential and that was this $\frac{1}{4\pi\epsilon_0}$ here we had $\frac{3(\vec{p}\bullet\vec{r})\,\hat{r}}{r^5}$ and then $-\frac{\vec{p}}{r^3}$ that was the expression we all we derived I mean that was the homework, but this is another expression, but in polar coordinates in r θ φ coordinate system. Now, what is the advantage of that? Because whatever the field line the dipole have you can find out the expression of that field line.

So, quickly let us understand that so, the field line if I want to calculate what is the expression of the field line due to the dipole because, you know if 2 dipoles are there plus and minus +q and -q. So, there are some field lines like this and like this so, this expression of this field line one can figure out what is the expression of this field line in polar coordinate because the electric field we know so, I can try to let me so, if I have a line element like this, this is the line element $d\vec{l}$ and if this angle is say $d\theta$.

So, this line element is the line element I take for this say whatever this is the line over these field lines the line element I take over this field line. So, $d\vec{l}$ in polar coordinates, it is simply $\hat{r}$ dr and +r $d\theta\ \hat{\theta}$. So, this is the way the line segment we write in polar coordinates this line segment I took over these field lines produced by this dipole the system of 2 equal and opposite charge particles.

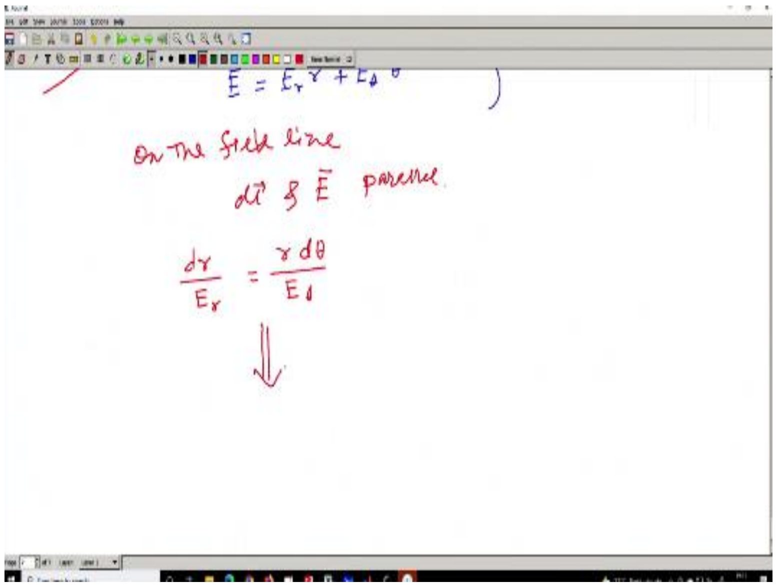
Now, the electric field the components are known $E_r\ \hat{r} + E_\theta\ \hat{\theta}$ what is E_r and E_θ because I just calculate here $\vec{E}(\vec{r})$ is here I can write it I need to check what was $\vec{E}(\vec{r})$ here?

(Refer Slide Time: 33:07)



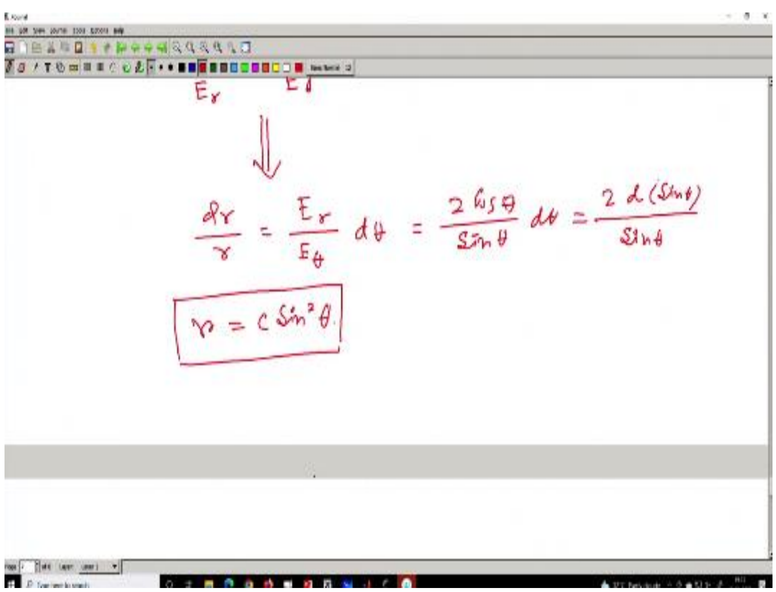
So, $\vec{E}(\vec{r})$ is $\frac{p}{4\pi\epsilon_0 r^3}$ and then 2 cos θ and E_θ is equivalent to the same quantity $\frac{p}{4\pi\epsilon_0 r^3}$ then sin θ these are the 2 components of $\vec{E}$. Now, if I want to find out the expression of this field lines I need to have a relation.

(Refer Slide Time: 33:54)



And the relation is on the field line so, this d$\vec{l}$ and this $\vec{E}$ they are parallel because that is why we calculate these on the field line are parallel. So, if 2 vectors are parallel, so, these vectors so, d$\vec{l}$ and $\vec{E}$ they are parallel. So, that means their components the ratio of their component should be same and that if I write $\frac{dr}{E_r} = \frac{rd\theta}{E_\theta}$ that basically gives us the relation.

(Refer Slide Time: 35:00)

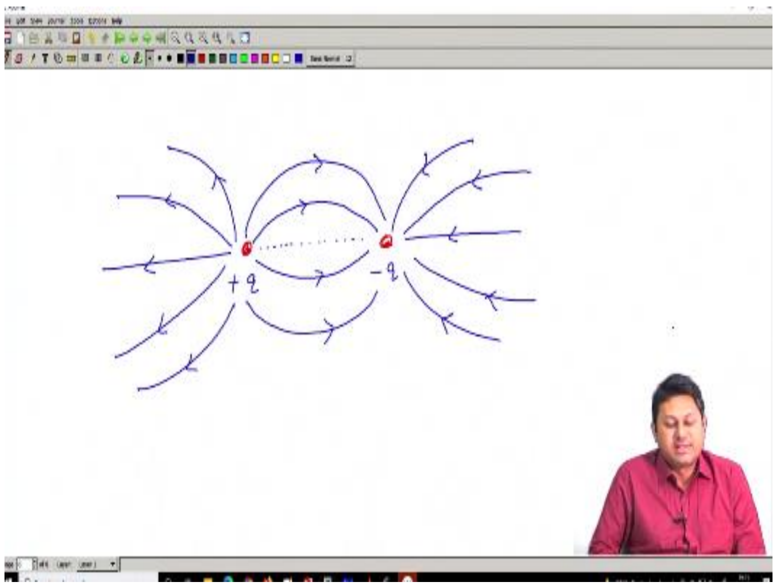


And the relation is saying that $\frac{dr}{r} = \frac{E_r}{E_\theta}$ dθ. And now, integrating this I can have so, what value we are having here so, this is dr is 2 $\frac{E_r}{E_\theta}$. So, p divided by these things will cancel out. So, we have

$\frac{2\cos\theta}{\sin\theta}$ and that dθ and that I can write that $\frac{2d(\sin\theta)}{\sin\theta}$ and now, if I integrate this entire stuff, I can simply have that is equal to some constant k.

So, I simply have r is equal to that constant multiplied by $\sin^2\theta$. So, that is the expression of the field lines that is created by the dipole. So, what kind of field lines so, the field line looks something like this.

(Refer Slide Time: 36:49)



So, I have 2 dipoles here, but I have a dipole here and the field line suppose this is +q and -q and the field line is going like this and the equation that we get is simply r is equal to the polar equation $r = c \sin^2\theta$ and then it goes like this going outward this is the way the field line is distributed and this is the dipole and this field line the expression of the field line we try to calculate here and which is this.

So, today we do not have much time to elaborate more on this issue. So, I would like to conclude here. So, today we will learn how the dipole is behaving in the external field how I can write the expression of the dipole say electric field due to the dipole in terms of polar coordinate and what should be the field line what is the expression of the field line by exploiting the polar coordinate expression of the polar coordinates of the electric field that is produced by the dipole etcetera. So, next class we will start another important thing, which is called the multipole expansion.

And in that expression and that calculation the concept of dipole will be there. So, on beforehand we just discussed about the general thing of the dipole, but our main aim is to find out that what is in the multipole expansion in charge distribution is there and If I want to find out the electric field then this electric field is nicely I mean systematically I can expand this electric field as a contribution of the monopole, dipole, quadrupole etc. so that thing we will do in the next class thank you very much for your attention and see you in the next class.