Foundation of Classical Electrodynamics
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Lecture – 29
Electrostatic Energy

Hello students to the foundation of classical electrodynamics course. So, under module 2, we today going to discuss about the electrostatic energy. But before that we will be going to calculate few problems related to electrostatic potential and then we will be going to start electrostatic energy. So, let us start today's class.

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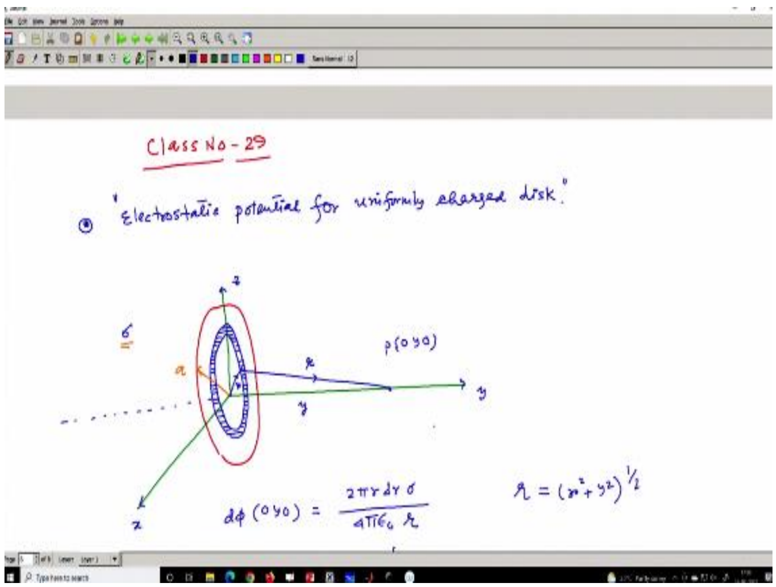

So, we today have the class numbers 29. So, in the last class, we calculated the electro static potential for infinitely extended wire for finitely extended wire and from that we calculated electric field. So, today we will be going to extend that calculation for another case and this case is for uniform charge disks, so we calculate the electrostatic potential for uniformly charged disk so, this is the problem, so, we have a disk here. Suppose, this is the disk let me draw the coordinates and strip I can draw here.

So, the problem is this is a disk, which is uniformly charged with say surface charge density σ and then what should be the potential at some point say over this axis if I say this is y axis and if it is say x axis and z axis. So, what should be the potential at some point here over y axis say the coordinate of this point say p is 0 y 0 so, over y axis I should find a potential for this disc. And suppose this disc is having a radius of a. So, this is a and charge density ρ uniform charge density I already mentioned.

So, normally in order to do this problem, we take a small section a ring of this disc, this is the ring and for that we calculate and then we are going to integrate that is the normal strategy. So, this is the region for which I will be going to calculate the potential energy at point p, and then I am simply going to integrate. So, this is the region tiny region. So, suppose the from here to here this is say r and from here to here this is Л and this is y I already have so, this is from here to here r this is Л and this is y.

So, the dϕ at the point (0 y 0) is simply due to this shaded region is 2πr if dr is the thickness dr and the surface charge density that is the charge total amount of charge divided by $4\pi\epsilon_0$.
And the distance from that is Л mind it Л is in terms of x and y is $(x^2 + y^2)^{1/2}$, this is the value of the Л in terms of x and y.

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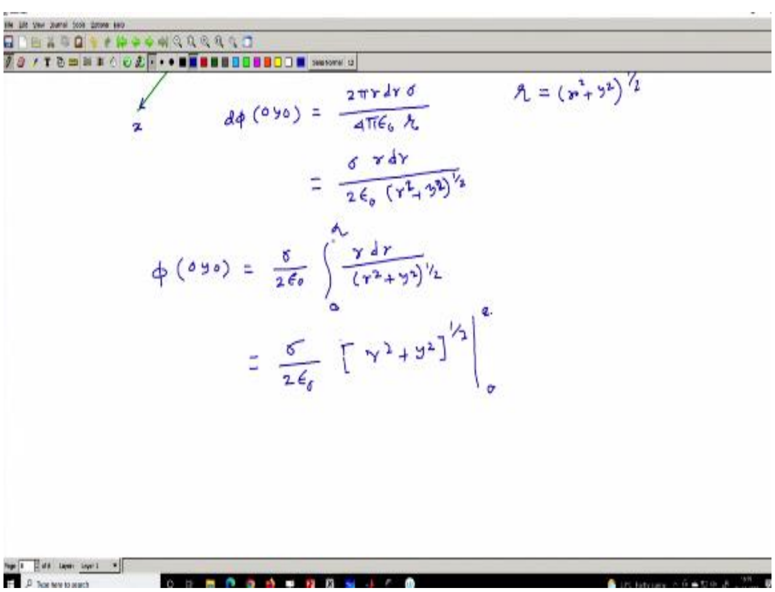

And I can simply write this in terms of the value so, 2π 4π will cancel out so, I can simplify it, it is $\frac{\sigma\ r\ dr}{2\epsilon_0 (x^2+y^2)^{1/2}}$, I mean in this case, it is simply r, $r^2 + y^2$ so I should write it r^2 because otherwise. So, better I should I write it simply r^2. Since, I already defined r here, so r^2 and whole to the power

$\frac{1}{2}$. So, now, if I want to find out what is my full ϕ at the point (0 y 0) then I simply need to integrate it.

And then $\frac{\sigma}{2\epsilon_0}$ and then when I integrate it should be $\frac{r\,dr}{(r^2+y^2)^{1/2}}$ so, this integration is simple. So, I have $\frac{\sigma}{2\epsilon_0}$ and this integration simply gives us $(r^2 + y^2)^{1/2}$ and now, I need to integrate this for the entire disk. So, the limit should be 0 to a, this is the limit. So, my limit here is 0 to a.

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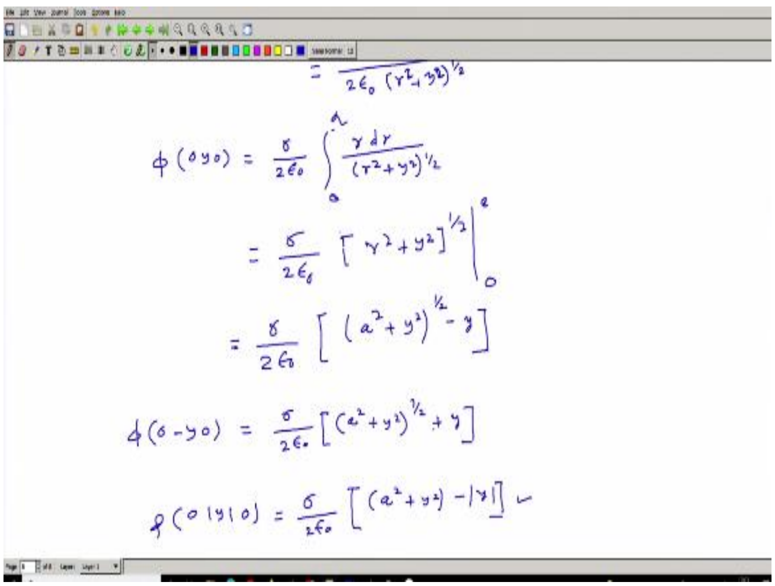

Now, this quantity is $\frac{\sigma}{2\epsilon_0}$ and then if I put the limits then it should be $(a^2 + y^2)^{1/2} - y$. And for minus if I go to the opposite direction if I want to find out what happened, the potential in the opposite side then the ϕ (0, - y, 0) this value simply comes out to be $\frac{\sigma}{2\epsilon_0}$ and then $(a^2 + y^2)^{1/2} + y$. So, combining these 2, I simply have that ϕ at any point $|y|$ is simply $\frac{\sigma}{2\epsilon_0}$ and now $(a^2 + y^2)^{1/2} - |y|$.

That is the value I have now, if I want to find out what is the corresponding electric field that we also calculated.

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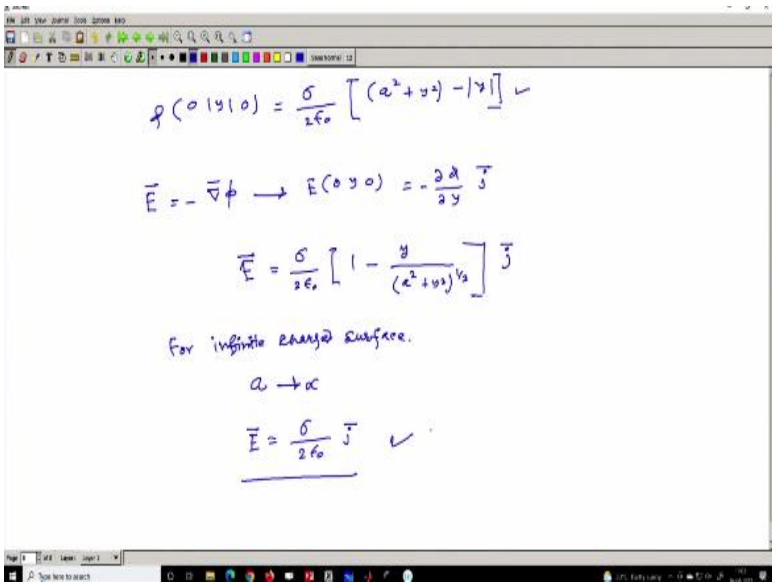

So, electric field is $-\vec{\nabla}\phi$. So, that value is simply E at (0 y 0) is the $-\frac{\partial\phi}{\partial y}$ in the direction of j and if I do then this value seems to be electric field comes up to be $\frac{\sigma}{2\epsilon_0}$ and then [1 - $\frac{y}{(a^2+y^2)^{1/2}}$] $\hat{j}$. Now, we know the result for infinitely extended surface. What is the electric field for infinitely extended surface that is known to us, if I now, make this surface infinite.

Then for infinite charged surface I simply have the limit *a* tends to infinity, because then it is infinitely extended and if we put that, then the value of the electric field that I get is simply $\frac{\sigma}{2\epsilon_0}$ $\hat{j}$ this is the result we already find already figured out using the Gauss's law and also using the simple calculation we find this result. So, this result again we figured out by calculating the potential. So, this part was left in the last class. So, I just conclude that I just complete this. Now, the next topic that we want to discuss, which is today's topic

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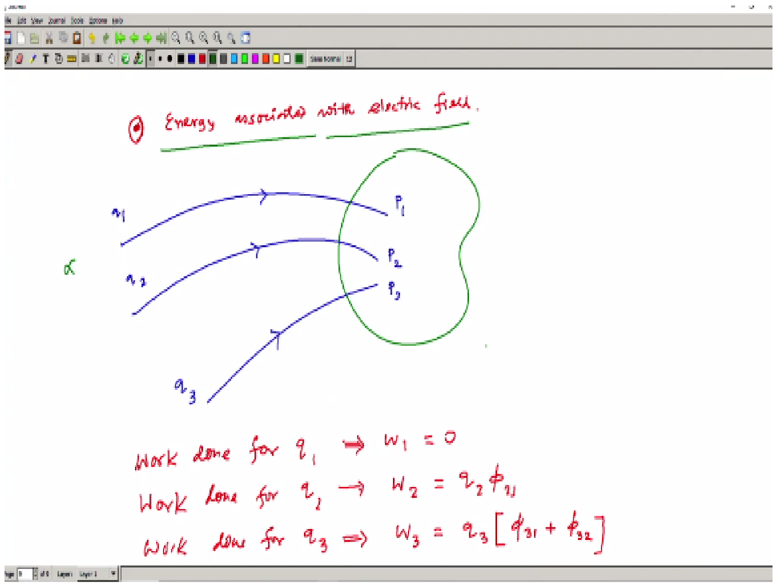

And that topic is energy associated with electric field so, when we have electric field, then some energies associated with that because, against the electric field if you bring a charged particle we need to do some work and that work needs to be stored as the electrostatic energy that thing we are going to calculate. So, the procedure is like this to do calculation procedure is like this very interesting way.

So, I bring a charged particle from infinite. So, this is say infinite and I bring a charged particle q_1 from infinite to some point so, this is my charged particle q_1 and I bring this charged particle from infinite to some point say P_1 in some domain, so, this is the domain where I bring all the charged particle. Then I do the same thing for another charged particle q_2, this is the charged particle and I bring this charged particle q_2 to P_2. So, mind it when I bring the charged particle q_2 to the point P_2 the charged particle q_1 at the point P_1 is already there.

So, we need to do some work against the electric field that is produced by the charged particle q_1, which is already there in the P_1 point that is the thing we need to consider, then I bring another charged particle say q_3 and this is the P_3 point where I bring and again when I bring this charged particle q3. Already the P_1 P_2 is staying there, I mean the charged particle at P_1 and P_2 are already there, so, we need to do the work against that. So, that will be stored. So, now if I calculate the work done.

So, the work done for q_1 that means to bring q_1 from infinity to point P_1. So, the work done for q_1 is simply 0 because there was so, if I write this is w_1 so, that value is 0 then the work done for q_2 that is w_2 that value is not q but the amount of charge you bring multiplied by the potential that is created due to the presence of charge 1 and that will be exerted on 2 so, it should be 2 1 try to understand that when the charge particle is here it is producing some kind of electric field or in other words some potential is generated.

And against this potential we bring the charge q_2. So, that potential will be experienced by q_2 and that will be stored as a work as an energy there then work done for q_3 is w_3 and that quantities q_3 is experiencing the potential due to the charge on q_3 plus potential due to the charge 2 on q_3. So, why the work done is q multiplied by ϕ, q multiplied the potential I quickly show.

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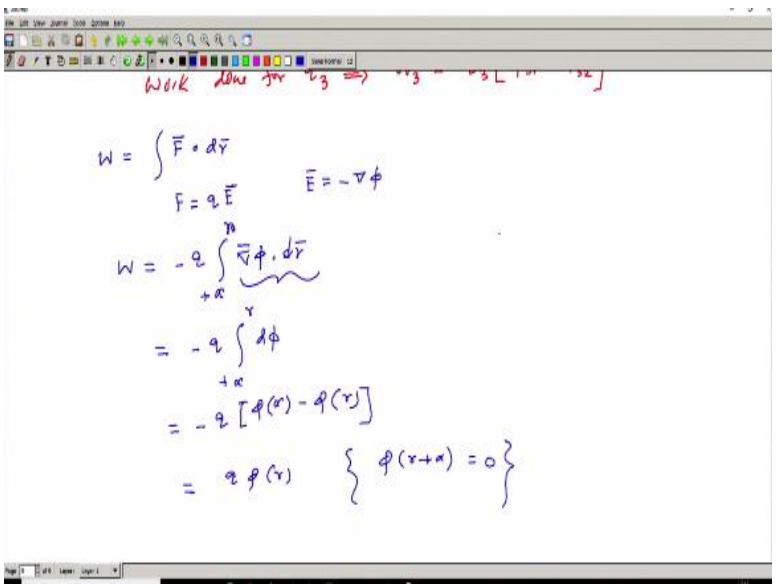

So that there should not be any confusion to work done we know that the work force dot dr and this is the Coulomb's force the force should be q into electric field $\vec{E}$ and again electric field $\vec{E}$ is associated with the potential with $-\phi$. So, now, work done is - q if I put everything together and then integration $\vec{F}$ is $q\vec{E}$ so, q is there. So, it is $\vec{\nabla}\phi \bullet d\vec{r}$ and I am taking the charge from minus infinity to some point say r.

So, that means this gives us - q and this quantity is simply minus of infinity to sorry, which is plus infinity to r. So, this is $d\phi$. So, that means it is simply - q potential at infinity minus potential at r.

So, that quantity is simply q as potential at r, because we consider that a potential at infinity to be 0. Now, the total work done so, that is why the work done is q and this quantity.

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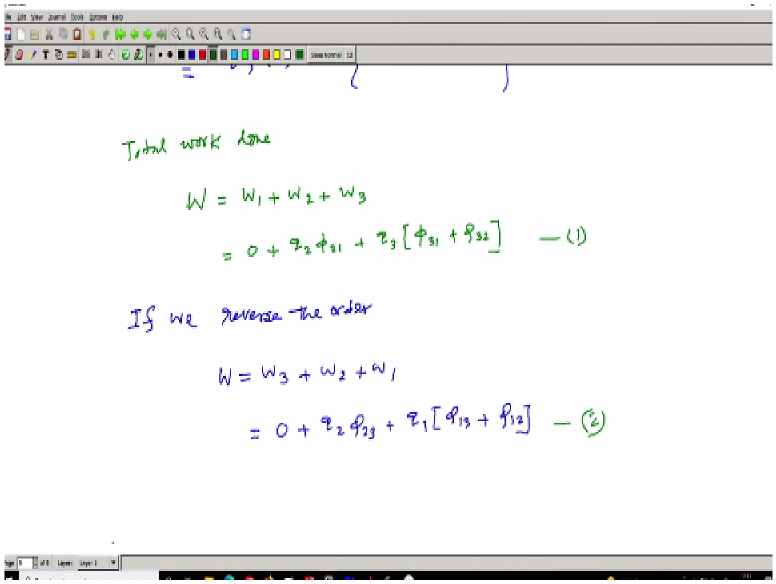

So, let us go back to the calculation then total work done W is $w_1 + w_2 + w_3$, which is simply $0 + q_2\ \phi_{21} + q_3\ (\phi_{31} + \phi_{32})$ that we already mentioned. Now, if I do the same thing in the reverse order, this interesting process I mentioned that is why if I do these things in reverse order, for example, before in this case, we just bring q_1 first and then q_2 and q_3, but if I do in the reverse order, let us first bring q_3 and then q_2 and then q_1.

Then, so, if we reverse the order that is bringing the particle in a different order in the reverse order then W will be simply $w_3 + w_2 + w_1$ and that is $0 + q_2\ \phi_{23} + q_1\ (\phi_{13} + \phi_{12})$. Now, if I add these 2s, so, whatever we have here in equation 1 and this is equation 2. So, if I add these 2 by adding equation 1 and 2.

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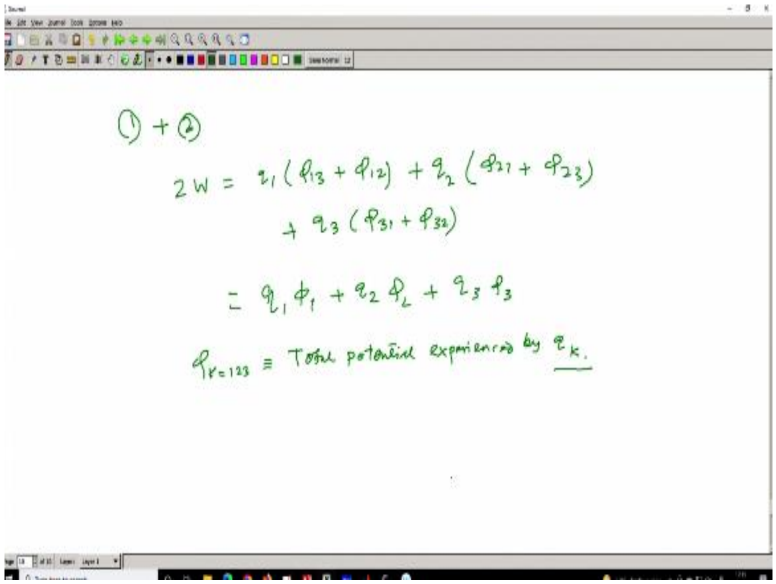

So, by making equation 1 + equation 2 we have $2W = q_1 (\phi_{13} + \phi_{12}) + q_2 (\phi_{21} + \phi_{23})$ and finally, q_3 $\phi_{31} + \phi_{32})$ so, that quantity 13 12 so, that is the total potential experienced by q so, I can write it simply q_1 ϕ_1 + q_2 ϕ_2 and q_3 ϕ_3. What is ϕ_k here, k = 123 so, ϕ_k, which is 1, 2 or 3 is simply the total potential experienced by the charge q_k this is the total potential experienced by the charge q_k.

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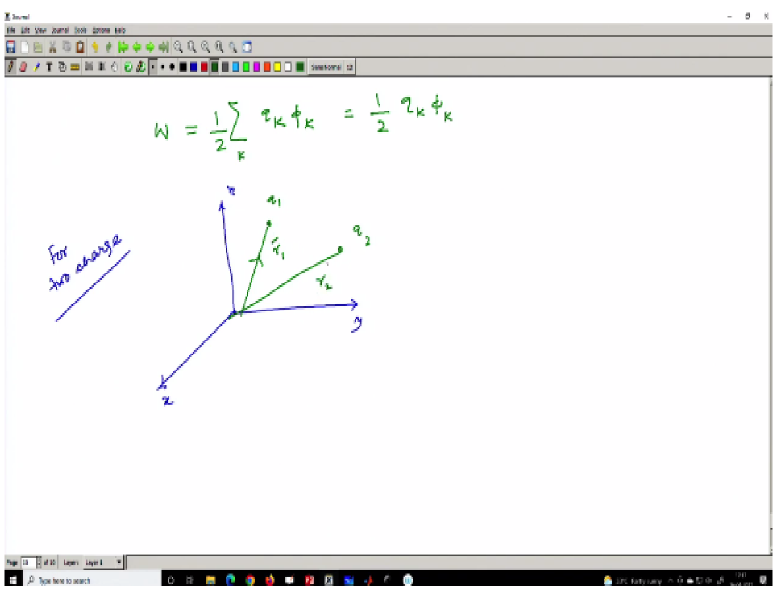

So, the W is simply $\frac{1}{2}$ of q_1 or if I write summation then it should be q_k ϕ_k, k is the number of particles that you are having. So, this quantity in Einstein notation, so, this is repetitive index so, I have $\frac{1}{2}$ q_k ϕ_k so, q_k as I mentioned q_k ϕ_k is it is a potential experienced by the charge q_k due to the all other charges. So, what happened for 2 charges if I have only 2 charges, so, simply for 2 charges suppose these 2 charges are placed in the coordinate system into 2 different points say, this is q_1 and q_2, this is r_1 and r_2.

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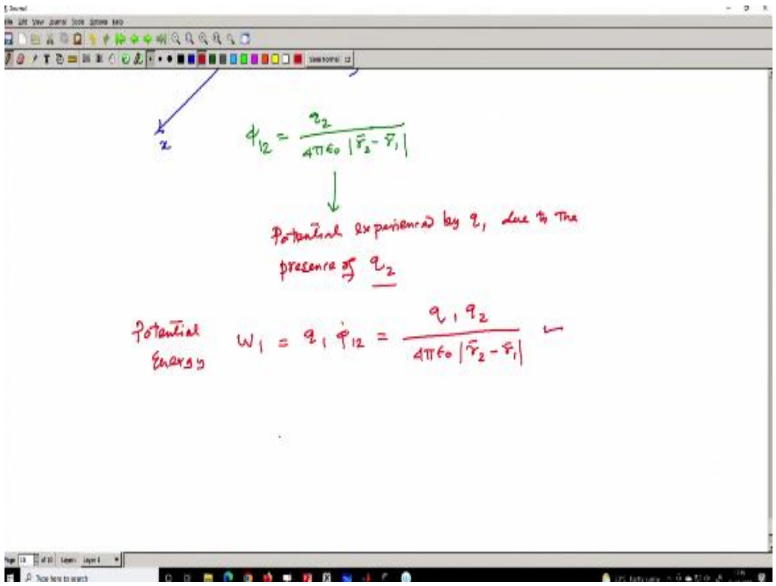

So, the ϕ_{12} is simply $\frac{q_2}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|}$ so, what is the meaning of that this is mind it potential experienced by q_1 due to the presence of q_2 so, the potential energy w_1 should be q_1 multiplied by ϕ_{12} so, it is simply $\frac{q_1 q_2}{4\pi\epsilon_0 |\vec{r}_2 - \vec{r}_1|}$ this is the potential energy that now for a set of charges, this is for 2 charges, I calculate.

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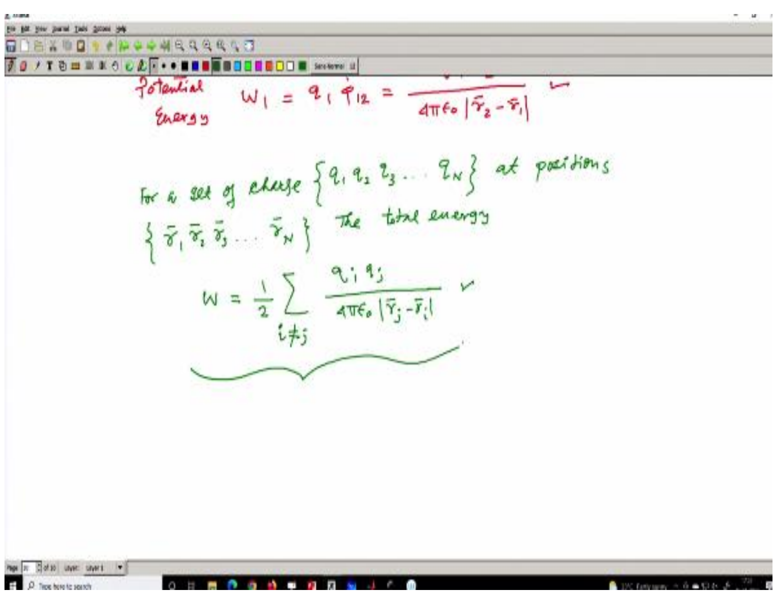

Now say for a set of charges what happened? For a set of charges say q_1 q_2 q_3 q_N at point these are at points for the set of charge these at positions say $\vec{r}_1$ $\vec{r}_2$ $\vec{r}_3$ $\vec{r}_N$ the total energy will be W equal to half of all the summation where i is not equal to j then $\frac{q_i q_j}{4\pi\epsilon_0 |\vec{r}_j - \vec{r}_i|}$. So, this will be the value for the total energy of when you have a number of points like q_1 q_2 q_3 distributed at the vector at the location $\vec{r}_1$ $\vec{r}_2$ $\vec{r}_3$ $\vec{r}_N$ etc.

And also you should note that this integration will be done for all the combinations of i and j but i and j are not equal. So, I just exclude all the condition when I add because when i and j is equal so, we are trying to you know find out the potential for the same particle itself so, that we exclude and then I will be going to get a result for the total energy and this total energy is associated with the set of charges sitting at the position $\vec{r}_1$ $\vec{r}_2$ $\vec{r}_3$ and then if that is the case discrete charge, if that is the case, we should have the expression like this.

So, this is the expression of the total energy when all the charges are placed in these locations. So, today I do not have much time to start something new. So, I like to conclude my class here. In the next class again we are going to discuss about more about the potential energy and then try to understand why the potential energy is positive. We will go to calculate and see that the potential energy should be positive it should not be negative value obviously, but we will see that and try to do few problems if time permits. So, with this note I would like to conclude. Thank you very much for your attention and see you in the next class.