Foundation of Classical Electrodynamics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology - Kharagpur

Lecture – 27
Electrostatic Potential

Hello students to the foundation of classical electrodynamics course. So, under module 2, today, we have lecture number 27. So, in today's lecture, we are going to understand something about electrostatic potential.

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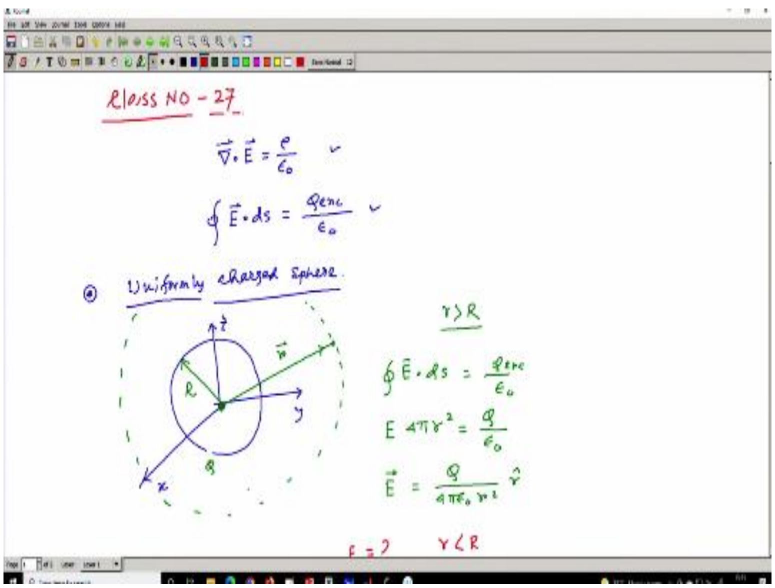

So, today, we have class number 27. So, in the last class, let me remind what we have in the last class and the last class we had the Gauss's law. So, we know this the $\vec{\nabla} \cdot \vec{E}$, where $\vec{E}$ is electric field is $\frac{\rho}{\epsilon_0}$. And if I want to find out the integral form, then it should be $\vec{E} \cdot d\vec{s}$ closed integral should be something like $\frac{Q_{enc}}{\epsilon_0}$, this is the differential form and this is integral form of the Gauss's law. So, we did few calculations also to find out the electric field for certain distribution of the charge.

So, today, we will be going to continue that and then we will try to understand what is electrostatic potential, so, we should do the problem related to uniformly charged sphere. So, when the charge sphere is given and you can try to find out the field there are 2 ways to find out the field, one is inside and another is outside. So, suppose this is a charged sphere with certain coordinate system and try to find out the electric field at any point outside suppose, the radius of this sphere is R and try to find out the electric field at some point outside, the condition is r is greater than R.

So, I simply use the Gauss's law that $\oint \vec{E} \bullet d\vec{s} = \frac{Q_{enc}}{\epsilon_0}$. So, that means, I should consider a bigger surface having the radius r like this so, this is my Gaussian surface and if I consider that Gaussian surface then for close surface integral it should be $\frac{Q_{enc}}{\epsilon_0}$. So, eventually we have E and the total area is simply $4\pi\ r^2$ and that should be equivalent to total charge suppose, the total charge is Q that is enclosed divided by ϵ_0.

So, simply my $\vec{E}$ should be $\frac{Q}{4\pi\,\epsilon_0 r^2}$ if I want to find out in the vectorial note, then that should be the electric field. So, that means, then it seems that the total charge is centered at the point this origin and due to that charge I am getting the electric field at point r, when the r is greater than R that means, it is outside the sphere. Well a same problem can be done for suppose I want to find out what is the field inside.

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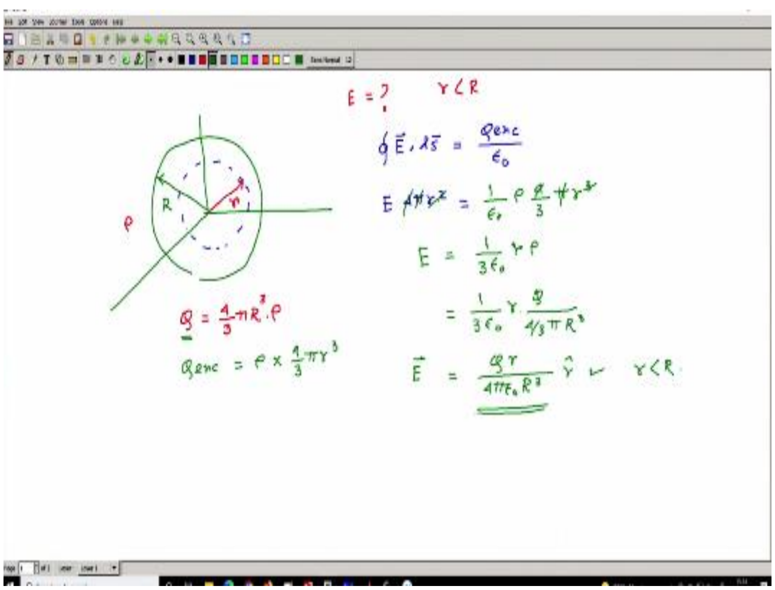

So, this is the sphere we have with certain coordinate system and the point is and this radius is like before R now my point is not outside, but inside this is r. So, the question is what should be the value of $\vec{E}$ at this point? Under the condition r is now less than R. So, we will be going to use the same process like we will make a Gaussian surface here and that surface is essentially a sphere as per the symmetry is given and for that now I have $\vec{E} \bullet d\vec{s}$ is charge enclosed you have to be careful enough because now the total charge enclose is not Q.

That is the basic difference you will be going to have here divided by ϵ_0 so, it should be E $4\pi r^2$ this is the area and then the charge enclose I can calculate but in order to calculate that I need the charge density suppose the total charge Q is uniformly distributed in this sphere. So, total charge Q; if it is uniformly distributed over this sphere then we have this, the relationship between Q and ρ should be this. So, ρ is given. So, in terms of ρ I can have the value here.

So, what should be the charge enclosed then? The charge Q_{enc} should be this volume ρ multiplied by the volume of this, which is this quantity. So, my right-hand side should be $\frac{1}{\epsilon_0}$ and ρ and $\frac{4}{3}\pi\ r^3$ now, I can simplify this it should be E is equal to these 4 and 4 seems to be cancelling out, π and π seems to be cancelling out, one r^2 r^2 is cancelling out having r only. So, eventually we have $\frac{1}{3\epsilon_0}$ and then r multiplied by ρ.

Now, if I want to write the entire electric field in terms of the Q that is total charge, I just simply replace ρ in terms of Q so, then it should be $\frac{1}{3\epsilon_0}$ then r and ρ should be $\frac{Q}{\frac{4}{3}\pi R^3}$. So, it should be $\frac{Qr}{4\pi\epsilon_0}$ and these 3 3 will cancel out and then $4\pi\epsilon_0$ should there and then R^3 so, that is the value of the electric field and if I write in terms of vector then that should be the field when r is less than R.

So, that means in the outside it varies as a function of $\frac{1}{r^2}$, but in the inside it simply linearly varies as a function of r.

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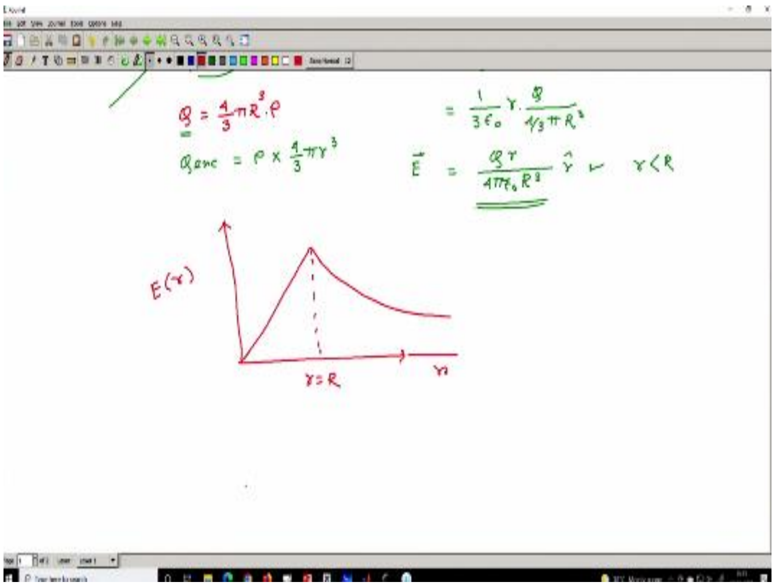

So, if I plot that quickly that how electric field should vary. So, electric field for the sphere as a function of distance r and then you can see that curve is something like it is linearly changing up to the point when r = R and after that is decaying like $\frac{1}{r}$ form and this side we are plotting r. So, this is the way the electric field is going to change this is a very standard problem I believe most of the students have already done it, but if you want to calculate the field due to uniform charged sphere in a rigorous manner, it should be very lengthy calculation.

So, in order to reduce this calculation, one can use this Gauss's law and using the Gauss's law these things will be very simple. Now, we jump to the next very important part where we calculate. So, let me use the name page here.

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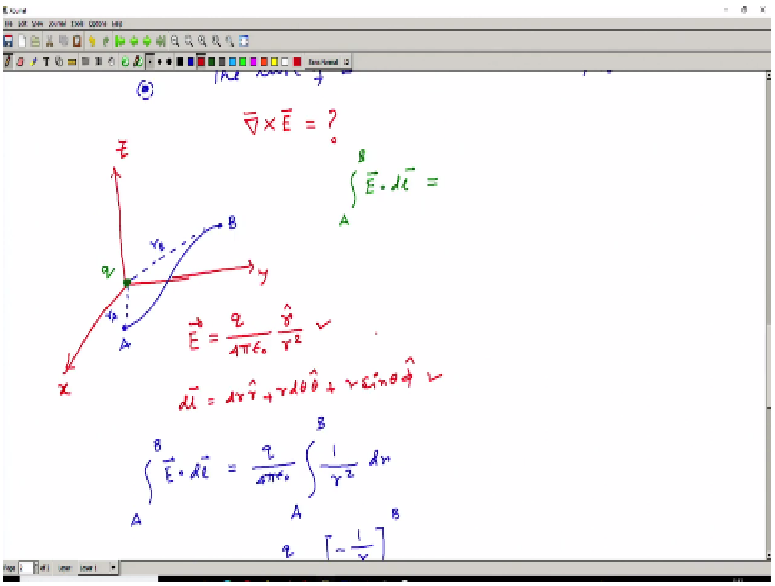

So, here we try to understand first what is the curl of the electric field? So, the $\vec{\nabla} \times \vec{E}$ so, $\vec{\nabla} \bullet \vec{E}$ mind it we already calculated just right here $\vec{\nabla} \bullet \vec{E}$ we figured out that it is simply a $\frac{\rho}{\epsilon_0}$ the question is what is this value? So, $\vec{\nabla} \times \vec{E}$ means, what is this value if I want to find out the curl then what are we supposed to get here the right-hand side. So, in order to understand that, let us consider a standard coordinate system say x y z and a line element here like this going from point A to point B.

So, now, one charge is placed here and origin q and due to the placement of the charge here at the origin it should produce some kind of electric field over the space and I need to calculate. So, my goal here is to calculate the line integral here simply I want to calculate first A to B the vector field $\vec{E}$ is given and that I calculate over the line. So, eventually I calculate the line integral. Well here these because the electric field we know here due to the placement of the charge.

The electric field that we will be going to have in this space is simply the magnitude of the electric field or in the vector if I write in the vector form, it should be $\frac{q}{4\pi\epsilon_0}$ and then $\frac{\hat{r}}{r^2}$. So, you can see that electric field is given as a function of r that means it is in spherical coordinate system, the vector field is given in the spherical coordinate system, so, because of that my line element whatever the line element I draw here should be in the cylindrical coordinate system.

And we know how to write the line element in cylindrical coordinate system it is dr $\hat{r}$ + r dθ $\hat{\theta}$ + r sin θ $\hat{\varphi}$ so, this is the line element I have because the vector field that is given in spherical polar coordinates my line element should be also in that coordinate system. So, now, we are in a position to calculate the integral. So, let us calculate because I know explicitly the value of E and dl. So, my $\vec{E} \bullet d\vec{l}$ should be $\frac{q}{4\pi\epsilon_0}$ that quantity is constant that will come outside.

And then integral point A to B and note it $\vec{E}$ is along $\hat{r}$ and $d\vec{l}$ is $\hat{r}$ associated with $\hat{\theta}$ and $\hat{\varphi}$. So, $\hat{r} \bullet \hat{\theta}$ and $\hat{r} \bullet \hat{\varphi}$ will not be going to contribute. So, the thing that we will have inside the integral is simply $\frac{1}{r^2}$ and dr.

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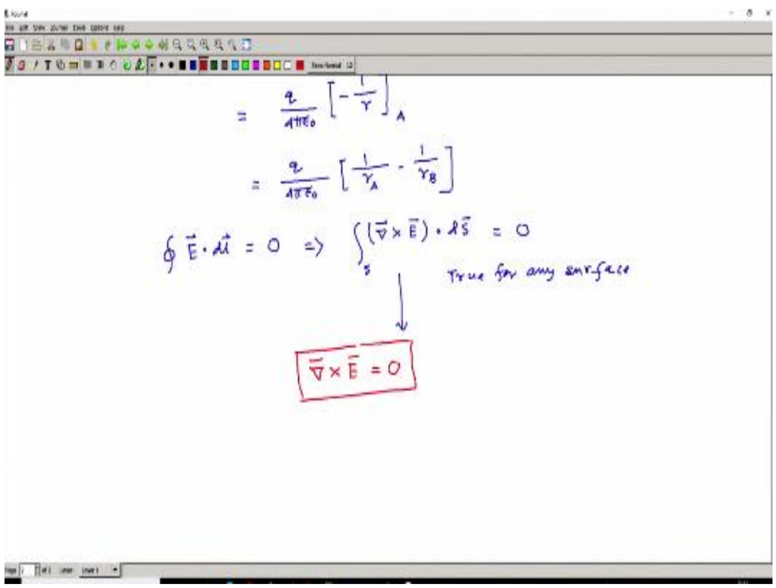

And that thing is simply $\frac{q}{4\pi\epsilon_0}$ $[-\frac{1}{r}]$ point A to point B. So, that quantity if I write so, $\frac{q}{4\pi\epsilon_0}$ then [$\frac{1}{r_A}$ $-\frac{1}{r_A}$] so, that is the value. So, what is r_A? So, r_A is if I join these 2 point in spherical coordinate so, this should be my r_A and it should be my r_B. So, now, you can see that the total integral only depends on the final point and that means, if I want to calculate the total integral close line integral of this quantity suppose, I have a closed loop and I calculate this quantity over this closed loop.

Then r_A and r_B these 2 points should be the same point that means, they coincide so, that means this quantity in the right-hand side should be 0. So, I should have simply 0 here because this is a path independent integral it just depends on the initial and final value. So, considering this thing I can write that if the integral is 0 then using the Stokes law from here I can have another important information and that is this quantity is nothing but $\vec{\nabla} \times \vec{E}$ mind it from the very beginning I want you to calculate the $\vec{\nabla} \times \vec{E}$.

And over the surface integral this has to be 0. So, from this equation this is true for any surface that simply gives me a very important information that $\vec{\nabla} \times \vec{E}$ is 0 so, the elliptic field should not have any kind of curl, this is a fundamental relation that we develop and we will be going to exploit this further to find out the concept of the potential. So, let us short down what we get so far.

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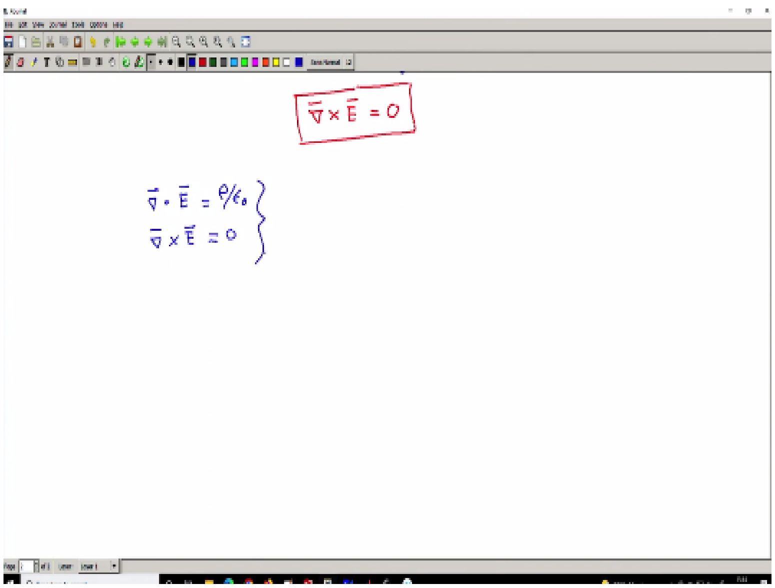

So, we have $\vec{\nabla} \cdot \vec{E}$ is $\frac{\rho}{\epsilon_0}$ and $\vec{\nabla} \times \vec{E} = 0$ these 2 information we have in our hand. So, from that we will be going to develop the concept of electric potential. So, let us write it here.

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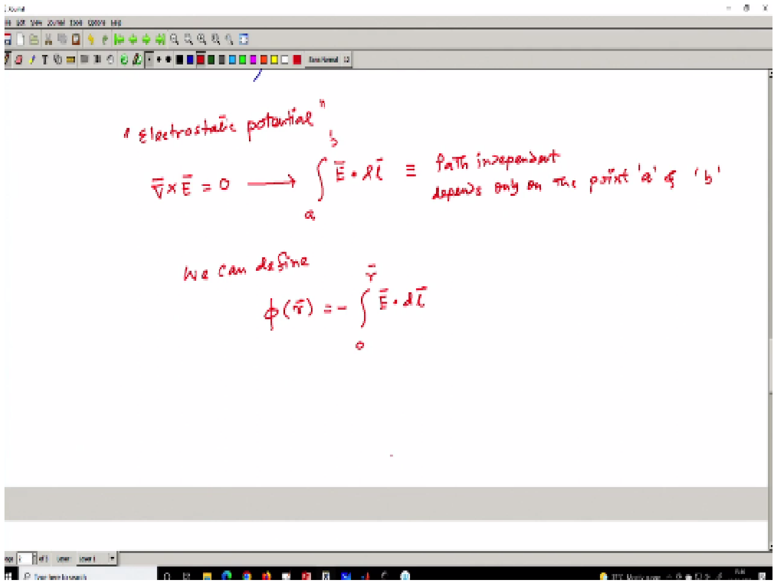

So, next thing is electrostatic potential. So, we have this information $\vec{\nabla} \times \vec{E} = 0$ that gives me because, I got it from the condition that if I calculate from point a to b the line integral $\vec{E} \cdot d\vec{l}$ then the result what I get is path independent, depends only on the point a and b it only depends on the initial and final point a and b only the end points. So, if this is the case, then I can define a scalar quantity we can define a scalar quantity, which should be a function of r as this. I just define this that is a scalar quantity. And these things mean, since it is a path independent.

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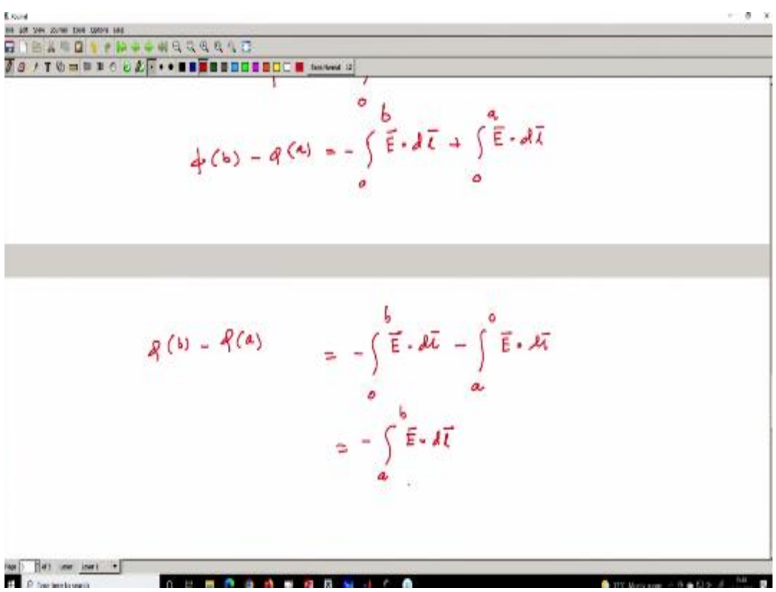

So, I can have $\phi(b)$ is - $\phi(a)$ if I calculate by defining this I can simply have $-\int_0^b \vec{E} \cdot d\vec{l}$ and then plus a to b because we have only minus sign here sorry 0 to a, 0 to a then $\vec{E} \cdot d\vec{l}$ this quantity we have so, then I can simply write it as $\phi(b)$ is - $\phi(a)$ as minus of 0 to b $\vec{E} \bullet d\vec{l}$, - a to 0 $\vec{E} \bullet d\vec{l}$ and that is essentially my minus of a to b $\vec{E} \bullet d\vec{l}$. So, that means, after making this integration I can have $\phi(b)$ is - $\phi(a)$.

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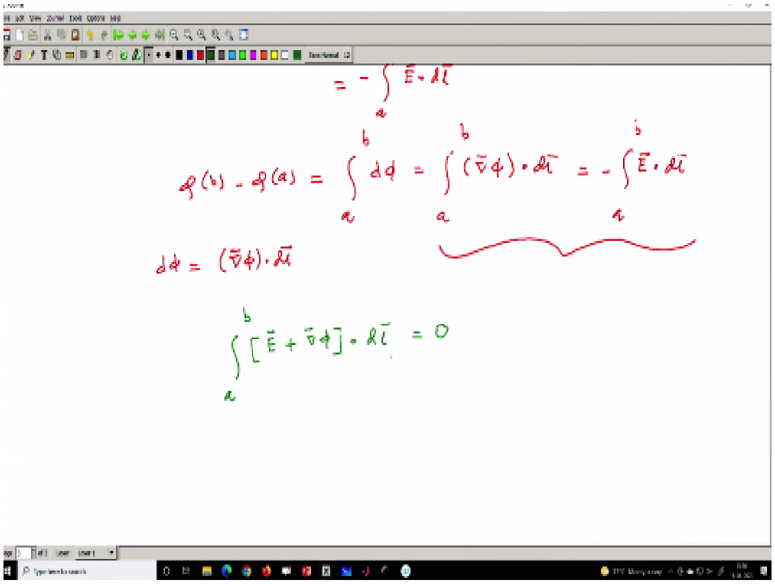

So, I can write $\phi(b)$ is - $\phi(a)$ because I am getting this as a result of the integration I can write it at simply a to b and then dϕ and we already know that dϕ I can write in terms of the gradient operator and it should be simply this we did it in our earlier classes. So, that I replace here and it should be a to b and then $\vec{\nabla}\phi \bullet d\vec{l}$ I just write in terms of dϕ and that quantity is nothing but minus of a to b and then $\vec{E} \bullet d\vec{l}$.

So, from this equation from here to here, I can simply write a to b then $\vec{E}$ plus this quantity is identically 0 and this is true for any line segment.

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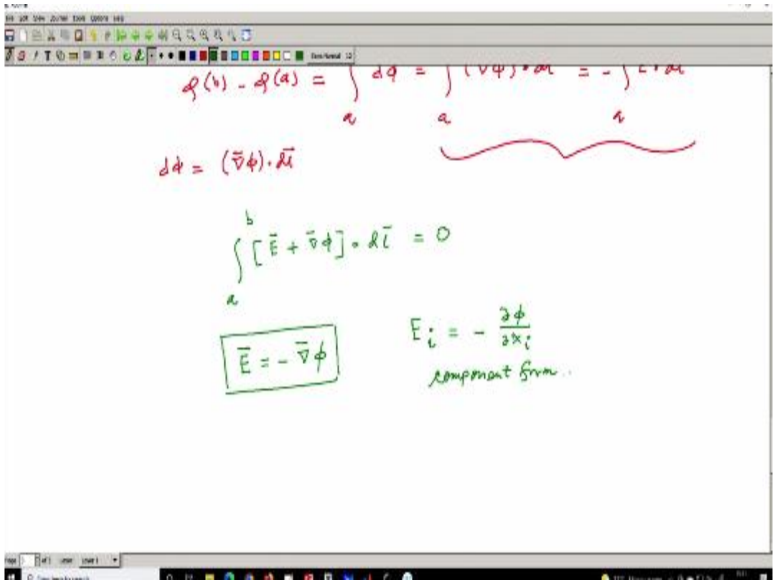

So, that means, I can define $\vec{E}$ in terms of a quantity ϕ, which we call the potential, this is called the electrostatic potential. So, I can define $\vec{E}$ electric field in terms of a scalar field ϕ whose gradient and negative of that gives me the total electric field and that is the electrostatic potential. Here if I want to find out i^{th} component it should be simply the scalar field divided by x_i this is the component form.

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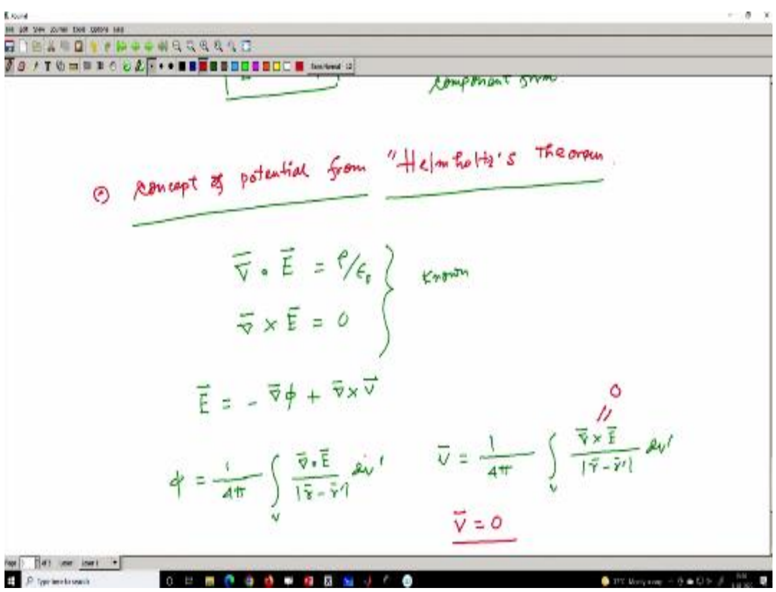

Now, we will do another thing quickly and that is concept of potential from Helmholtz's theorem because if you remember the Helmholtz's theorem so, the Helmholtz's theorem was suggesting that if I know the divergence and curl of the vector field the question was what should be the $\vec{E}$ in terms of these known parameters, can I expand can I write $\vec{E}$ in terms of these things uniquely? And the answer was yes, we can do that. So, in general $\vec{E}$ can be written as this form according to the Helmholtz's theorem.

That I can write I can decompose my total electric field into 2 parts one is the gradient of a scalar field and another is the curl of a vector field say $\vec{V}$ here and the scalar field and vector field is defined by the curl and these things. So, what should be my ϕ here? My ϕ according to these things $\frac{1}{4\pi}$ integration whatever the curl I am having whatever the divergence I am having, so, that divergence values so, $\frac{\vec{\nabla} \bullet \vec{E}}{|\vec{r}-\vec{r}'|}$ and dv' that should be my ϕ.

And what is my $\vec{V}$ according to the Helmholtz's theorem, my $\vec{V}$ should be $\frac{1}{4\pi}$ and integration of whatever the curl I am having at r point and then $\vec{r}$ - $\vec{r}$' and then dv' with this. Now you can see this quantity is 0. So, the vector potential for the electric field is 0, but the scalar potential is not equal to 0.

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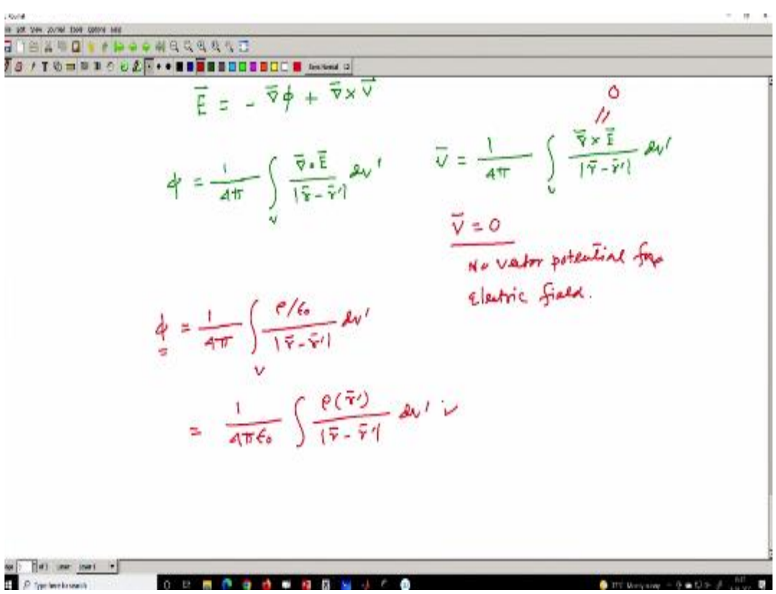

What is the scalar potential ϕ? ϕ according to the Helmholtz's theorem is simply $\frac{\rho}{\epsilon_0}$ because this is known. And It should be ($\vec{r}$ - $\vec{r}$') dv' mind it, this ρ should be calculated at the point $\vec{r}$', I can write in a more compact way, this is $\frac{1}{\epsilon_0}$ integration ρ calculated at $\vec{r}$ ' and ($\vec{r}$ - $\vec{r}$ ') dv'. Note it all these calculation or whatever the derivation we did it from on the basis of Helmholtz's theorem.

Because in Helmholtz's theorem, this thing was already mentioned that if you have the information of the divergence and curl of a given vector field $\vec{E}$ then the vector field $\vec{E}$ in general decomposed by these 2 that it should be minus of gradient of the scalar field and the curl of a vector field where the scalar field and vector field can be defined in this way where these inside the integral I have the information of this known thing that the divergence and curl.

For electric field we find that the $\vec{\nabla} \times \vec{E}$ since it is 0 so, there should be no vector potential. So, no vector potential for electric field but the scalar potential is there. And the scalar potential is simply this and that is precisely today's task we wanted to define the vector potential and the scalar potential for the electric field. And since $\vec{\nabla} \times \vec{E} = 0$ that gives me that there is no vector potential associated with the electric field but we have the scalar potential for the vector field, which is called the electrostatic potential.

For magnetic field we will be going to see that the vector potential is non zero but there should not be any scalar potential so with that note I like to conclude today's class. In the next class may be we will be going to discuss more about this potential thing and if possible few problems. So, thank you very much for your attention and see you in the next class.