Foundation of Classical Electrodynamics
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Lecture – 25
More on Gauss's Law

Hello students to the foundation of classical electrodynamics course, under module 2, we will be today going to learn more about the Gauss's law.

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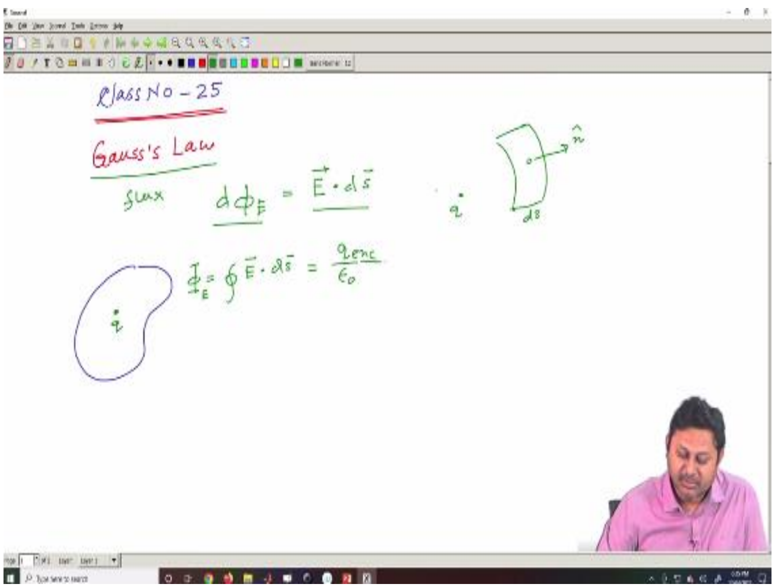

So, today we have class number 25. In the last class, we started the Gauss's law and in Gauss's law we mentioned that if we have the flux dϕ for a given charge having electric field $\vec{E}$ is $\vec{E} \cdot d\vec{s}$. I have a charge here and I have a surface here and say this charge is q and this is the direction say $\hat{n}$ of this and this is $d\vec{s}$ surface area then the amount of flux if somebody calculate is this one.

Now, if I have an enclosed surface and this point charge q is inside that and if I calculate the total flux is say ϕ that is the total surface integral of that quantity $\vec{E} \cdot d\vec{s}$ and that quantity we figure out is simply $\frac{q}{\epsilon_0}$ exploiting the Coulomb's law and since it is enclosing the volume is this total surface is a surface that is enclosing this given volume. So, q is inside this volume, so, I should write it as $q_{enclose}$. So, the natural extension of these Gauss's law for a scattered charge.

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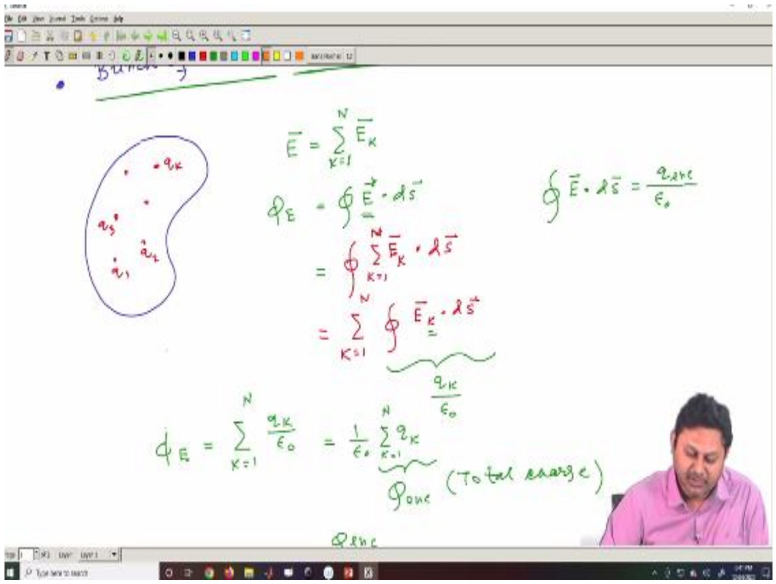

So, now, we will do the same thing for a bunch of charges that are scattered around so, we have bunch of charges scattered around. So, instead of having one charge now, say I have a region where all the charge are scattered. q_1 q_2 q_3 q_k these are the scattered charges and for the scattered charges we know if I want to find out the total electric field that should be the summation of the electric field for each individual charges at a given point r. So, k goes to say 1 to N if there are N number of charges.

So, this is my total electric field by superposition principle we can say that. Now if I want to find out what should be the total flux ϕ_E? That is integration of $\vec{E} \bullet d\vec{s}$ and this is a closed surface integral because I am enclosing this volume with this surface and now, this E is this one, because I am taking the value for all the charge point. So, it should be $\sum_{k=1}^{N} \vec{E}_k \bullet d\vec{s}$. So, simply we have for one charge so, I can have this like summation of k, 1 to N that I put outside and then it is integration of $\vec{E}_k \bullet d\vec{s}$.

So, this quantity is for each and every point charge, which is here and for that we know that value should be $\frac{q_k}{\epsilon_0}$ according to the Gauss's law. So, eventually I have my total flux ϕ_E to be summation of k = 1 to N and this quantity we know because this is the Gauss's law that total close surface integral $d\vec{s}$, flux = $\frac{q}{\epsilon_0}$ and you should write $q_{enclose}$ if this is single charge and that is enclosed by this surface area.

So, that quantity I just replaced by $\frac{q_k}{\epsilon_0}$. Now, this is nothing but $\frac{1}{\epsilon_0}$ and $\sum_{k=1}^{N} q_k$. Now, this quantity is my total charge that is enclosing I write Q this is the total charge that we are having here.

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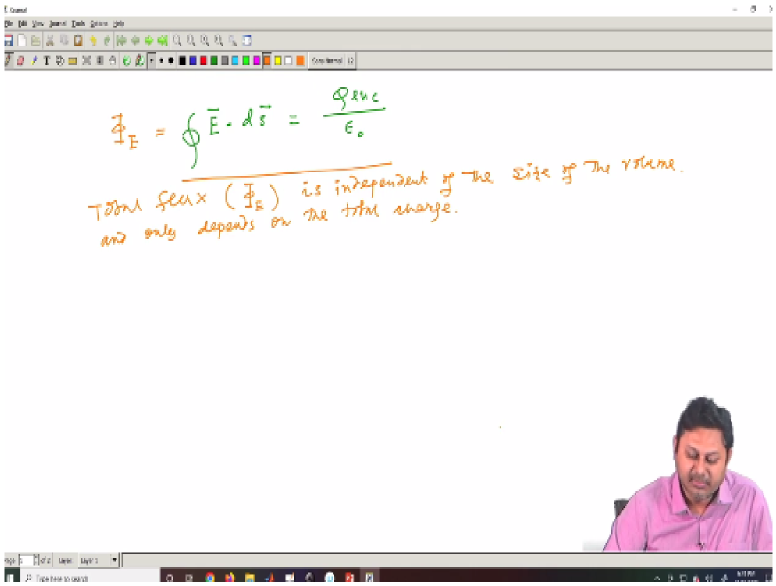

So, that means, I simply have that the integration of the total field $\vec{E} \cdot d\vec{s}$ for the scattered charge is simply $\frac{Q_{enclose}}{\epsilon_0}$ that is the expression we are having this is the extension of the Gauss's law for scattered charges. Now, we will go so, I can write that the total flux from here we can see that. The total flux that is equal to ϕ_E the total flux ϕ_E is independent of the size of the volume and only depends on the total charge.

So, I can make different volume by just enclosing all these charge by drawing different kind of closed surface but that does not matter whatever the volume is there for all cases the total amount of flux is same and it only determined by these term $Q_{enclose}$. What is the amount of charge is enclosing that matters.

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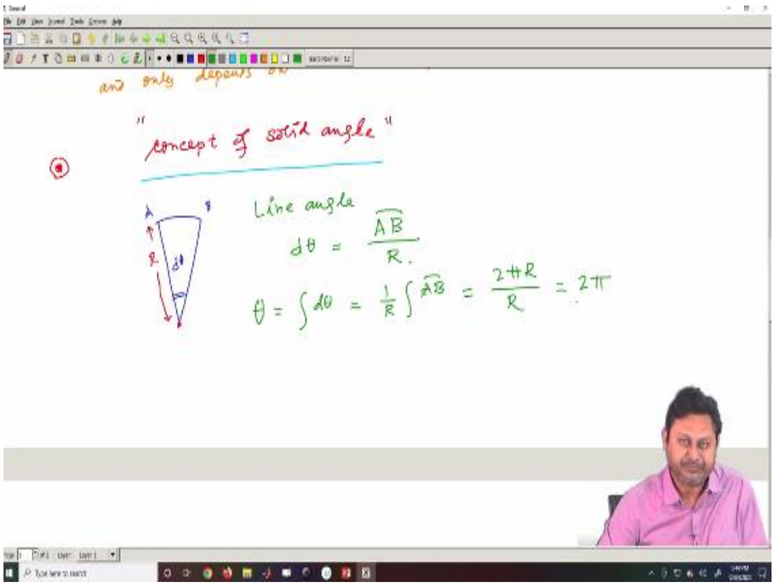

The next thing we are going to discuss more about the Gauss's law and we will see how these things happen when the charge point is inside or outside the closed surface. So, before doing that, let us try to understand the concept of solid angle. The concept of solid angle is this suppose I have say line angle this is called a line section AB and this angle is say dθ and this is R from here to here from the point. So, the line angle is dθ, which is AB this line segment divided by R, let me write this A properly.

This is $\frac{AB}{R}$ and if I want to find out the total line angle it should be the integration of dθ and that eventually give us $\frac{1}{R}$ integration of whatever the line segment I am having total integration of this quantity and we know that this quantity is 2πR because that is the periphery divided by R and we have the total line angle like 2π that we know.

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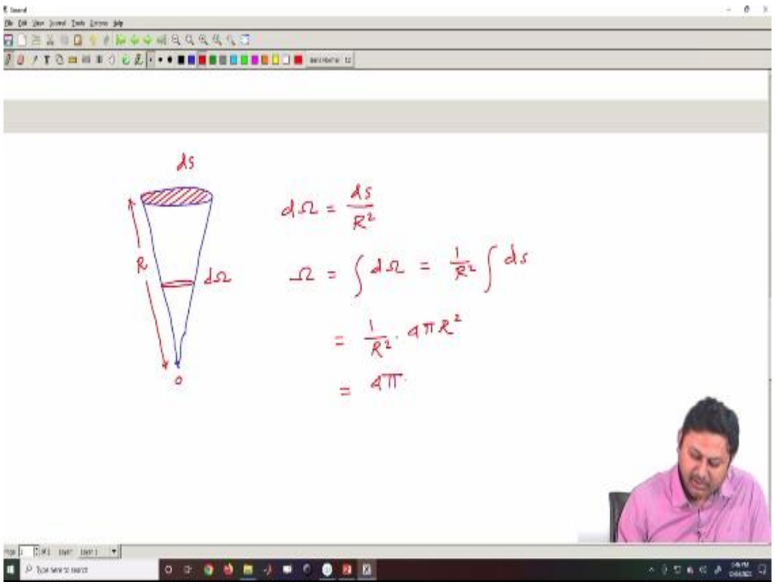

In the similar way, if I do the same problem for a surface suppose I have a surface here and then I have suppose this is the area and this is ds, from here to here the length is radius R and this is say O and we have an angle here we call the solid angle, which is $d\Omega$ say now, how you define this $d\Omega$? This $d\Omega$ is equal to this area $\frac{ds}{R^2}$. Now, if I want to find out the total Ω total solid angle then it should be integration of $d\Omega$ and it should be $\frac{1}{R^2}$ and then we have the integration of this ds.

And ds is now the total area total surface whatever the if I have a closed surface then it should be a total surface then it should be simply $\frac{1}{R^2}$ then $4\pi R^2$, which give us simply the value 4π. So, the solid angle seems to be 4π for the surface, which is included which is forming a close region.

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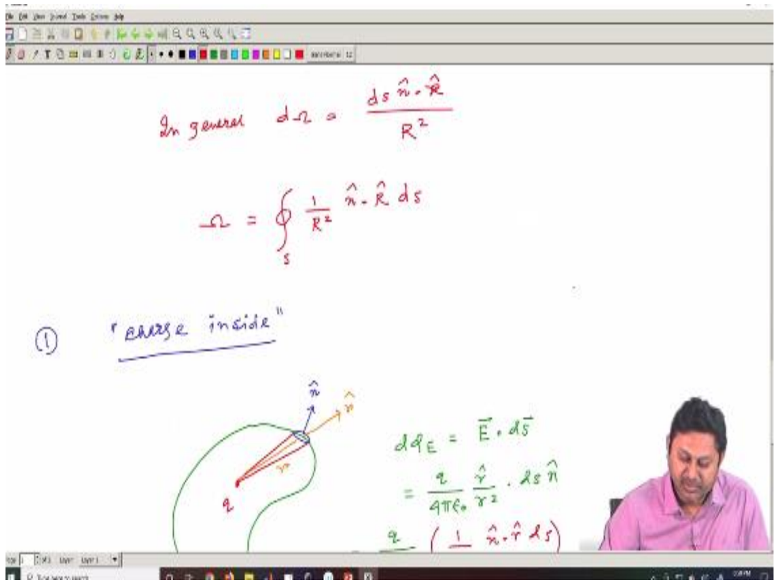

So, in general dΩ is simply ds surface and then the unit vector of the surface dot unit vector of the R. So, let me put this $\frac{\hat{R}}{R^2}$ and if I want to find out the total solid angle Ω then I need to integrate it and it should be $\frac{1}{R^2}$ and then $\hat{n} \bullet \hat{R}$ ds and that should be if it is a closed region then I should have a closed surface integral here. So, that is the definition we know now, let us try to understand the Gauss's law in this way.

Suppose, we have a charge, case 1, when the charge is inside the region, inside the volume, inside the closed region so, this is a closed region suppose I draw and suppose the charge is sitting here some point q. So, if I want to find out what is the solid angle it will make from this point so, I should have an area here and the r from this from say here to here this is along unit vector it is r from here to here the value is r and the unit vector of the surface is along this direction. This is the direction of the surface, which is a vector quantity and I write n.

So, what is the flux for this small section? So, the flux is $d\phi_E$ and if I calculate it should be $\vec{E} \bullet d\vec{s}$ where $\vec{E}$ is the electric field exerted by this charge q. Now, from the Coulomb's law, I can know what is the value of electric field here? If the distance is r and that value is simply $\frac{q}{4\pi\epsilon_0}$ and then $\frac{\hat{r}}{r^2}$ that is this quantity and then dot ds with the $\hat{n}$ if it is having a unit vector this side.

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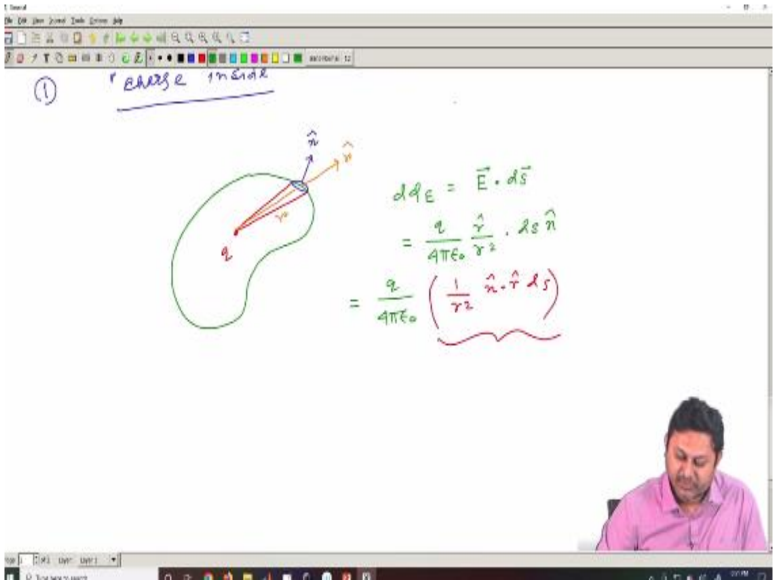

Now, that quantity I rewrite in a different way like $\frac{q}{4\pi\epsilon_0}$ and the rest of the term I write in this way in the bracket and that is $\frac{1}{r^2}$ and then $\hat{n} \bullet \hat{r}$ and ds. Note that this quantity I just defined few minutes ago that this quantity is this dΩ.

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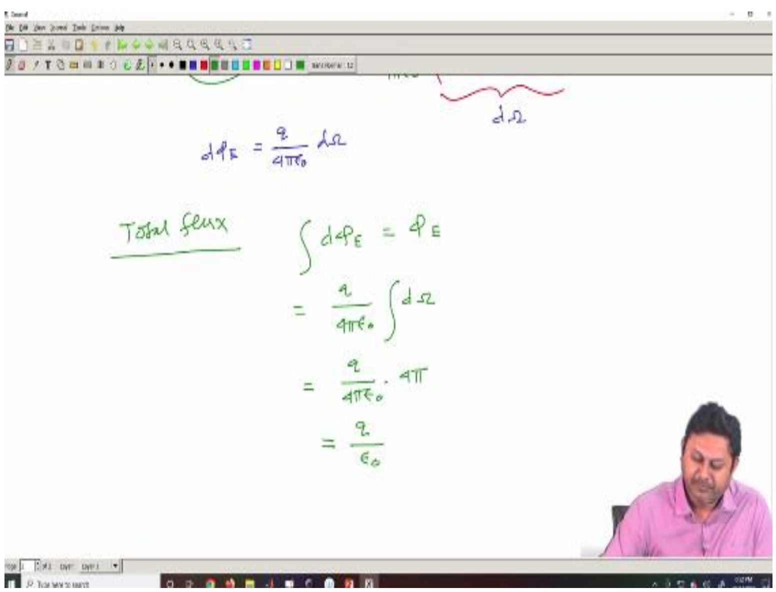

So, this quantity is simply the solid angle that we know and that is dΩ. So, flux $d\phi_E$ is simply $\frac{q}{4\pi\epsilon_0}$ and dΩ that we have. Now, if I want to find out the total flux is integration of $d\phi_E$, which is nothing but ϕ_E at this quantity if I integrate then it should be $\frac{q}{4\pi\epsilon_0}$ and integration of dΩ and that dΩ integration is nothing but the total solid angle. So, I have $\frac{q}{4\pi\epsilon_0}$ multiplied by 4π. So, 4 this 4π will cancel out and we have $\frac{q}{\epsilon_0}$ that is the total flux.

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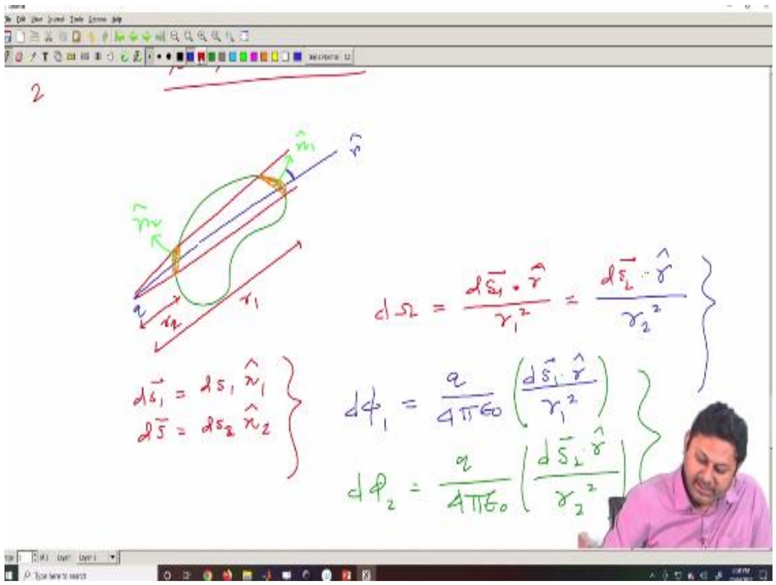

Now, the next thing is what happened if the charge is outside. So, case 2 when charge is outside what happened. If the charge is outside the picture that we have like before I have a closed volume region and suppose the charge is placed somewhere here, so, this is my charge q and if I draw a line here so, this is the r vector I am drawing this is my $\hat{r}$ and then what is the solid angle it is exerting for the surface that is this one and this one so, 2 solid angles are here one is the area that we are having this area and another is this area.

So, what is the unit vectors of this area I quickly draw that one unit vector is along this direction this is $\hat{n}_1$ and another unit vector because it is opposite directions. So, in this direction this is n_2 this is the unit vector we have this as $\hat{n}_2$ now from here to here I have this length from here to here this is r_1 and from here to here this length is say so, suppose the big one is say r_1 and small one say r_2. So, what is my $d\vec{s}_1$? This is the vector $d\vec{s}_1 = ds_1\,\hat{n}_1$ and $d\vec{s}_2 = ds_2\,\hat{n}_2$ this we know.

Now, what is $d\Omega$ here? So, $d\Omega$ is same because it is exactly the same thing. So, I can write it, it is $\frac{d\vec{s}_1 \cdot \hat{r}}{{r_1}^2}$ that is again equal to $\frac{d\vec{s}_2 \cdot \hat{r}}{{r_2}^2}$ that quantity. So, $d\phi_1$ is the flux due to this quantity and that is this charge $\frac{q}{4\pi\epsilon_0}$ and then I have the area here $\frac{d\vec{s}_1 \bullet \hat{r}}{{r_1}^2}$ and dϕ_2 the flux due to this ϕ_2 s_2 area that is due to this area is $\frac{q}{4\pi\epsilon_0}$ this quantity will remain unchanged.

Then we have this ds so, this is ds_1 so, this is ds_2 again $\hat{r}$ dot vector sign divided by r_2 square. So, now, you can see 2 interesting thing that this quantity and this quantity they are same I mean both are dΩ, but, the angle here is if I calculate $d\vec{s} \bullet \hat{r}$ and $d\vec{s}_1 \bullet \hat{r}$ then we need to take the angle here, but if I want to take the other ds_2 $\hat{r}$, r is in this direction, so, then my angle should be this one. So, that is simply suggest that this dϕ_1.

So, whatever the dϕ_1 I calculate and whatever the dϕ_2 I calculate this quantity and this is same quantity, but this quantity if I execute then these are opposite because there will be a negative sign associated with this both are dΩ. So, one case is dΩ is positive and another case it is negative.

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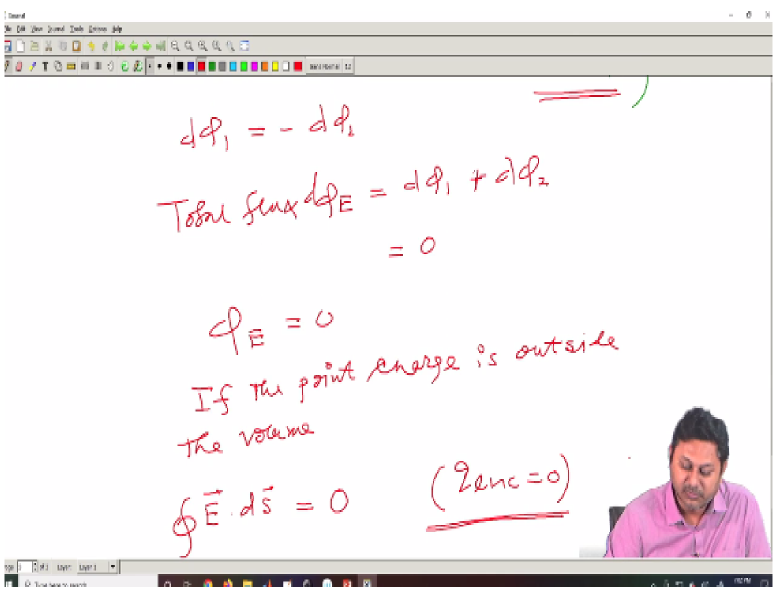

So, eventually we have dϕ_1 = - dϕ_2 because one is negative to other here this portion. So, total flux if you calculate under this condition, which is dϕ_E due to this small segment then it should be simply dϕ_1 + dϕ_2 and this quantity is 0. So, that means, if I calculate the total flux ϕ_E it will be simply 0 if the point charge is outside the volume so, that means, whatever the flux we have here in the same amount of flux is going out that is the physical meaning.

So, since the amount of flux that is going in and the amount of flux that is going out they are same for the case when the charge is placing outside of this volume element then that basically makes the total flux calculation 0. So, we do not have any electric field if we are having I mean the total flux is equal to 0 if the point charge is outside. So, that is also consistent with the Gauss's law because in Gauss's law you calculate if you calculate the close integral $\vec{E} \cdot d\vec{s}$ then we say that it is the total integral is the $\frac{q_{enclose}}{\epsilon_0}$.

But in this case, you can see that q is sitting outside this is my q, which is sitting outside. So, since it is not q enclose we can say $q_{enclose} = 0$. So, that means if I calculate this quantity close integration of $\vec{E} \cdot d\vec{s}$ for this charge then I will find 0 because $q_{enclose} = 0$ here, which is consistent with the Gauss's law, and that we also mentioned here we proved that with this figure that it is also 0. So, with that note, I would like to conclude today's class because I do not have much time today.

So, in the next class, we will try to understand the differential form of the Gauss's law and the corresponding calculation. So, thank you very much for your attention and see you in the next class.