Foundation of Classical Electrodynamics
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Lecture - 24
Charge Distribution Problem, Gauss's Law

So, welcome students to the foundation of classical electrodynamics course under module 2, we today are going to understand the few problems regarding the different kind of charges distribution and then we will introduce the Gauss's law.

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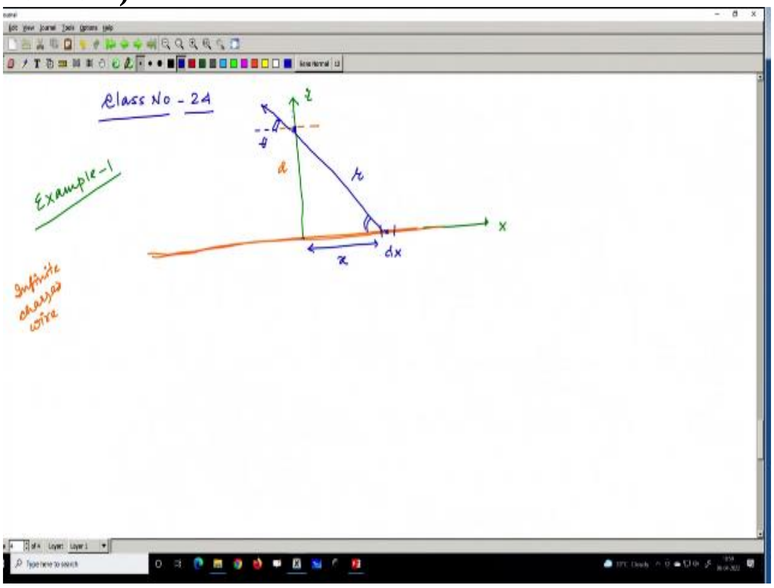

So, we have class number 24. So, let us directly start because last day we discuss about the charge distribution how electric field can be calculated from different charge distributions. So, let us take an example 1 how to execute this in a problem. So, a very standard example and the problem is if I have an infinite wire and then this is an infinite charged wire. So, for infinite charged wire, suppose I put my origin here and along this direction is my z direction and suppose it is placed over x direction.

So, this is my x coordinate and this is the way it is distributed over x axis and it is extended infinitely. The problem is what should be at any point say over here at any point d over z axis what should be the electric field because of the presence of this wire, which is infinitely extended and charged particles. I mean it is a charged wire. So, in order to execute this problem first let us because this is infinite wire so, the contribution of each and every point should come here all that all the points.

And then there should be vector summation and we will see that due to the symmetry these components will cancel out only along the z component will have the nonzero values. So, let us take a small segment here we call it dx and from here to here this is x distance say and now for dx segment let us try to find out what should be the field for this dx segment at this point. So, suppose this distance is л according to our notation, we always put this л when we want to find the value.

When we define the distance from this source point to this field point. And let us consider that this is making an angle with this point this is making an angle say θ. So, the electric field of dx section will be along this direction if you divide this electric field into 2 parts along this and this by symmetry you can see that and this side also we have a dx here and this portion will cancel out. So, whatever we have is only z value the z direction. So, this is roughly the geometry if this is θ, then this has to be θ as well and this length is d from here to here, this is O origin, this field point is d.

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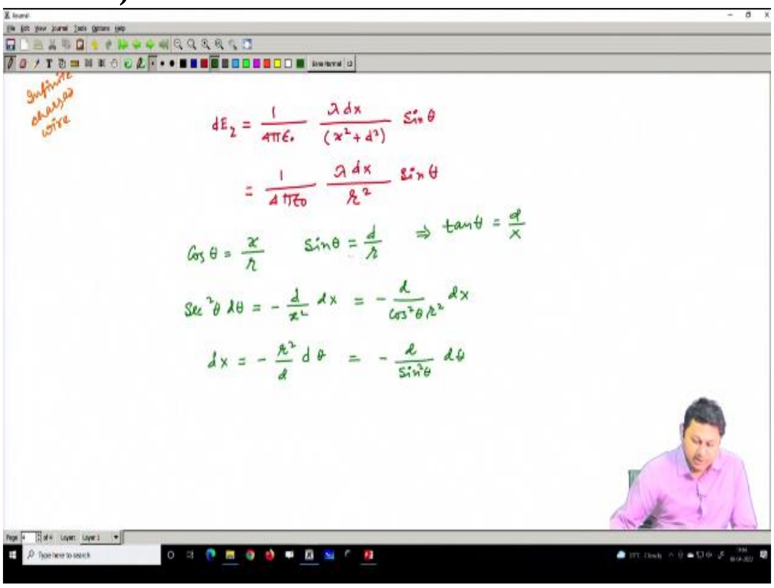

So, straight way we calculate the z component for this dE z component. So, the z component if I calculate, it should be $\frac{1}{4\pi\epsilon_0}$ and then the charge distribution over line is λ and for dx it is dx. So, λ dx is the amount of charge we are having here which is dq, instead of writing dq I straight way write λ dx. Now what is the distance? Distance is л directly I write л in terms of the given x plus say d^2 that should be the value of $л^2$ this л.

So, this л is equivalent to $(x^2 + d^2)^{1/2}$. So and then the cos component no not the cos component because this angle is θ. So, then the sin component different way you can do this problem in different way. So, now this thing is $\frac{1}{4\pi\epsilon_0}$ λ dx, let us put in terms of л, because it seems it should be easier if I execute in terms of θ by taking the range now, the cos θ. So now, the next thing is to find out θ in terms of л that is all.

So, cos θ it is simply $\frac{x}{л}$ according to this geometric because this is θ if I want to find a cos θ it should be $\frac{x}{л}$ and sin θ is $\frac{d}{л}$ that makes tan θ = $\frac{d}{x}$ these are the information we are having right now. Now, if I make a derivative of this quantity, so, I know that $\sec^2$ θ dθ = $-\frac{d}{x^2}$ and then we have dx, which is $-\frac{d}{x}$ I know in terms of so this is $-\frac{d}{cos^2\theta л^2}$ dx.

So, dx in terms of θ if I execute, it should be simply - $л^2$ cos square, this become $\frac{1}{d}$ and dθ, which is in terms of sin θ, if I just eliminate this л is $-\frac{d}{sin^2\theta}$ dθ. So I just write dx in terms of sin θ and I am going to put it an $л^2$ I can write in terms of sin θ as well. So, everything is now in place.

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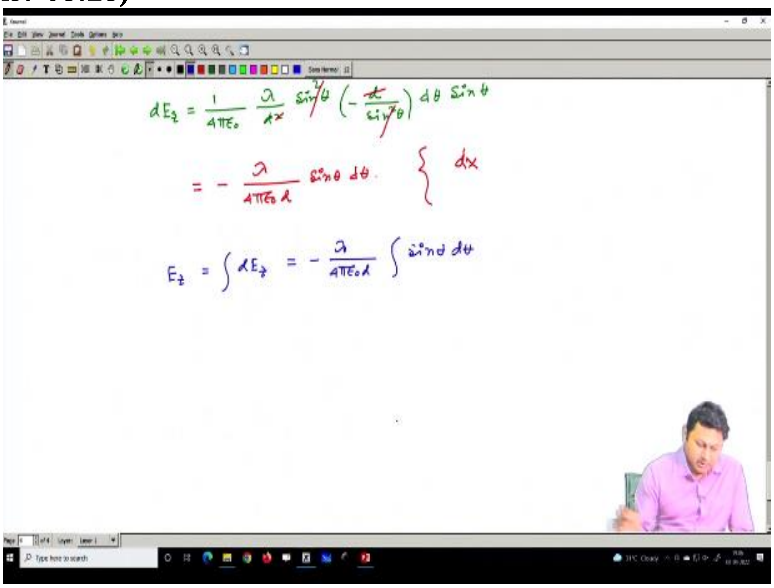

So, I simply write dE_z is simply $\frac{1}{4\pi\epsilon_0}\frac{\lambda}{d^2}$ because I am writing this $л^2$ in terms of sin and d. So, $л^2$ should be $\frac{d^2}{sin^2\theta}$ so I should have one sin θ here sin θ that is the component d to z and let us put the $\sin^2$ θ and later I put sin θ and dx is $-\frac{d}{sin^2\theta}$ and then dθ and the z component I will have a sin θ already I am talking about this sin θ, which is already there.

So, these sin θ and $\sin^2\theta$ will $\sin^2\theta$ $\sin^2\theta$ will cancel out. So, that basically leads me to $-\frac{\lambda}{4\pi\epsilon_0}$, one d seems to be cancelling out here as well. So, you should have a d here sitting and then simply have sin θ dθ. So, this is for the segment dx. Now, if I want to find out the entire field then I need to integrate it and this integration, so, for entire field E_z should be integration of dE_z and that value is simply the integration of that quantity $\frac{\lambda}{4\pi\epsilon_0 d}$.

And the integration of sin θ dθ now, you have to be careful enough to put the limit here, because now, I am taking all the infinite length so, there should be limit of the θ and if I go from if I now move this dx from here to the infinite distance here to some infinite distance from this point to some infinite distance then the θ will vary from $\frac{\pi}{2}$ to 0. So, I should $\frac{\pi}{2}$ to 0 this is a variation of the θ we should have when I extend all the length, not only that this is for one direction this is for in this direction from here to here.

If I go from this side also then by symmetry I can just simply multiply it to that is for other side by symmetry I can do that.

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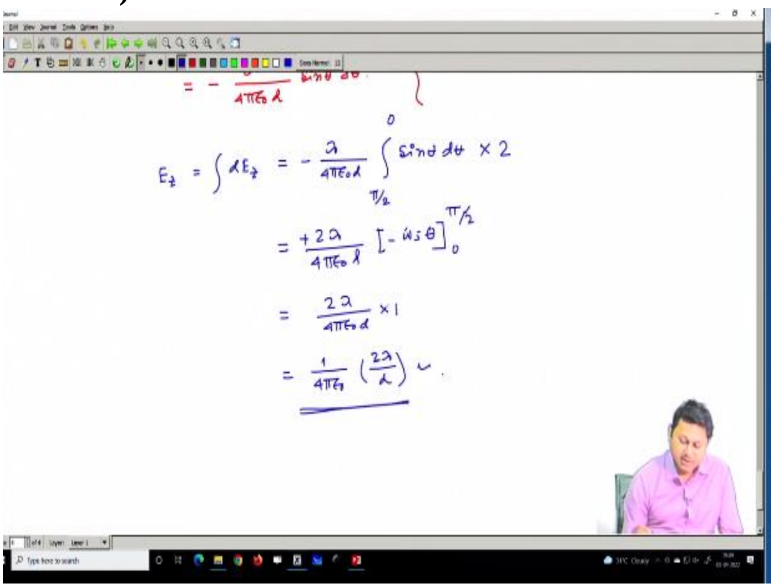

So, now if I execute this stuff, I should simply have $\frac{2\lambda}{4\pi\epsilon_0 d}$ it should be the minus sign is there so, you should find it I should have a cos θ with a negative sign after integration and if I absorb this minus sign by the way I can go from 0 to $\frac{\pi}{2}$ and that eventually gives me because cos $\frac{\pi}{2}$ is 0 then it should be like $\frac{2\lambda}{4\pi\epsilon_0 d}$ that is all multiplied by 1.

So, this quantity is simply $\frac{1}{4\pi\epsilon_0}$ into 2λ this is a constant value we are getting when you are getting a constant value when you are having a fixed d. Now, if we increase the value of the d that is if you go more and more over the z axis there should be a decay and it only depends on the charge density λ. So, this is the value of the electric field for infinite distribution of the charge over line. Now, with the same note I can go forward for another problem I will say example.

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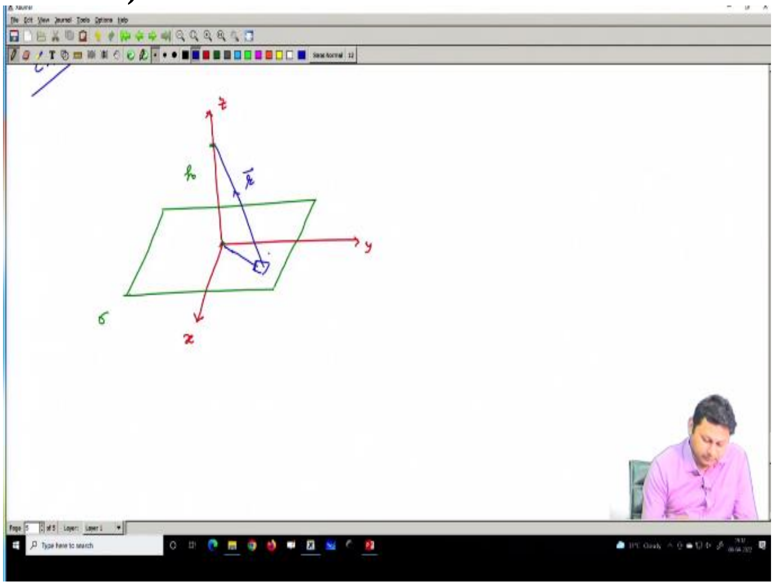

So, in this example, what we take is the surface charge for infinite surface so, surface infinite we should have an infinite surface charge density σ so, what is the problem so, I am having so, let us first draw my coordinate system here and I need to draw in a different way because this is surface. So, maybe it could not be easier this is x say this is y and this is z suppose the surface is in this plane so, the surface is in xy plane and it is charged the surface is charged with a surface charge density σ. The question is over a height h from here to here over a height h.

If I want to find out the electric field what should be the value of that electric field, very standard problem later we will be going to solve this with much ease using exploiting the Gauss's law that maybe today I am going to introduce, but let us first do that in a standard way, standard way means in just doing everything rigorously exploiting the Coulomb's law. So, let us have a point here small section like this small surface like this.

And from here to the surface this point say $\vec{\pi}$ and from here to here on this coordinate this length is say ρ so, this is a ρ vector and this is say φ. So, I am using a cylindrical kind of system because I can change my φ and ρ together to find out the value here because this is a surface.

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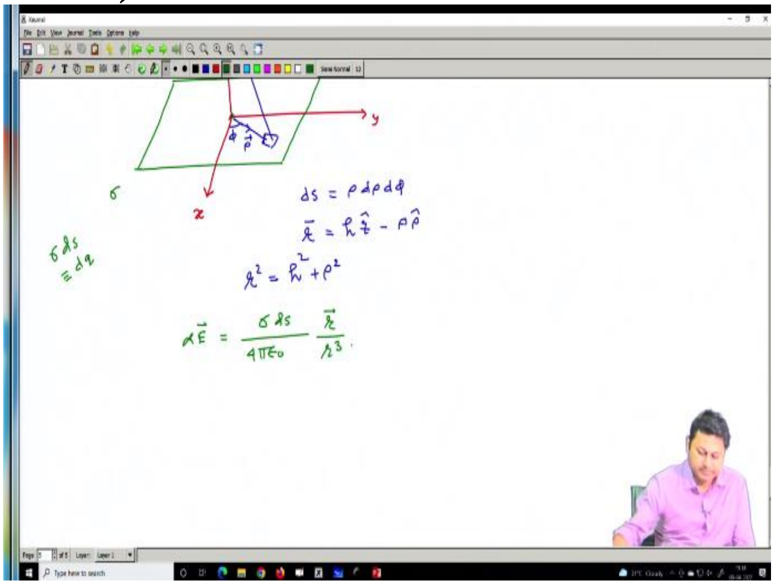

So, the surface element now, we know the surface element ds the surface element, which is ρ dρ and then dφ this is the surface element I can have. What is $\vec{\mathscr{r}}$ here by the way $\vec{\mathscr{r}}$? $\vec{\mathscr{r}}$ is h because this is h. So, h with $\hat{z}$ - ρ with $\hat{\rho}$ because this is ρ this is from here this is h the length so, I can find out $\vec{\mathscr{r}}$ in this way. So, what is $\mathscr{r}^2$ then? $\mathscr{r}^2$ is simply $h^2 + \rho^2$ just make a dot product and you will be going to get the result.

Now, what is $d\vec{E}$ here? $d\vec{E}$ that is σ ds due to ds if I want to find divided by usual $4\pi\epsilon_0$ that is there this is dq σ ds is my dq and for dq I am getting this and then $\frac{\vec{\mathscr{r}}}{\mathscr{r}^3}$ that is my $d\vec{E}$.

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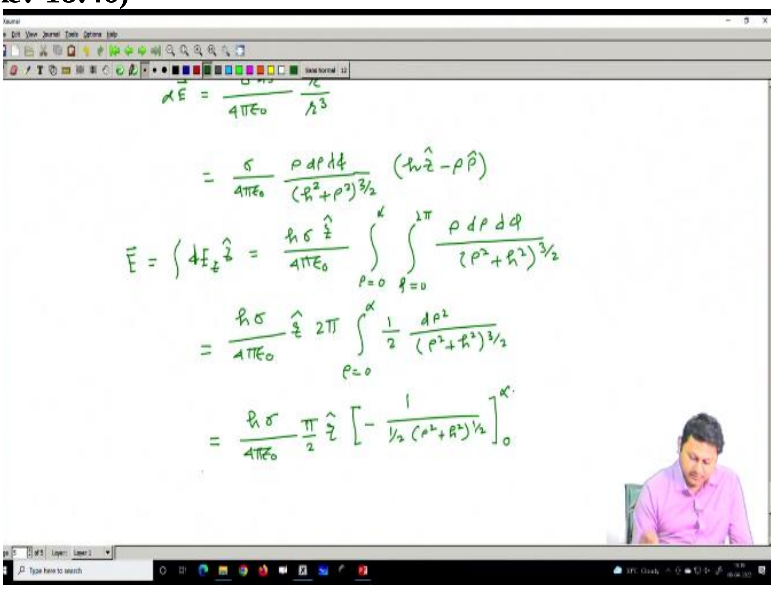

Now, if I put everything so, it is $\frac{\sigma}{4\pi\epsilon_0}$ and then ρ surface element if I write ρ dρ dφ looks good that $\frac{\rho\, d\rho\, d\varphi}{\mathscr{r}^3}$ in terms of h and ρ it is $(h^2 + \rho^2)^{3/2}$ that we have and $\hat{\mathscr{r}}$ is $h\hat{z} - \rho\hat{\rho}$. So, now if I try to find out the total electric field E it should be integration of dE again it should be the z component.

Because by symmetry you can see that all the components that we have should vanish all the component that is parallel to this xy plane should vanish and only we have this z component that sustain. So, it is $\hat{z}$ along this direction I want to find out then let us write this value. So, this ρ component I can eliminate only thing that I have in my hand is h and then σ then there should be an $\hat{z}$ direction and then I have $4\pi\epsilon_0$.

Then I integrate it over ρ because now ρ and φ are going to change so, by integration ρ goes to 0 to infinity and φ goes to 0 to 2π I am having this problem 0 to 2π. So, then I have ρ then dρ then dφ whole divided by this quantity say $(\rho^2 + h^2)^{3/2}$. Now, I have $\frac{h\sigma}{4\pi\epsilon_0}$ and then $\hat{z}$ and then φ integration I can do because this is not a function of φ. So, you have 2π outside and the rest of the integration is only over ρ.

ρ should be 0 to infinity with this value, so, here we are having ρ dρ so I can have one $\frac{1}{2}$ here to write it is as $\frac{d\rho^2}{(\rho^2+h^2)^{3/2}}$ just manipulate it here by just writing d ρ^2. So, now, my integration is easier, I can simply have $\frac{h\sigma}{4\pi\epsilon_0}$ and then this $\frac{1}{2}$ I should have $\frac{\pi}{2}$ the unit direction should be z I should keep it and then I should simply integrate $\frac{3}{2}$ if I now d ρ so, this is I can just write this as d $\rho^2 + h^2 = x$.

Then I simply have the integration like minus of this is $\frac{3}{2}$ so, I can have like $\frac{1}{\frac{1}{2}(\rho^2+h^2)^{1/2}}$ this is $\frac{3}{2}$. So, -3 by 2 + 1 divided by - 3 by 2 + 1 so that is why $-\frac{1}{2}$ here and this quantity and now, the limit is 0 to infinity.

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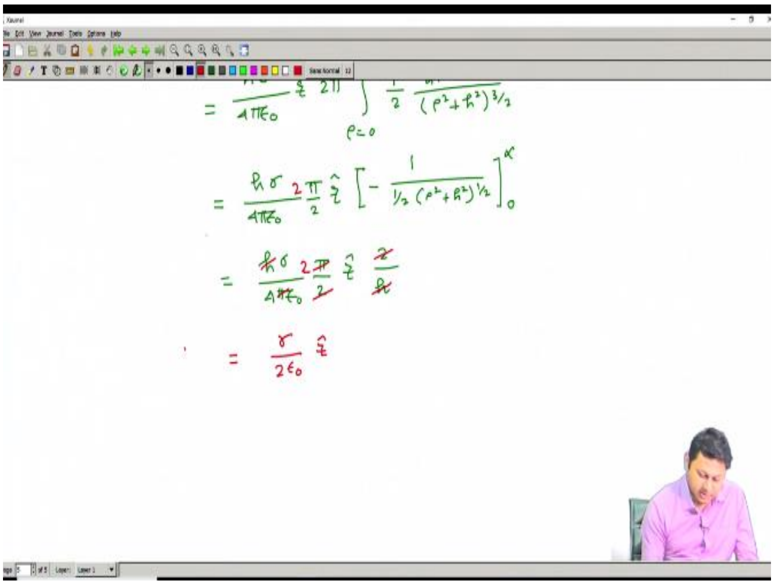

And with this limit we simply have $\frac{h\sigma}{4\pi\epsilon_0}\frac{\pi}{2}$ is here $\hat{z}$ is here when I put infinity, so, this quantity will no longer be there and when we put 0 then minus sign will go to absorb and we simply have 2 here and then divided by h. So, these 2 2 will cancel out, this π will cancel out, h seems to be cancel out which is interesting, because it does not depend on h any more. So, that is a very significant so, simply we have σ so, these $\frac{\pi}{2}$ is here so, I am having a 2 here 2 so, this 2 will cancel out, so σ so I should have 2π.

So, previously I should have 2π here and then I have a $\frac{1}{2}$ outside so, this $\frac{1}{2}$ I take, but this 2 I forgot to cancel. So, there is another 2 sitting here so, that I forgot to cancel. So, please note it, it was 2π h σ 4π ϵ_0 $\hat{z}$ 2π and this $\frac{1}{2}$ is coming out and the rest of the thing I invented $\frac{1}{2}$. So, this $\frac{1}{2}$ is coming up then I should have another 2 sitting here. So, we should have simply σ divided by these 2 and 4 should be 2.

We should have ϵ and that is all $\hat{z}$ this is a very important result the electric field that for infinite extended surface. The interesting thing is that it is not depending on h. So, I started the calculation that what should be the field at each point and finally, I find that for infinitely extended sheet of charge, a charge sheet the electric field is a constant it is $\frac{\sigma}{2\epsilon_0}\hat{z}$. So, after that maybe we can naturally it is natural that we can extend the problem for.

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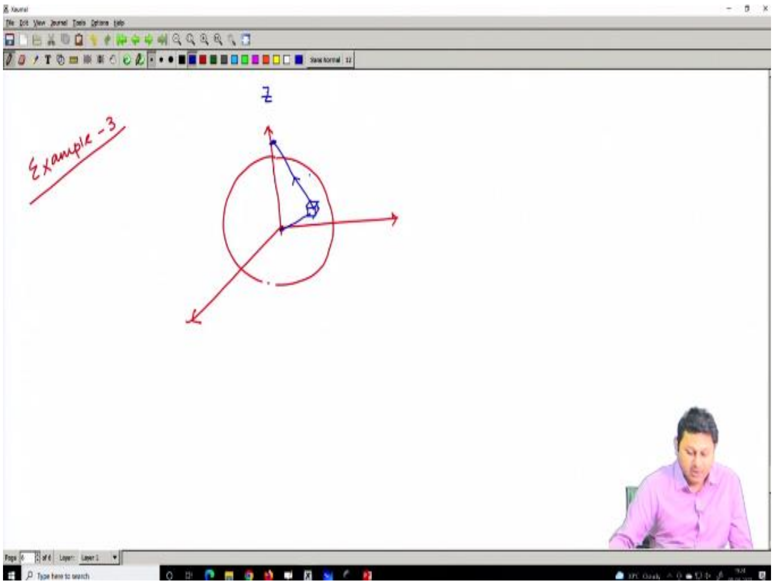

So, example 3 it is natural that we extend the problem for a spherical surface spherical not spherical surface, a sphere charged sphere and try to find out over say some z point. So, this is my z at some point what should be the electric field due to a volume charge element sitting inside this sphere. So, this is a natural extension of all this problem I started with the you know line charge density then I have the volume charge density and now, I can have an extension of this.

But if you want to do this problem in a rigorous way, by using whatever the rule we are having whatever the law we are having which is Coulomb's law it will be very extensive problem you can do that I mean in principle you can do that by taking the proper integration and you will get the result, but you can do this problem with much easier way if you know the Gauss's law. So, that is the thing today I like to introduce I am not going to do this problem, because it will be very lengthy problem.

If I want to calculate in a standard way the way we did for other 2 problems. On the other hand, if I use the Gauss's law for this problem, it will be very simple. So, let us now introduce the Gauss's law today.

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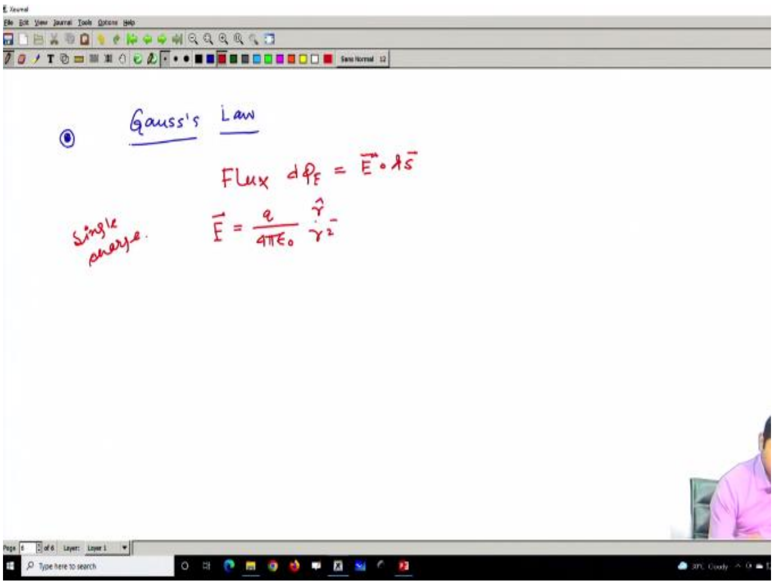

So, the next thing the problem I am not going to do here as I mentioned, because this is a very lengthy problem and more sophisticated way and more simple way to do this problem by using this Gauss's law. So, I will just start Gauss's law maybe we will continue this in the next class as well. So, Gauss's law is saying, before Gauss's law let us try to understand what is flux? Already we exposed with this term flux. So, the flux is $d\Phi_E$ and that is $\vec{E} \cdot d\vec{s}$, so, that is flux.

So, for a single charge, $\vec{E}$ electric field we know this electric field for a single charge is simply say sitting in the origin it is $\frac{q}{4\pi\epsilon_0}$ and then $\frac{\hat{r}}{r^2}$.

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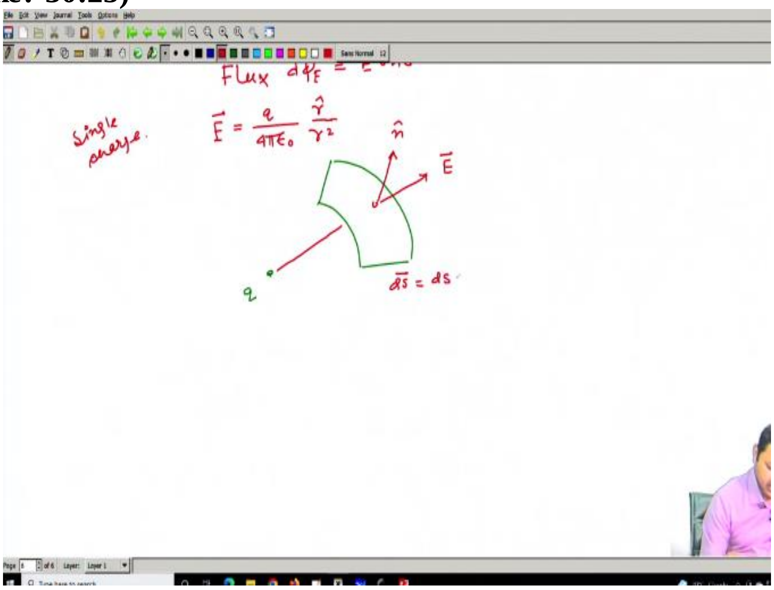

Now, if I want to find out the flux here for this, so, I should have a surface like this. So, this is the surface and I am having a point here sitting q. So, the electric field is going here and should cut the surface like this and this is my direction on the electric field. But also the surface should have a direction here and the direction is say $\hat{n}$ and ds surface whatever the surface we are taking here the surface element is simply ds $\hat{n}$.

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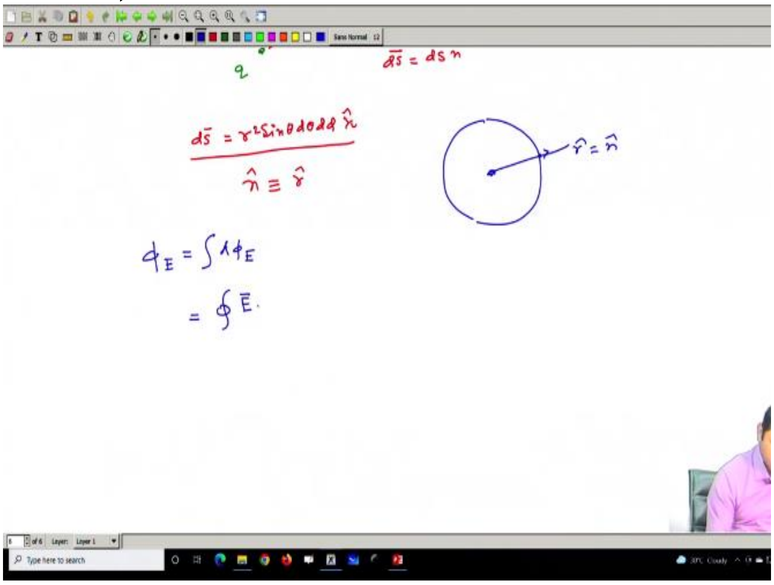

So, for a spherical surface, if I have a spherical surface my ds should be r^2 then sin θ then dθ dφ with $\hat{n}$ this is the surface element first sphere now $\hat{n}$ should be equivalent to $\hat{r}$, since I am taking a sphere where the charge is sitting at the center point here. So, whatever the $\hat{r}$ we have that should be equivalent to the direction of the surface.

So, the total flux if I want to call calculate for the single charge sitting inside the sphere it should be integration of $d\Phi_E$ and that quantity is nothing but the $\oint \vec{E} \bullet d\vec{s}$.

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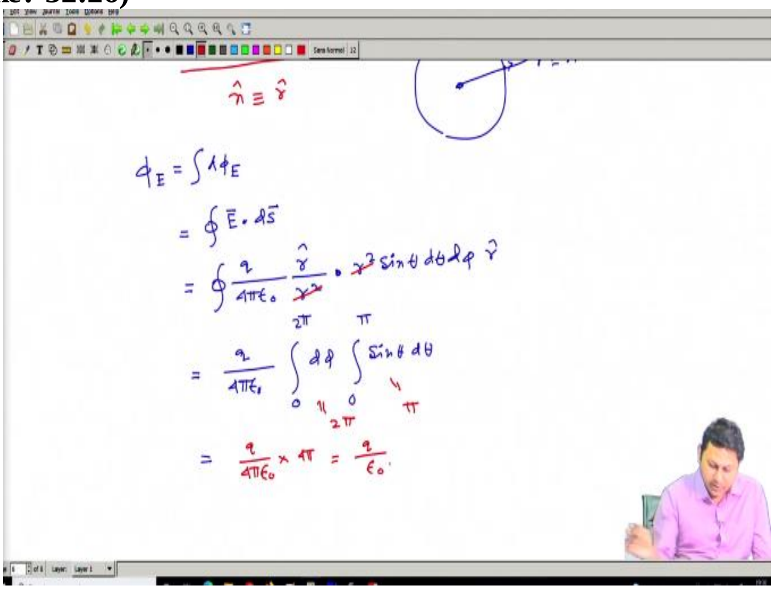

Now, $\vec{E} \bullet d\vec{s}$ I can calculate and by putting the value because for a single charge I know all the all the values, so, this is $\frac{q}{4\pi\,\epsilon_0}$, I just write my value of $\frac{E}{r^2}$ and then the surface element is r^2 sin θ dθ dφ that is all then $\hat{r}$ this is a dot product here. So, this quantity simply $\frac{q}{4\pi\epsilon_0}$ I can take it outside the integral and then dφ will be integrated from 0 to 2π and r^2 here you can see this r^2 will be going to cancel out there will be no function of r.

And I should have sin θ whose limit for close circle is 0 to π and then dθ. So, this quantity gives us 2π and this quantity gives us π. So, if I put this you should have $\frac{q}{4\pi\epsilon_0} \times 4\pi$, which is $\frac{q}{\epsilon_0}$.

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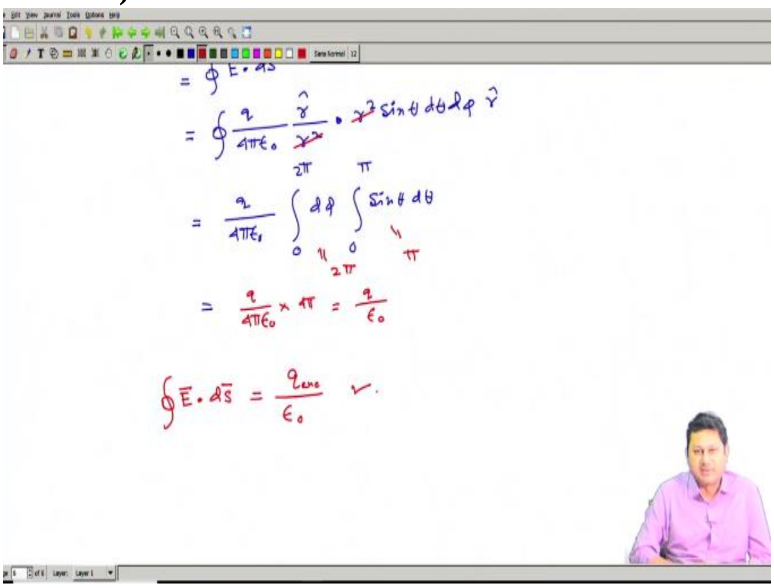

So, that means if I want to find out this $\oint \vec{E} \bullet d\vec{s}$ then that value is $\frac{q}{\epsilon_0}$ but this q mind it is inside this surface in this volume element the surface is enclosing so inside this surface you are having q so I should write here something called $q_{enclosed}$ the surface is enclosing the charge q. So, that is one of the statement of Gauss's law and we will discuss more about, today I do not have that much of time. So next class we will discuss more about the Gauss's law and try to find out the more interesting result. With that note I like to conclude here thank you for your attention and see you in the next class.