Foundation of Classical Electrodynamics
Prof. Samudra Roy
Department of Physics
Indian Institute of Technology – Kharagpur

Lecture - 23
Coulomb's Law (Contd.), Charge Distribution

Hello student to the foundation of classical electrodynamics course, so under module 2, we are going to discuss today the Coulomb's law that we started last class and also try to understand about the different kinds of charge distribution.

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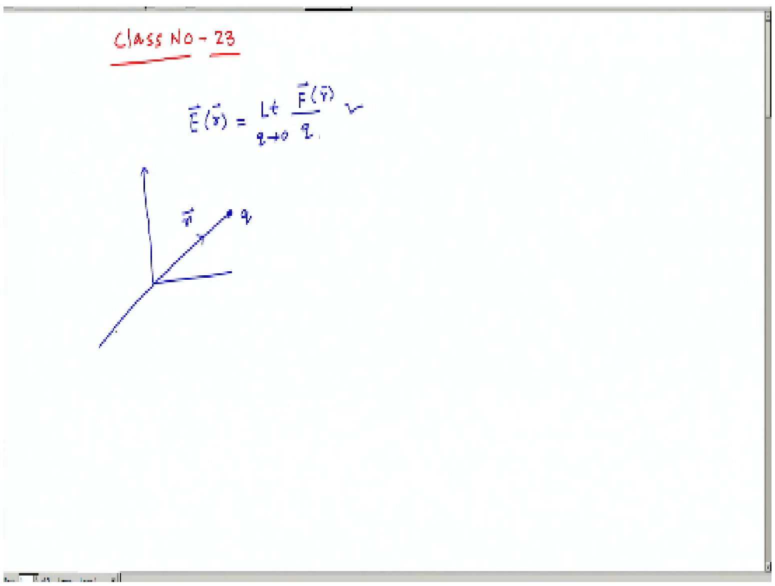

Today we have class number 23. So, let me remind that what was the electric field last week we defined electric field at some point $\vec{r}$. So, the electric field is the force per unit charge as shown in here that would exert on a small test charge q, if it were at a distance some distance $\vec{r}$ with the limit q tends to 0. So, I should find the force experienced by a test charge q by some unit charge say sitting in the origin and with the limit that this tends to 0. So, if there are 2 charges, we also find that the field at the point P due to some charge q

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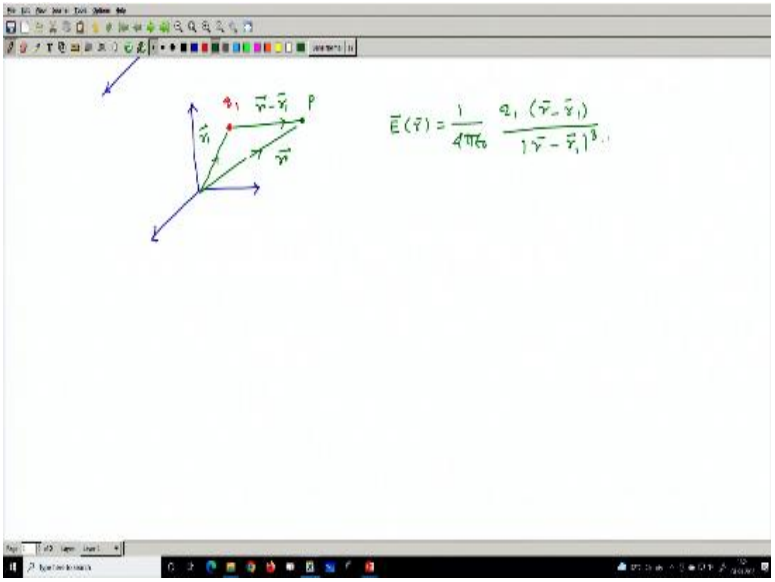

In general, so, this is my coordinate point and suppose I am having a charge q_1 here and the coordinate of this q_1 is say $\vec{r}_1$, if I want to find out the field at some point here at point P, this is $\vec{r}$ and this distance we normally write ($\vec{r}$ - $\vec{r}$ ') and that is $\frac{1}{4\pi\epsilon_0}$and then $\frac{q_1(\vec{r}-\vec{r}_1)}{|\vec{r}-\vec{r}_1|^3}$ this is for. And now, if there are many charges, there are multiple charges.

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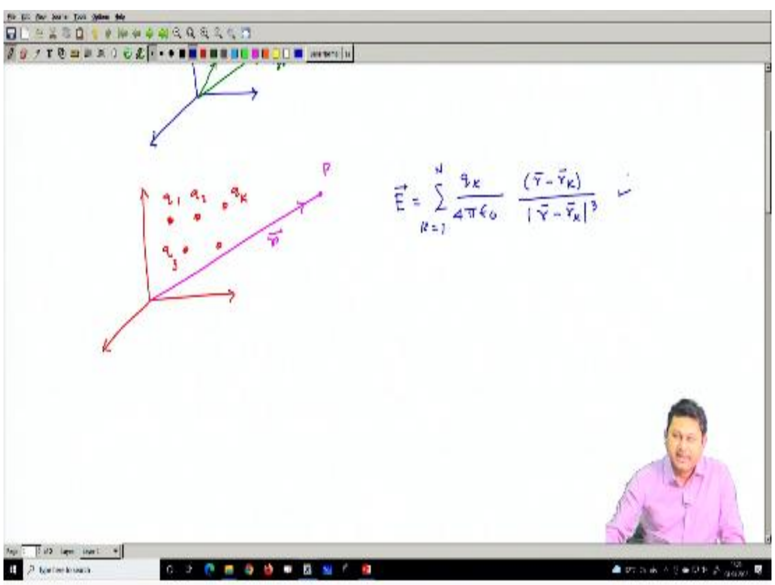

So, this is my coordinate system now, in some coordinate system there are many charges are there distributed q_1, q_2, q_3 so on q_k so, the electric field at some point $\vec{r}$ due to all these charges some point, this is some point $\vec{r}$ and here this is the point where I try to find out the electric field due to the presence of all these charge points. Then I should write it like this. This is the superposition of electric field of all the individual charges, it should be summation of $\frac{q_k}{4\pi\epsilon_0}$, k can move from 1 to n.

If there are n number of charges are there and let us put it as N, because normally we put the total number as N and then it is $\vec{r}$ - $\vec{r}_k$, where $\vec{r}_k$ is the position of the k^{th} charge point and then cube of that. So, we should have a vector addition here. So, that thing we did in the last class. So, now, we will go on with that we just put one simple try to find out one simple example very simple example.

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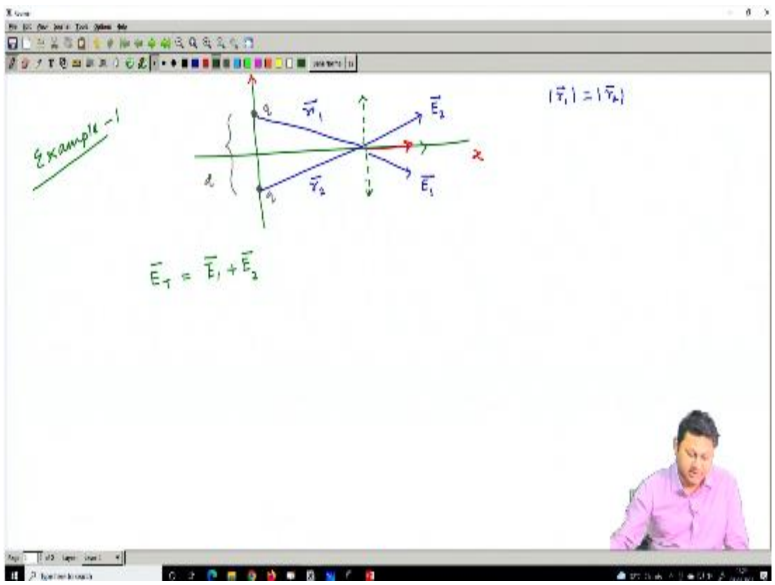

How to execute the example 1 how to execute the field for different charge distribution in this case the charge distribution is simple, suppose, I have a coordinate system here along this direction it is x, along this direction it is z and 2 charge points are sitting here and here equal distance so, the distance between 2 charges is say d. So, this is q and this is q both are q. Now, the question is what should be the electric field over any point here in x axis.

So obviously, it should be simply so, the electric field due to this q it should have a direction like this, this is say $\vec{E}_1$ and the electric field is this charge should go along this direction. So, this is $\vec{E}_2$ and whatever we have is a resultant, it should be a resultant because I am doing the vector summation it is expected that by symmetry it should be in this direction. So, this length in general I can write it as $\vec{r}_1$, this is $\vec{r}_2$ under the condition that $\vec{r}_1$ and $\vec{r}_2$, the magnitude is same, because of this symmetry.

So, the total electric field we should have is simply the vector sum of $\vec{E}_1 + \vec{E}_2$ is a vector sum. So, you can see that these components whatever the component we will have, if I just make a vector decomposition along this direction and this direction, so, this component will go to cancel out so, $\vec{E}_x$ component that is important here that will matter.

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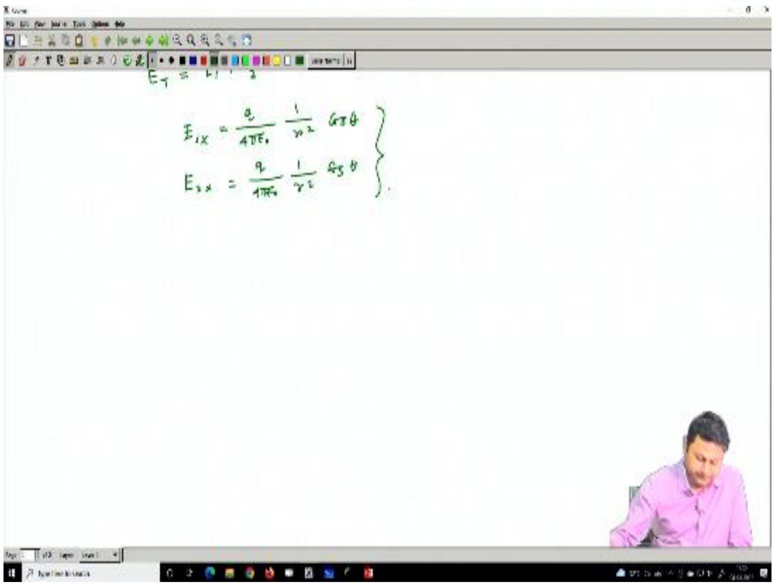

So, $\vec{E}_{1x}$ If I want to find it is simply charge whatever the charge I have here $\frac{q}{4\pi\epsilon_0}$ and then the distance and this distance say this distance I just simply write it is $\vec{r}$. The distance r^2 of that and then the component and this component is a cos component if I consider this angle to be θ. So, it is simply the cos component. So, very straightforward problem we are doing cos θ. In a similar way, $\vec{E}_{2x}$ should be simply $\frac{q}{4\pi\epsilon_0}\frac{1}{r^2}$ cos θ same value.

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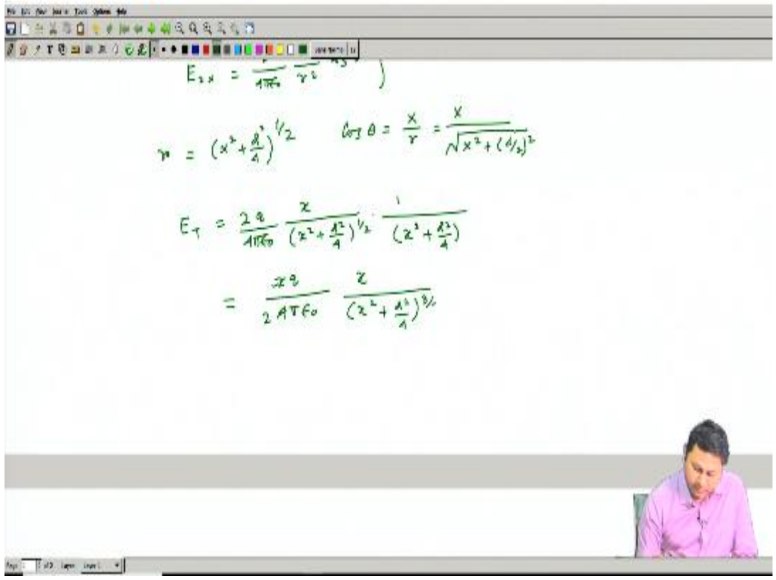

Now, if I want to find out what is r because this is at x distance because I want to find out at this point. So, suppose this is a distance, which is x. So, r is simply $(x^2 + \frac{d^2}{4})^{1/2}$ and cos θ is $\frac{x}{r}$ that means, it is simply $\frac{x}{\sqrt{x^2+(\frac{d}{2})^2}}$. So, now, we have all the value in terms of x and d, which is given and now, we just simply find out the value.

And it is E_{total} (E_T) should be $\frac{2q}{4\pi\epsilon_0}$ and then $\frac{x}{\sqrt{x^2+(\frac{d}{2})^2}}$ is there for cos θ and another is r^2. So, that means, I should have another value $(x^2 + \frac{d^2}{4})$ and that is the value of r^2. So, overall this value is $\frac{2q}{4\pi\epsilon_0}$ if you want you can cut these 2 2, you have 2π here $\frac{x}{(x^2+\frac{d^2}{4})^{3/2}}$. So, this is a very straightforward and simple problem I just do as a warm up.

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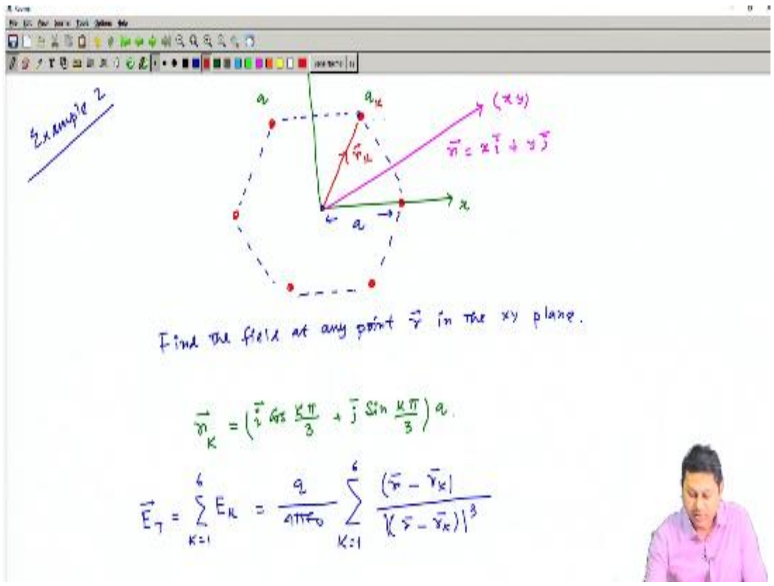

Now, a similar kind of problem let me give you one example like example 2. So, you can expect different kind of charge distribution and based on the symmetry you need to find out some the electric field at some point. So, suppose I am having a coordinate system like this, this is x and say this is y and the 6 charge are placed in this way. So, I have one charge here let me put in other colours, one charge here, one charge here and then maybe one charge sitting here, one charge sitting here, one charge sitting here and other charge sitting here.

So, all these distances are equal. So, there is a symmetry you can see that so, this is the way the charge is distributed in xy plane. So, there are 6 charges and this is my origin so, and this length is say a from here to here so, now you want to find out the field at any point $\vec{r}$. So, the question should be, find the field at any point $\vec{r}$ in the xy plane. So, in the xy plane I want to find, so simply I just need to you know calculate the electric field for any given point and then add for all others. So, by the symmetry you can write the position of any kth charge.

So, I should put the charge here. So, this is charge q, all the charges are q, q_1, q_2, q_3, q_4, q_5, q_6 all are equal to q. So, the position of the kth charge is simply $(\hat{\imath}\cos\frac{k\pi}{3} + \hat{\jmath}\sin\frac{k\pi}{3})\,a$. So, that is the position if you just calculate the position of all these k then it should be simply by symmetry you can see that it should be simply like this. And now, once you know the $\vec{r}$ then my total field should be the summation.

So, total field $\vec{E}_T$ should be the summation of all the fields, k running from say 1 to 6 and that is simply all the charges q this is $4\pi\epsilon_0$ and then we can have a summation sign with k running from 1 to say 6 and then it should be $\vec{r}$ - $\vec{r}_k$. So, let me write it here $\frac{\vec{r}-\vec{r}_k}{|\vec{r}-\vec{r}_k|^3}$.

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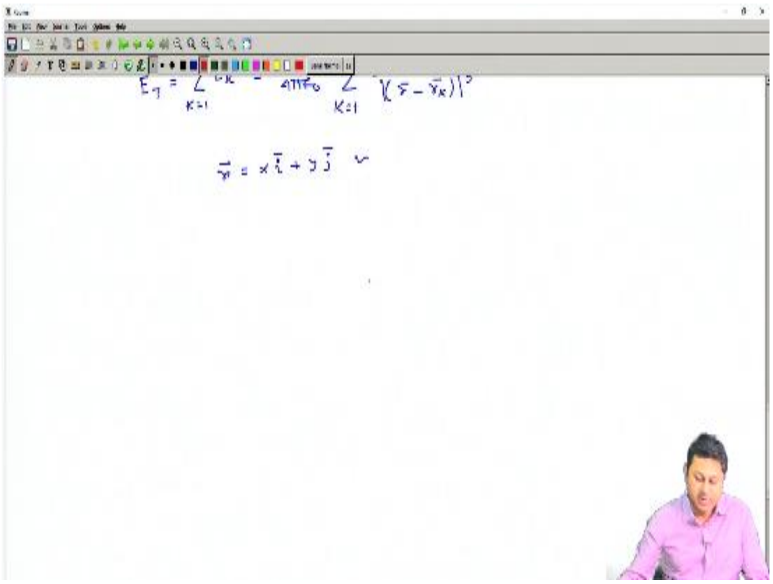

Now $\vec{r}$ is $\vec{r}_k$ I find and what is $\vec{r}$? $\vec{r}$ is simply x $\hat{\imath}$ + y $\hat{\jmath}$. So, if you put this value here then you will get the result that all. Now, if r is specifically mentioned then we can calculate this and that is your result that's all. You just show that how you can add for all these 6 points, how vectorially you can find out the point any point say I want to find out the field here at some point and this $\vec{r}$ value vector in this plane if the coordinate here is x y.

So, $\vec{r}$ is x $\hat{\imath}$ + y $\hat{\jmath}$ that is all and then the location of this point is $\vec{r}_k$. So, this point is $\vec{r}_k$, this is $\vec{r}_k$ if it is k and then I am just trying to find out the resultant of all these points and that basically summation sign will take care and get the value.

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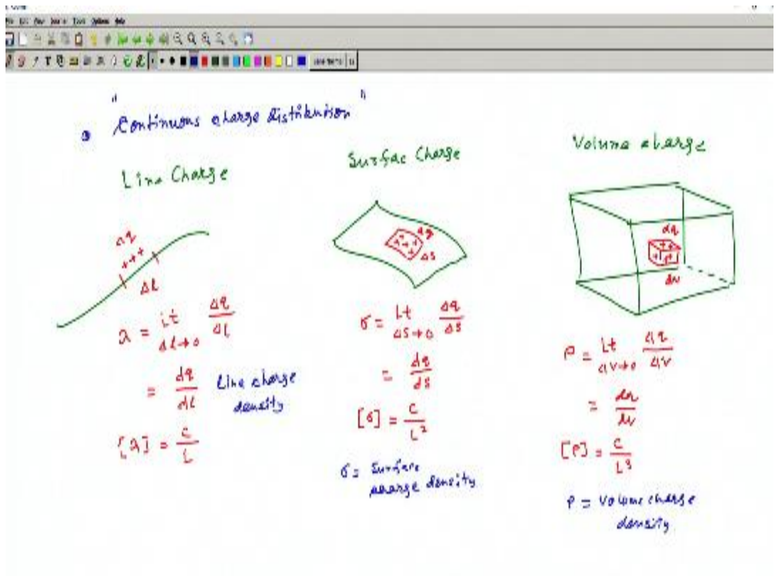

Now, after that we will discuss about the continuous charge distribution. So, far we are dealing with discrete charge now, continuous charge distribution is rather important, so first dimension wise what I have? We have first is line charge, what is line charge? The line charge is if I have a line here like this and if I have a small section of this line say Δl and if I have uniform charge distribution of over this line, then the line charge density I can write it as λ with the limit that Δl tends to 0.

Whatever the charge we have if this charge amount is Δq then $\frac{\Delta q}{\Delta l}$ or in other words, it is simply $\frac{dq}{dl}$. And dimensionally if I want to find out the unit of these things it should be Coulomb divided by unit of length $(\frac{C}{l})$ and λ is called the line charge density. Similarly I can have a surface charge density. So, surface charge is something suppose you consider a surface like this and if you take a small segment here and the charge is uniformly distributed there.

So, you can have the charges here like this for this small segment and Δs is the amount of area small area and Δq is the amount of charge you are having here in this small segment. Then the surface charge density defined by σ is limit Δs tends to 0 then $\frac{\Delta q}{\Delta s}$ and that value is simply $\frac{dq}{ds}$ and if I want to find out the dimension of these things, this should be Coulomb divided by length square $(\frac{C}{l^2})$ and it is called the surface charge. σ is called the surface charge density.

So, we can go on and we can now understand that if there is a line charge, surface charge I should have something called volume charge already we discussed this. And in volume charge suppose, we are having a volume, let us just consider a block here so, we have a block here and inside the block we have a small block, this is my volume element and we have charges here so, these amount is small amount dv and the charges dq.

So, like surface charge and volume charge we can have like surface charge and line charge we can have the volume charge ρ, which is limit Δv tends to 0 then $\frac{\Delta q}{\Delta v}$, which is $\frac{dq}{dv}$ and want to find out the dimension, which is Coulomb divided by L cube ($\frac{C}{l^3}$). So, this quantity is called ρ is called volume charge density, so, this is the way the charge distribution for starting from line charge and all.

Now, the next question is if this kind of continuous charge distribution is there, then how to define the electric field at some point previously it was a discrete one then I can use this form. Now, if it is not discrete if it is continuous then how to define it?

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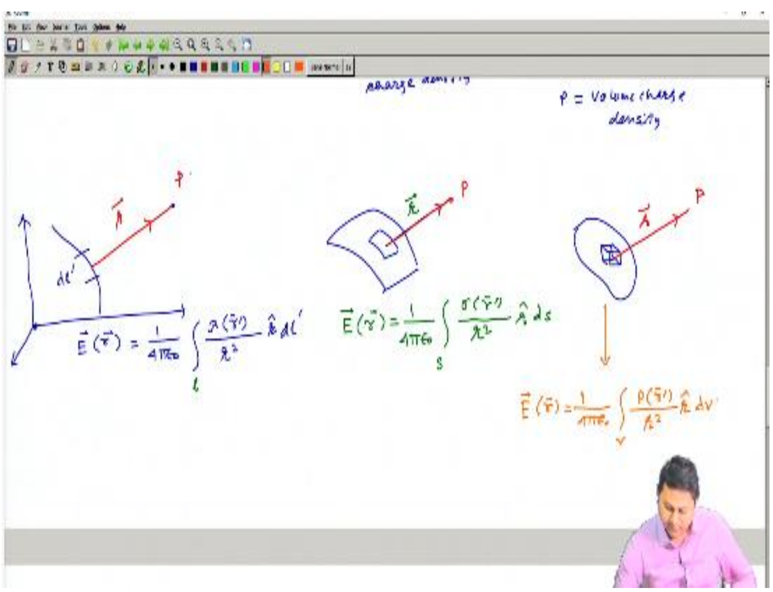

So, the electric field if I want to define here for example, we are having a line here and for this dl, so, this is a dl' and if I want to find out the electric field at some point P and this distance is say $\vec{r}$ then I can write my electric field as this, electric field at point $\vec{r}$, so, this $\vec{r}$ is from a coordinate system suppose, I am having a coordinate system here and the location of this P is at $\vec{r}$ so, if I this is the origin, so, if I join a line from origin to P that is my $\vec{r}$.

So, at this point, if I want to find out the electric field it should be $\frac{1}{4\pi\epsilon_0}$ then the integration of λ because now, we are having a continuous charge distribution it should be λ($\vec{r}$') divided by the length here which is $л^2$ and it should be the direction along this and dl' because of this dl' I am getting the electric field here instead of having the summation now, I am having the integration because this is a continuous charge distribution what happened for surface? If I have a surface here.

So, this is my surface and this is just the small segment of the surface try to find out the electric field at this point P and the position of the P is $\vec{r}$. So, $\vec{E}(\vec{r})$ would be the electric fields at $\vec{r}$ so, this point is as before $\vec{r}$ should be this is a line integral by the way I should have $\frac{1}{4\pi\epsilon_0}$ integration a surface integral with σ, which is a function of say $\vec{r}$' because from here to here, if this is $\vec{r}$', the location of the charge distribution then $л^2$ then $\hat{л}$ and then ds.

So, this is the way we can define. Exactly in the similar way, if I have a volume element suppose, this is a volume element and I am having a volume here tiny volume having a charge density volume charge density ρ and want to find out the field at this point P. So, this is my $\vec{r}$ and if I want to write the $\vec{E}$ for this case my $\vec{E}$ that should be a function of $\vec{r}$ $\frac{1}{4\pi\epsilon_0}$I should have a volume integration and then it should be $\frac{\rho(\vec{r}')}{л^2}$ $\hat{л}$ and then dv.

So, this is the way you can represent for these 3 different system you can represent the electric field in general, I have say.

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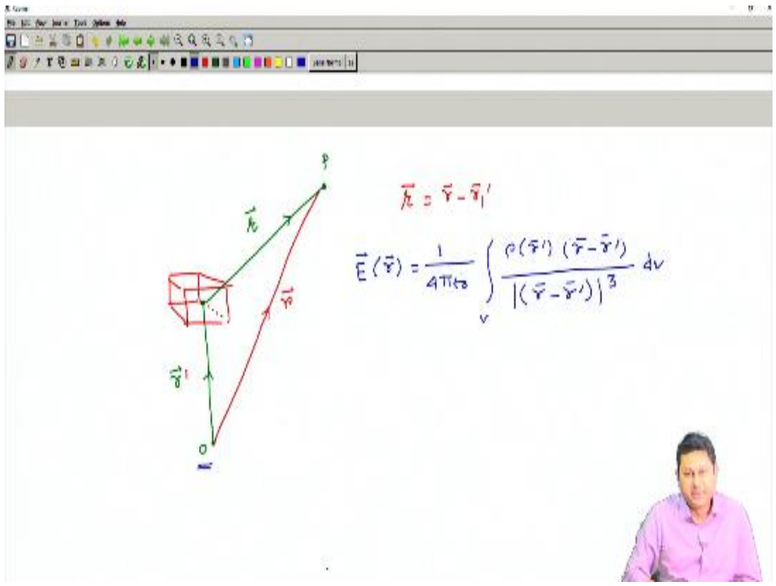

So, let me just draw it properly because normally we are dealing with the volume charge so suppose I have a volume here and this volume due to this volume I can have a field here try to find out the field at point say P this is my $\vec{\pi}$ and I am having some origin here some point this is my origin say O, from origin to this point so the middle point say so this vector this is my origin so this vector I am so this is my origin so this vector say $\vec{r}\,'$ and from here to here this is the point and that is $\vec{r}$.

So, this is the geometry we are having so my $\vec{\pi}$ is $\vec{r}$ - $\vec{r}\,'$, $\vec{r}\,'$ is the source point sorry here I should write prime so $\vec{r}\,'$ is a source point with respect to this origin. So, now if I want to write this $\vec{E}$, $\vec{E}$ is at point $\vec{r}$ is $\frac{1}{4\pi\epsilon_0}$ integration that should be a volume integration of $\rho(\vec{r}\,')$ and then we should have $\frac{(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$ and dv.

So that is the general way to write because if you have the knowledge of this $\vec{r}\,'$ that is the source point and $\vec{r}$ that is the field point with respect to this origin then you can write it in this way. So, this is the way we write so today I do not have much time so in the next class what we do that we try to you know based on this knowledge whatever we have that how the electric field should be executed for continuous charge distribution, what should be the expression.

So, exploiting this expression we try to find out the electric field for some given charge distribution, so 2 problems will be done one is when the charge is distributed over a straight line and in another case when the charge is distributed in some surface. So, thank you very much for your attention and see you in the next class and then we will discuss all these aspects of charge distribution thank you.