Foundation of Classical Electrodynamics
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Lecture – 22
Coulomb's Law

Hello students welcome to the foundation of classical electrodynamics course. So, we are in module 2, understanding electrostatic and today we are going to discuss about the Coulomb's law. The last class also we intend to discuss, but due to the time constant we could not. So, let us now do it in this class.

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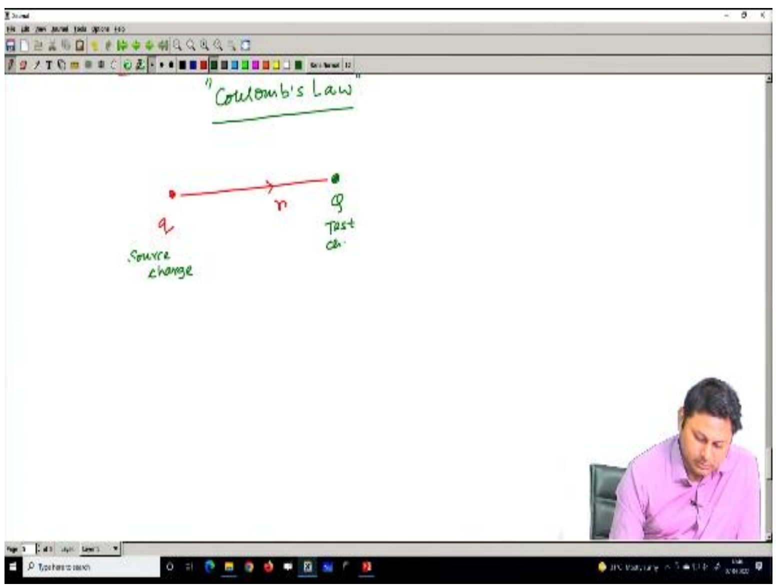

So, it is class number 22, where we discuss the famous Coulomb's law. So, I think most of the students is already aware of the Coulomb's law nothing new here. So, let me go through this law once again. So, suppose you have a source charge here say q and at some distance r you have another charge, which we call the test charge Q.

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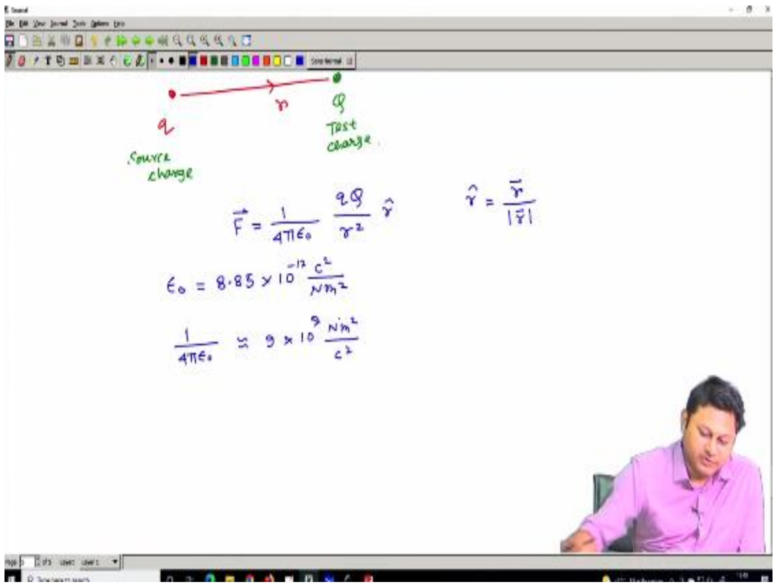

So, then the force between them is simply written this like inverse square law. So, force between them is $\frac{1}{4\pi\epsilon_0}$is constant and then $\frac{qQ}{r^2}$ $\hat{r}$ when $\hat{r}$ is simply$\frac{\vec{r}}{|\vec{r}|}$. The value of the ϵ_0, which is called the permittivity of this free space is roughly 8.85×10^{-12} then coulomb square per Newton meter square ($\frac{C^2}{Nm^2}$) and $\frac{1}{4\pi\,\epsilon_0}$if I calculate instead of calculating ϵ_0, it is you know of the order of nearly equal to say 9×10^9 then $\frac{Nm^2}{C^2}$. So, this is for single charge.

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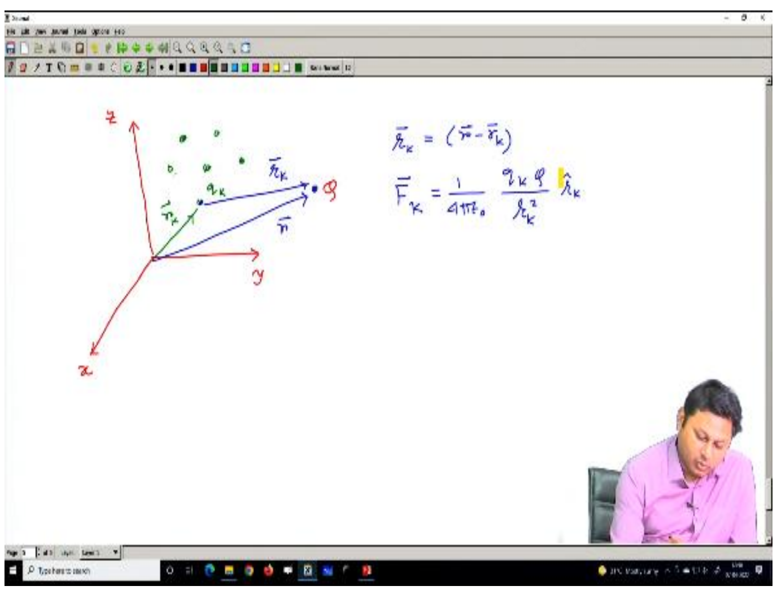

What happened if there are multiple charges are there? So, I need to put a coordinate system now simply x y and z and instead of having one charge we have a charge distribution, but, these are the discrete charges. So, suppose this is my q_k and from here to here the coordinate the distance is r_k and my test this is my source charge, my test charge is sitting somewhere here by Q, somewhere here.

So, the location of this charge from this coordinate system is say r and from here to here this is the distance between the source charge and this I can write $\mathscr{r}_k$. So, $\vec{\mathscr{r}}_k$ is nothing but $(\vec{r} - \vec{r}_k)$. So, for this particular charge q_k, if I want to find out what is the force, then the force due to the charge q_k so, that is why I put a k here it should be $\frac{1}{4\pi\epsilon_0}$, then $\frac{q_k Q}{\mathscr{r}_k^2}$ and then $\hat{\mathscr{r}}_k$.

Now the total force because there are not only one charge, other charges are also sitting here. So, if I want to find out what is the total force on this test charge Q.

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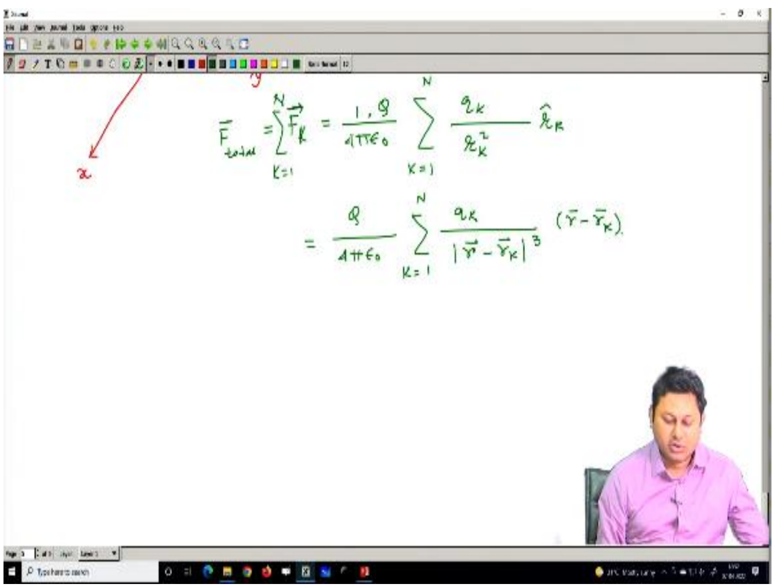

So, then the total force $\vec{F}_{\text{total}}$ should be the vector sum of all the poll, there is some sort of superposition. So, I should have vector sum where say k goes to 1 to N, because there are N number of charges. So, this is $\vec{F}_k$ and that is equivalent to $\frac{1}{4\pi\,\epsilon_0}$ this is constant for all the cases. Then the sum over k goes to 1 to N and also I can take this charge Q outside because Q is nothing to do with the summation. So, then I have $\frac{q_k}{\mathscr{r}_k^2}$ and $\hat{\mathscr{r}}_k$.

Now, if I know the coordinate in r_k for all the point charge, so, this I can write $\frac{Q}{4\pi\epsilon_0}$ and then some over q_k, k goes to 1 to N for all charges divided by $(\vec{r}-\vec{r}_k)$ because now I am writing in terms of normal r_k and then it should be cube and $(\vec{r}-\vec{r}_k)$. So, this is the way you can represent the total charge on a test charge Q when the charge distributions are there.

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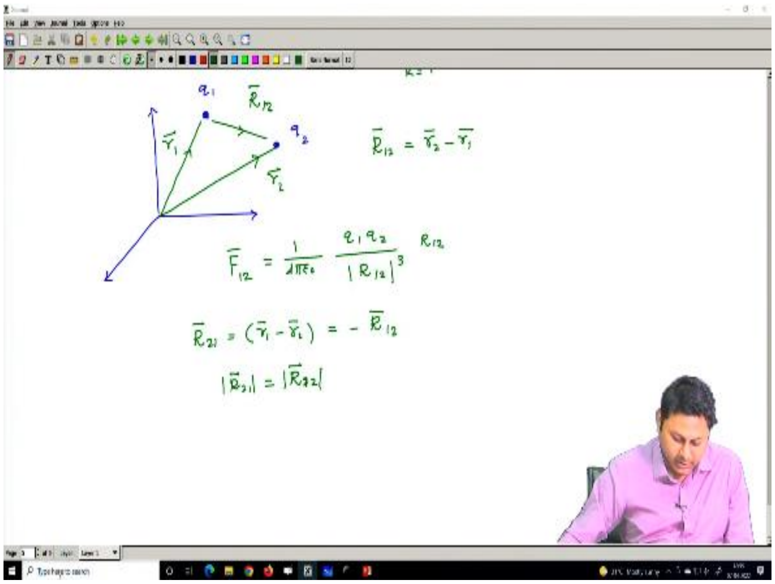

So, also from this one can understand when there are 2 charges, which is the coordinate system and 2 charges q_1 and q_2 are there. So, this is q_1 and this is q_2. So, we can have the coordinates of q_1 like $\vec{r}_1$ and coordinate here is $\vec{r}_2$ so that these I can now write say $\vec{R}_{12}$. So, $\vec{R}_{12}$ is simply $(\vec{r}_2 - \vec{r}_1)$. So, now, if I want to find out what is the force between these 2 charged particle, so, force on 2 due to 1 is $\frac{1}{4\pi\epsilon_0}$ then $\frac{q_1 q_2}{|R_{12}|^3}$ and then $\vec{R}_{12}$ this is one force on 2 due to 1.

Now, you can check that $\vec{R}_{21}$ is simply $(\vec{r}_1 - \vec{r}_2)$, which is $-\vec{R}_{12}$. So, the magnitudes are same. The magnitude of 21 and 12 is same. It is obvious. But the question is, if I want to find out the force on 1 due to 2.

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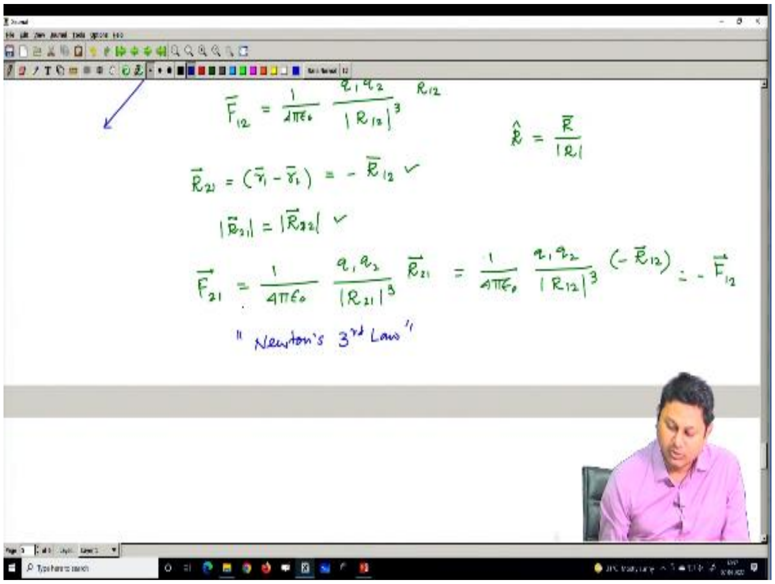

Then it should be force on 1 due to 2 is this $\frac{1}{4\pi\epsilon_0}$ and same $\frac{q_1 q_2}{|R_{21}|^3}$ but now, R will be $\vec{R}_{21}$ mod cube and unit vector $\vec{R}_{21}$ not unit vector, this is simply $\vec{R}_{21}$, because I am making cube here $\vec{R}_{21}$. Since I am making cube here, so, I just write the unit vector. So, $\hat{R}$ is simply represented by $\frac{\vec{R}}{|R|}$. So, one additional $|R|$ is here on top of the R^2, then that is why it is R^3. But the thing is, here $\vec{R}_{21}$ and $\vec{R}_{12}$ has a relation.

So, I can write this as $\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|R_{12}|^3}$ and then $-\vec{R}_{12}$ because that relation we know these 2 relation we know and I just put this. So, this quantity is nothing but - $\vec{F}_{12}$, which simply tells that this is nothing but the Newton's third law and that we can simply check. Every action has its equal and opposite reaction. So, the force 21 and force 12 they are equal and opposite.

Now, last class we were discussing about the you know, how this gravitational force and the electromagnetic force how they are I mean, what is the weightage between these 2, what is the ratio between these 2 we discuss. So, let us do that quickly try to find out the order.

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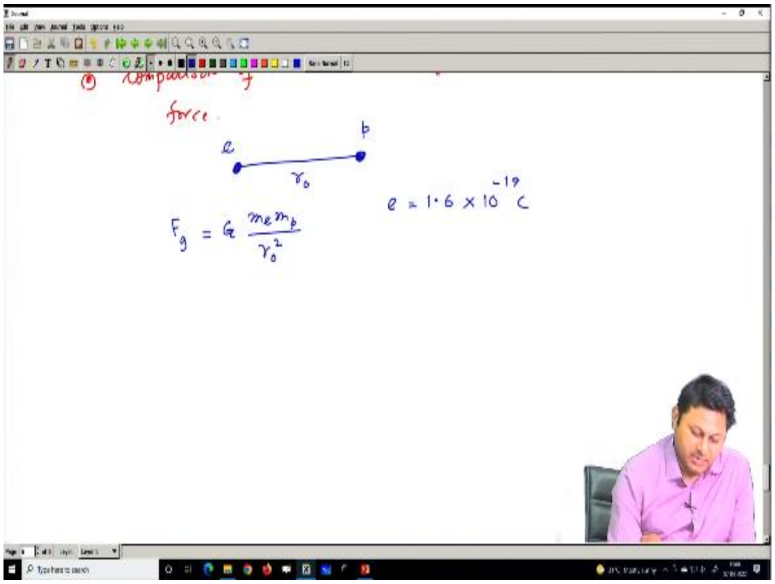

So, we will go to compare quickly I mentioned in the last class. So, comparison of electrostatic and gravitational force. So, suppose I have an electron and proton sitting here. So, this is my electron and this is my proton and the separation between these 2 is r_0. Now, the question is what should be the gravitational force and what should be the gravitational force between these 2 is simply if I want to calculate the magnitude only because both are having the same vectorial form.

So, this is $\frac{m_e m_p}{{r_0}^2}$ where the charge of the electron I should write it here somewhere 1.6×10^{-19} C and m_e is the mass of the electron and m_p is the mass of the proton that we are going to use later.

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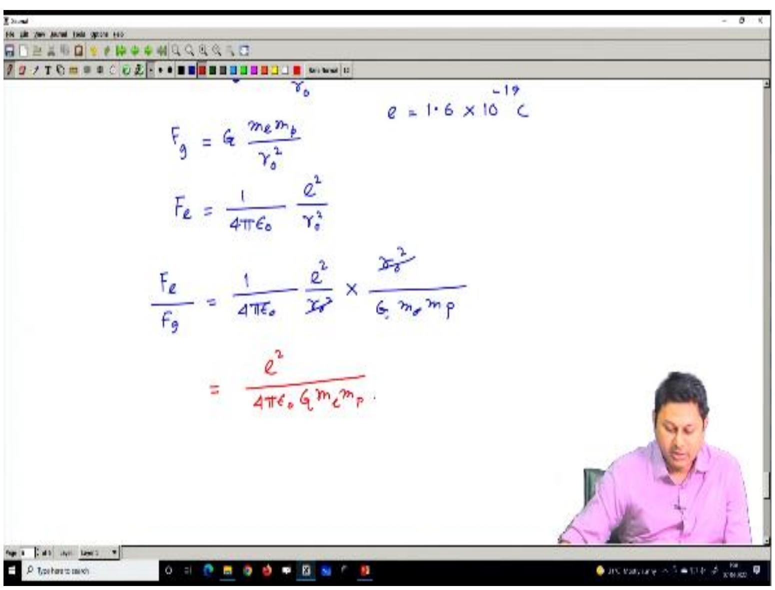

So, what is the electrostatic force? Electrostatic force should be $\frac{1}{4\pi\epsilon_0}$. And then e^2 because both the cases the amount of charge electron and proton is same that is e and r_0^2. Now, I have to find out $\frac{F_e}{F_g}$, the ratio between the force and this is $\frac{1}{4\pi\epsilon_0}\,\frac{e^2}{r_0^{\,2}} \times \frac{r_0^{\,2}}{Gm_em_p}$, and then I can see that r_0 r_0 are going to cancel out and what else that is all.

So, eventually, I get this quantity that $\frac{e^2}{4\pi\epsilon_0 Gm_em_p}$, all the values are known. If I put all the values, e I already mentioned.

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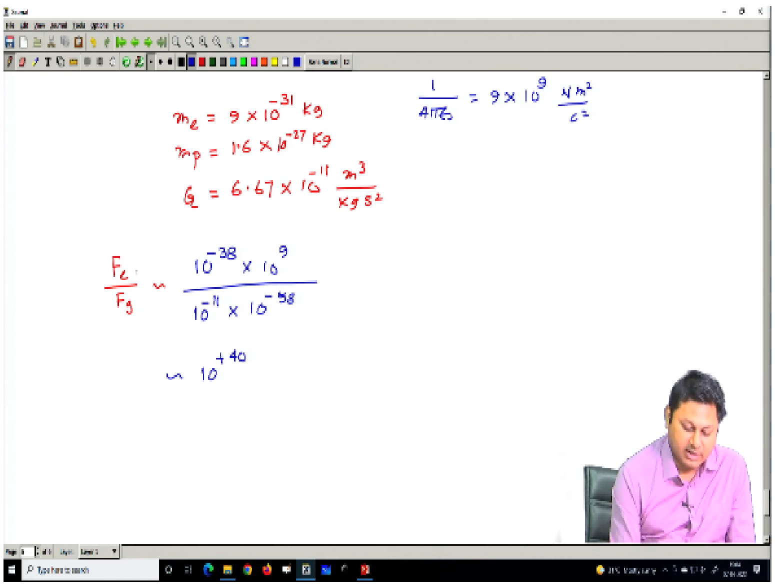

So, m_e is around 9×10^{-31} kg, m_p is around mass of the proton is around 1.6×10^{-27} kg because it is heavier than electron and G gravitational constant is around 6.67×10^{-11} in the unit of $\frac{m^3}{kg\ s^2}$ and all the values and e is 1.6, I already written here 10^{-19}. So, if I start putting all this value here in this equation.

So, $\frac{F_e}{F_g}$, if I start putting, just try to understand all the orders, so, I just remove all these terms, 9, 1.66, just to get a rough idea, it should be say 10^{-38}. And then 10^9 divided by what else we are having, because here, e^2 is 19. So, that is why 10^{-38}. And then we have $\frac{1}{4\pi\epsilon_0}$ that value I did not write $\frac{1}{4\pi\epsilon_0}$ this is $9 \times 10^9\,\frac{Nm^2}{C^2}$.

So, that 10^9 should be here and rest of the term in the denominator it should be 10^{-11} then into so, what we have in the denominator G this is 10^{-11} and then $m_e \times m_p$, so, if I just add these 2, so it should be 10^{-58}. So, now, if I calculate all these things, it should come off the order of say 10 to the power roughly say -40 or -39. So, you can see that how plus, so you can see that how strong the electromagnetic force compared to the gravitational force.

So that you should realize that it is a very, very strong force, compared to the gravitational force even though it is weaker than the strong force, nuclear strong force. But still, it is very, very strong, because there is a long range force and is a very, very strong force.

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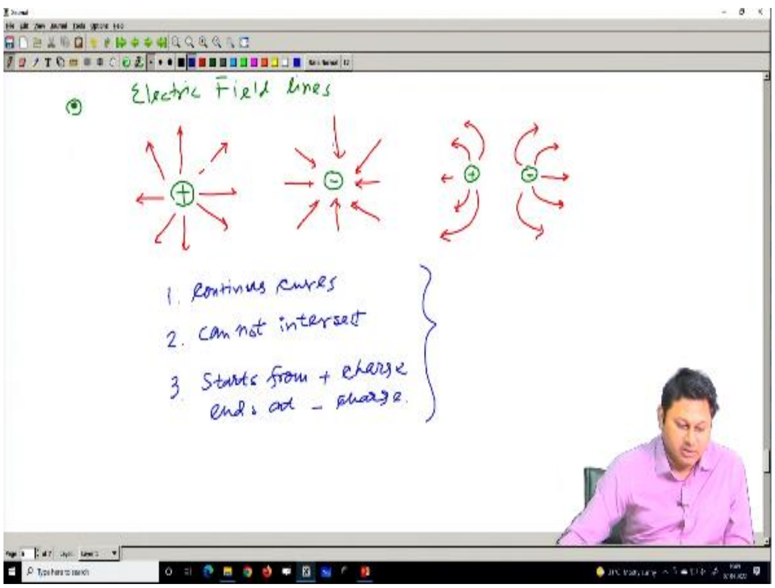

The next thing that we are going to discuss is the electric field lines. So, if I have a positive charge here sitting and draw the field lines that means, if I make a unit positive charge here then the direction of the force if I measure so, we will find that the field lines will like this. When we put a negative charge here the field line will be inward and I can have the field line like this this is roughly the field line I am just drawing by hand it will go into play an important role later on.

So, this is the way the field line one can draw. So, there are a few properties of the field line quickly let us write it down. So, first thing is these are the continuous curves these field lines are nothing but the continuous curves, second thing is they cannot intersect each other why they cannot intersect? If they intersect each other then at the same point they have 2 different you know direction for the force which is not accepted.

So, that is why they will not be going to intersect each other. Then the third point is it starts from plus charge and ends at negative charges so, these are the well-known properties. I am just writing it down that these are the continuous curves they are not going to intersect and they start from the positive charge and ends at the negative one. So, already I mentioned that when we have the electric field. So, this electric field let me the force the Coulomb's law, when we discuss today we discuss that there is a superposition.

And we have the total force that is a superposition of all the lines so, F_k is the summation, so F_{total} is a summation of F_k now if we try to understand that what happened for the electric field, electric field for multiple points.

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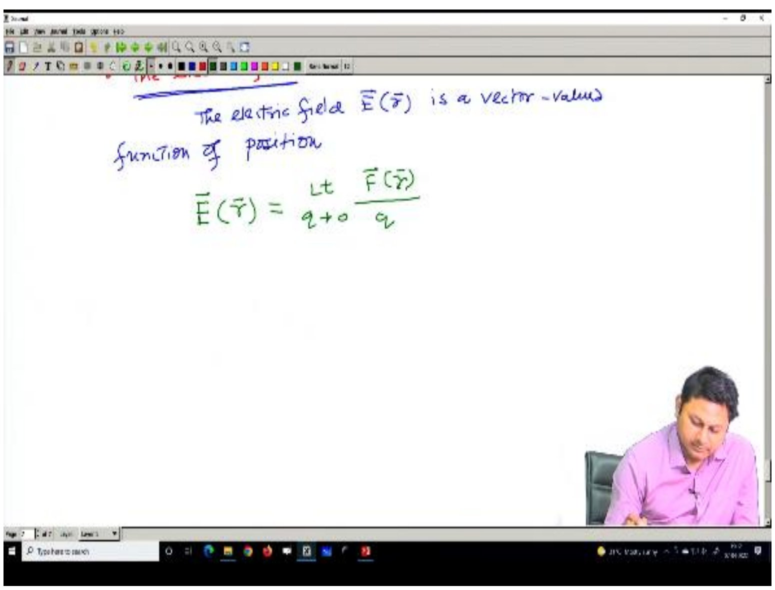

Then the same thing will happen will appear here. So, the electric field the same thing we do for, there is some issue with the pen again. The electric field is a vector values. So, let me define first what do you mean by electric field? Because electric field line I define so the electric field E the function of r is a vector valued function of position is a vector and is it is a function of position at different point. So, by definition I already defined that in the last class.

So, $\vec{E}$ electric field is defined as limit q tends to 0 and then we have the force here and under the condition that this q tends to 0 so, this is by definition the electric field and I explained in the last class why I should take this limit.

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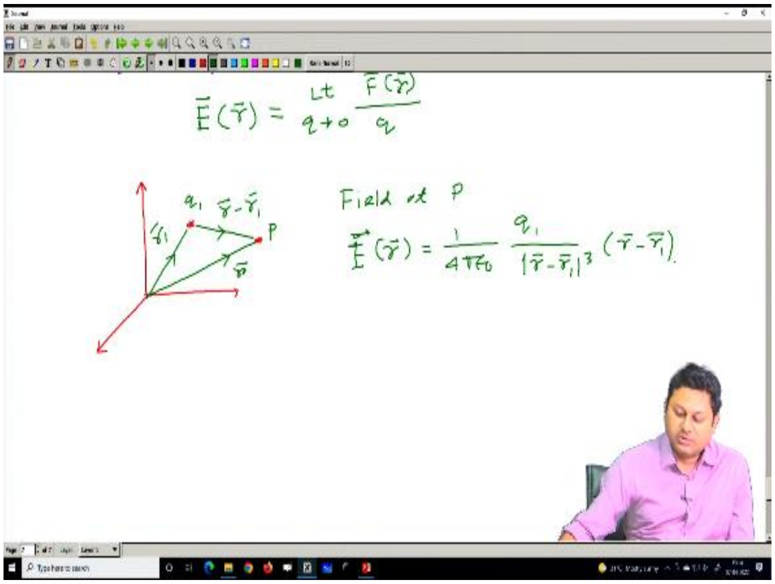

Now, if I want to find out the field between 2 discrete points like the force we calculate earlier it will be something similar. So, this is my 1 this is so I am having $\vec{r}_1$ here $\vec{r}_2$ vector here and the distance between these 2 is let me if this is the point where. So, $\vec{r}_1$ is a charge point and this is $\vec{r}$ the point where I try to find out the field and the charge point is here q_1 is sitting here. So, due to the q_1 what should be the field here that is the thing we calculate the field at point P should be $\vec{E}$.

That I want to calculate at some point $\vec{r}$ is simply $\frac{1}{4\pi\,\epsilon_0}$ then whatever the $\frac{q_1}{|\vec{r}-\vec{r}_1|^3}$. So, I should write $|\vec{r} - \vec{r}_1|^3$ multiplied by ($\vec{r}$ - $\vec{r}_1$) this is field. The force is exactly the same for force we have exactly the same you know this expression and for field we are having the same one. Now, if there are many charges, so, what should be the expression of the field?

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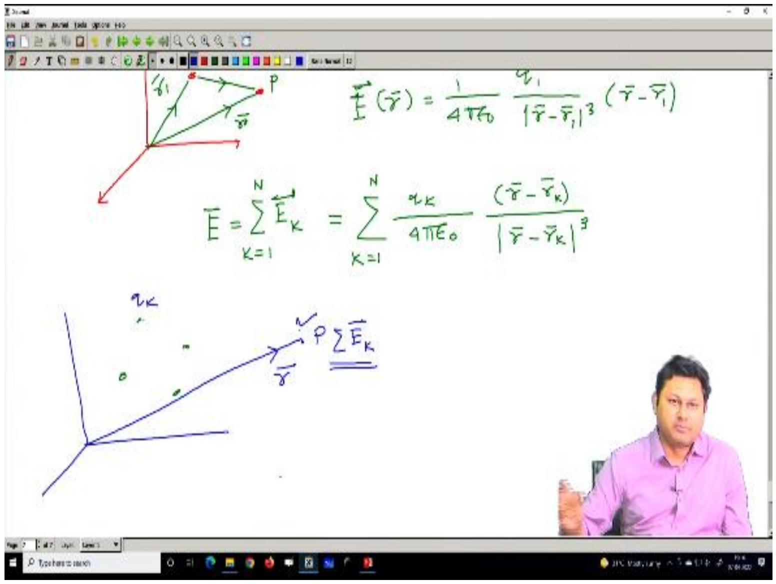

So, then the total field will be the summation of all the fields like before $\vec{E}_k$ and k goes to 1 to N. So, that eventually gives me summation of k from 1 to N and then $\frac{q_k}{4\pi\epsilon_0}$ and then ($\vec{r}$- $\vec{r}_k$) and then $|\vec{r}-\vec{r}_k|^3$. So, the point is if I want to calculate the field for a discrete charge sitting at different points and want to find out. So, these are the points where the charges are there and I want to find out some distance here at P point what should be the total field.

So, in this coordinate system, this is my coordinate system. So, this point is my $\vec{r}$. So, I need to calculate all the fields here due to each q_k point and then make a vector sum of that, so, that will give me the total field at this point. So, in the next class, we will do few problems with the superposition principle, this is called the superposition principle. If you know the field here for each point, then if you just add all these fields there, then you will get the result and it should be a vector addition mind it.

So, you just need to make a vector addition of all the fields there exerted by this point charges located at a different position and we can calculate the total field there. So, the next class will do few problems related to that. So, with that note, I would like to conclude here. Thank you very much for your attention and see you in the next class.