Foundations of Classical Electrodynamics
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Lecture - 20
Tutorial (Contd.,)

So, welcome students to the foundation of classical electrodynamics course. So, today we have lecture number 20. So, in the last class we discuss about few problems. So, today also we will be going to continue with certain problems. So, the tutorial will continue here in this class as well.

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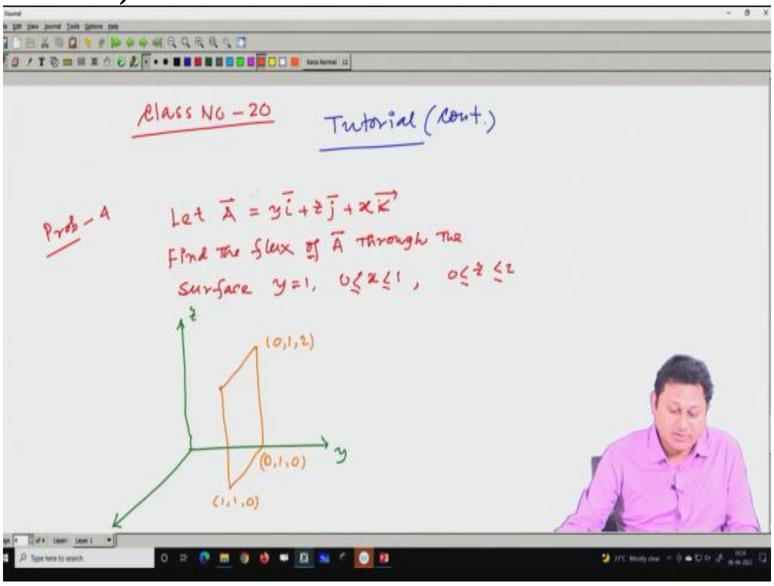

Today we have class number 20 and we are discussing tutorial, so, this is tutorial and this is continued whatever we started, so, few more problems we are going to discuss. So, 3 problems we discussed last class. So, let us do more so, today we will start with problem number 4 and the problem number 4 is this let $\vec{A}$ = yi vector field is given yi + zj + xk a vector field is given in this way.

Now, the question is find the flux of vector field $\vec{A}$ through the surface y = 1 and for x 1 to 0 and z 2 to 0. So, this is the surface for which we need to calculate the flux, flux is nothing but we need to do the surface integral here for this $\vec{A}$ vector field, but before that we need to first find out the surface through which we need to calculate this surface integral. So, this is my coordinate system so, this is my x, this is y and this is z.

So, the surface is such that y is constant so, it is parallel to xz plane and also the value of z is goes to 0 to 2. So, I can first draw that, so, x is 0 to 1 from here to here. So, I can have 0 to 1, but z is 0 to 2 like this. So, this is the surface we are talking about. So, this point is obviously, 1, this point is the coordinate of this point is 0, 1 coordinate of point is 1, 1 and coordinate of this point here it is 1, 2, 3 dimension I mean I need to prove the 3 dimension also.

So, it is 0 here so, this is 1, 1, 0 here make it correct it is 1, 1, 0 here and this coordinate is x is how much? x is 0, y is 1 and z is 2. So this is the surface also we have this coordinate but anyway, so this is the surface through which I need to calculate the flux of this vector field.

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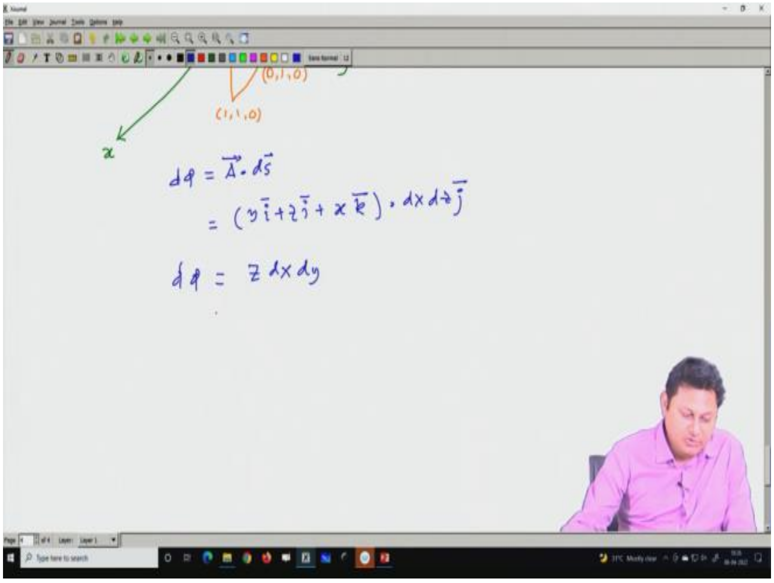

By the way what is flux? The flux dφ is $\vec{A}$ dot this elementary area. So, here what kind of area we are having? d$\vec{s}$ here is how much dx dz in the direction of j because this is the direction of the surface, which is j unit vector. So, that quantity I can calculate and $\vec{A}$ we know this is yi + zj + xk dot. So, this is precisely this is the $\vec{A}$ value I wrote here and d$\vec{s}$ here is dx and dz with the unit vector j.

So, from here you can see that i • j will vanish j • j will stay k • j will vanish. So, eventually I have z dx dy this is the amount of flux tiny amount of flux dφ.

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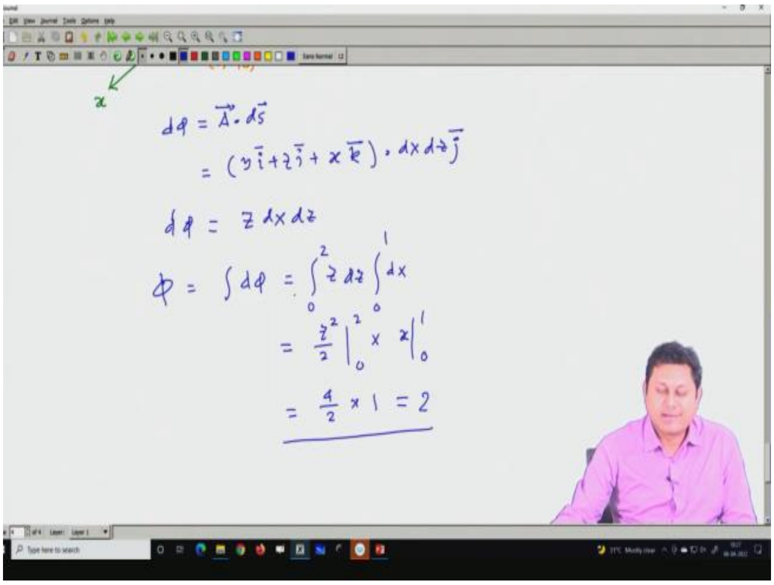

Now, I need to calculate the total flux if I calculate the total flux φ, which should be the integration of dφ. And if I do the integration here, so, there is a z so, sorry this is not dx dy this is I am making a mistake here. So, this is dz so, I can have this integration, I can have 0 to 2 and then z dz and this integration dx x is changing from 0 to 1 and we are done. So, this value if I integrate it is $\frac{z^2}{2}$ with the limit 0 to 2 multiplied by x with the limit 0 to 1.

So, it is simply $\frac{4}{2} \times 1$ so, the value is 2. So, the amount of total flux for this given vector field for the surface is this 1. So, eventually we just calculate the surface integral here this this was our problem 4. Let us now continue with more typical problem. So, let us take a little bit lengthy problem.

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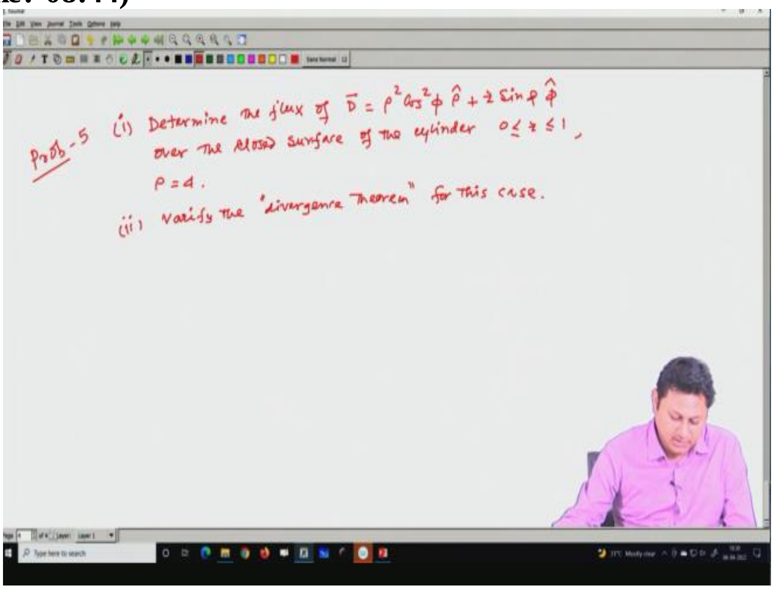

So, the problem is this, so, this is problem number 5 and the problem number 5 is like that determine the flux of a vector field $\vec{D}$, but this vector field is given in a different coordinate system note it $\rho^2 \cos^2 \varphi \, \hat{\rho} + z \sin \varphi \, \hat{\varphi}$ over the closed surface of the cylinder. And the cylinder is defined in this way z is changing from 0 to 1 and $\rho = 4$ that is the thing that is part 1 problem.

The second part of the problem is saying that verify that is interesting verify the divergence theorem for this case that part is interesting because we need to verify everything.

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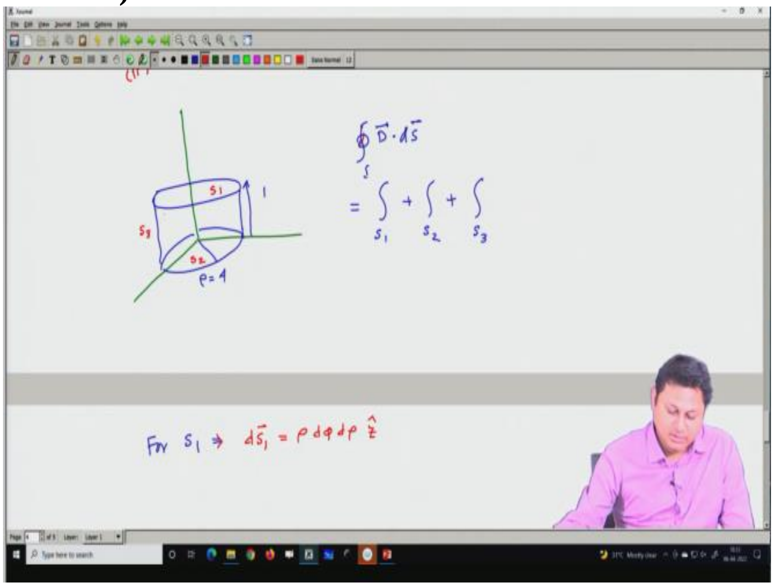

So, first do we want to find out what kind of surface it is? So, as usual I have a coordinate system here and this is a cylindrical coordinate so, and the cylinder and the closed surface is a cylinder with this so, I can just draw it and it should be something like this. So, this is the cylinder where this height it is z it is unit and the ρ whatever the ρ is saying this is 4 and this is the height, which is z roughly this is the cylinder and this is a closed surface and we need to calculate first the flux of the closed surface.

So, how many surfaces are there first we need to calculate. So, first so, what I need to calculate the closed surface integral that is the flux. So, my calculation is this I need to calculate $\vec{D} \bullet d\vec{s}$ I need to calculate this with the closed surface integral. So, this quantity know that it is with combining 3 surface one is this one upper one and lower one. So, I need to divide these 3, surface the total surface can be divided into 3 parts s_1, s_2, s_3.

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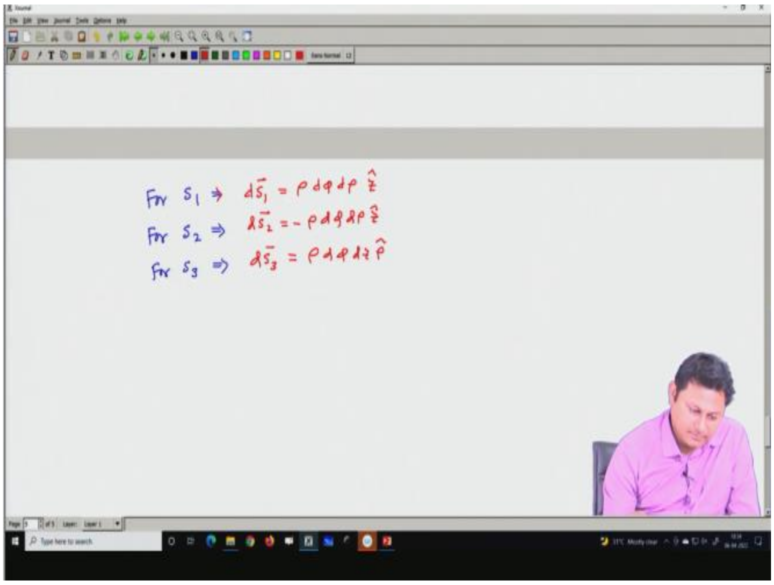

Now, for surface s_1 I can have the surface element $d\vec{s}_1 = \rho\ d\varphi\ d\rho\ \hat{z}$ upper surface, for lower surface that is for s_2. So, let me write it otherwise you may so, my s_1 is this one, my s_2 is this 2 and this is my s_3 this entire surface. So, next for s_2 I have $d\vec{s}_2$, which is equal to minus of because there is a lower surface $\rho\ d\varphi\ d\rho\ \hat{z}$ and finally, for s_3 we have $d\vec{s}_3$ and that is $\rho\ d\varphi\ dz\ \hat{\rho}$.

These are the 3 surface elements of the cylinder you should go back and check the surface element. And now, I am going to execute the entire integral for the surface.

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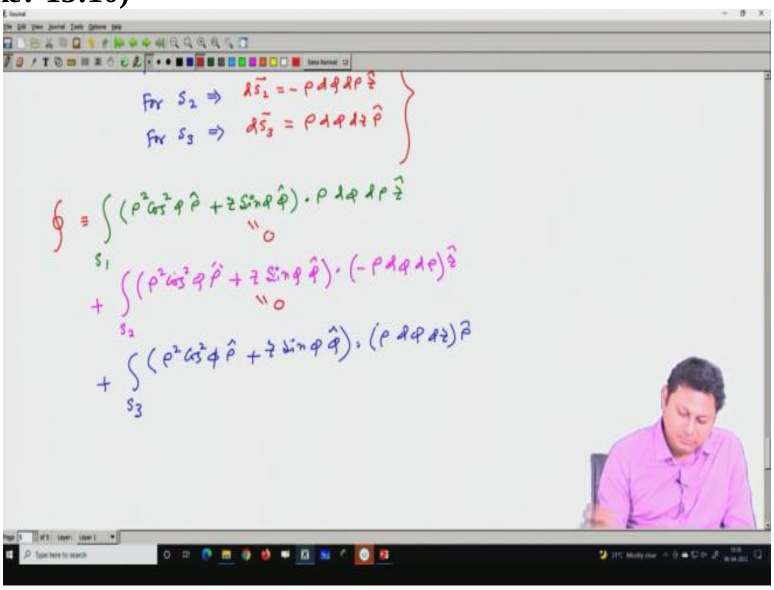

So, my total closed integral will be equivalent to this surface 1 and then field and the field is $(\rho^2 \cos^2 \varphi\ \hat{\rho} + z \sin \varphi\ \hat{\varphi}) \bullet d\vec{s}$ surface, which is $\rho\ d\varphi\ d\rho\ \hat{z}$ that is surface. What is surface 2? plus surface 2 same quantity here $\rho^2 \cos^2 \varphi\ \hat{\rho}$ the vector field, which is given here I am just writing that z sorry not cos sin, $z \sin \varphi\ \hat{\varphi}$ dot.

There is a negative sign mind it ρ dφ dρ $\hat{z}$ and finally, the third one ($\rho^2 \cos^2 \varphi\, \hat{\rho} + z \sin\varphi\, \hat{\varphi}$) • ρ dφ dz $\hat{\rho}$. Now, it looks very clumsy and very big but you can see that here $\hat{z}$ should be dot with ρ and φ so, these elements will be 0, and simply I can write this as 0. z will be dot with ρ z will be dot with φ 0 so, the contribution that we have here ρ, ρ and ρ φ.

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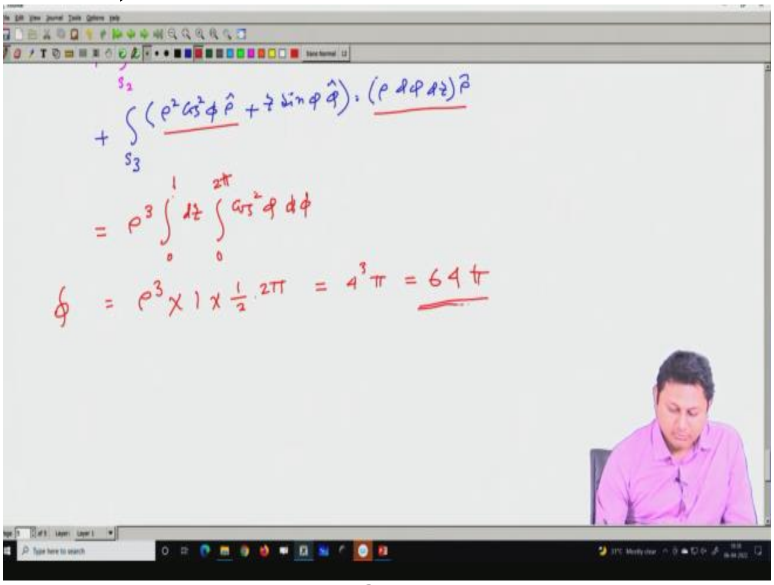

So, the contribution what we have here is ρ^3 integration of so, this contribution this and this contribution will be there. So, ρ is constant here so, I can put it outside then we have dz so, I just put dz, which is having limit 0 to 1 and then integration $\cos^2 \varphi\, d\varphi$, which has a limit 0 to 2π. So, if I execute that it simply gives me the closed line integral, closed surface integral simply gives me $\rho^3 \times 1 \times \frac{1}{2} \times 2\pi$ if we execute that you will get this.

So, this quantity ρ is 4, so, it should be 4^3 and π, so, it should be 64π so, that is the result of the problem 1 now, the next part of the problem is saying that please verify the divergence theorem so, you need to verify it. So, what is divergence theorem? Let me write it first.

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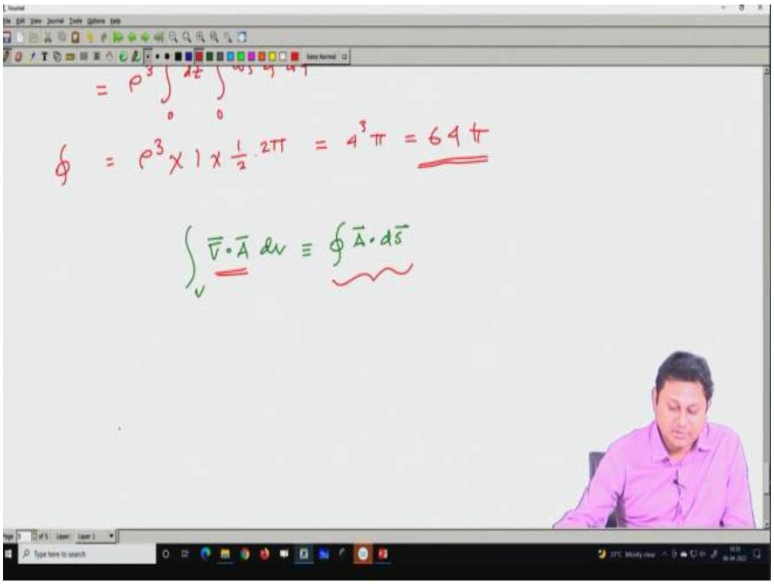

So, divergence theorem is if I have divergence of a vector quantity over a volume that should be equivalent to the close surface integral of the surface that is enclosing the volume. So, please check it that we have already figure out this part this is 64π that is a thing precisely we are calculating so far the closed surface integral the flux of so, now if I calculate this quantity over the given volume and this value if 64π if it comes like this, then we are done.

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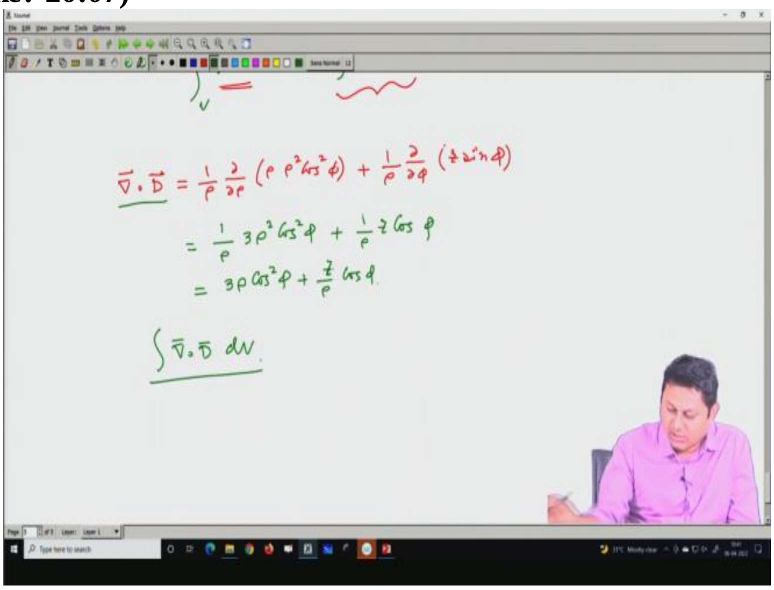

So, first we need to calculate this quantity divergence of the given vector field $\vec{D}$ whatever the vector field is given that. So, it is in cylindrical coordinate system, so, you should be careful enough so, that quantity is $\frac{1}{\rho}\frac{\partial}{\partial\rho}$ and then $\rho\ \rho^2 \cos^2 \varphi$, which is the first component plus $\frac{1}{\rho}\frac{\partial}{\partial\varphi}$ and then $z \sin \varphi$ and the φ component is not there.

So, you can see that this is there is no z component, so, I should not have the z derivative here. Now, if you execute it, it should be simply $\frac{1}{\rho}$ you are making partial derivative with respect to ρ here so, ρ^3 so, you have 3 ρ^2 and then you have $\cos^2$ φ what about this one? $\frac{1}{\rho}$ is already there we have z and partial derivative with respect to φ, so, you have cos φ.

So, eventually this is cos φ. So, eventually you have 3ρ and then $\cos^2$ φ + $\frac{z}{\rho}$ and then cos φ so, after that so, this is the value so, dv so, this calculate you calculate, but you need to calculate this quantity over dv. So, first you so, this I calculate here.

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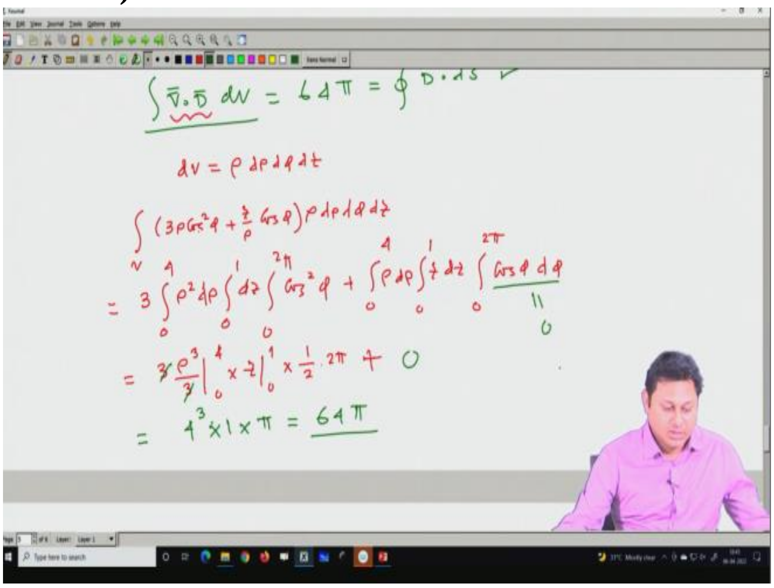

So, dv we need to calculate and this is a volume element in cylindrical coordinate system and it should be ρ dρ dφ dz this is the volume element. So, now, I calculate the total volume integral that means, I calculate integration volume and then this quantity that we derive which is 3 write it nicely so, I should have 3ρ and then $\cos^2$ φ and + $\frac{z}{\rho}$ and then cos φ and then ρ dρ dφ dz this is the integral and now, this integral I need to calculate.

So, if I now gather everything so, 3 then the ρ part so, this is ρ^2 dρ with the limit 0 to 4 then we have integration of 0 to 1 then dz then we have integration 0 to 2π $\cos^2$ φ that is the first part, what is the second part? Second part is 0 to 4 and then ρ dρ and then z dz in the limit is 0 to 1 and then 0 to 2π we have cos φ dφ we are almost there. So we have 3 $\frac{\rho^3}{3}$ 0 to 4 multiplied by z 0 to 1 multiplied by $\frac{1}{2}$ × 2π and plus here, if you look carefully that you will see this is cos function.

And I am integrating 0, 2π. So, it should give me 0 so, it is simply 0. So, what is this value if I calculate, so, this is 3, 3, will cancel out. So, we have 4^3 plus sorry not plus multiplication. So, multiplication by 1 and multiplication by π, so, which is nothing but 64π. So, this is the same result we get earlier. So, this value I calculate 64π, which is equal to this quantity $\vec{D} \cdot d\vec{s}$ so which proves which basically establish the divergence theorem.

So, we can prove we can show that the divergence theorem is correct we verify the divergence theorem here. So, whatever the time we have let us quickly do the last problem.

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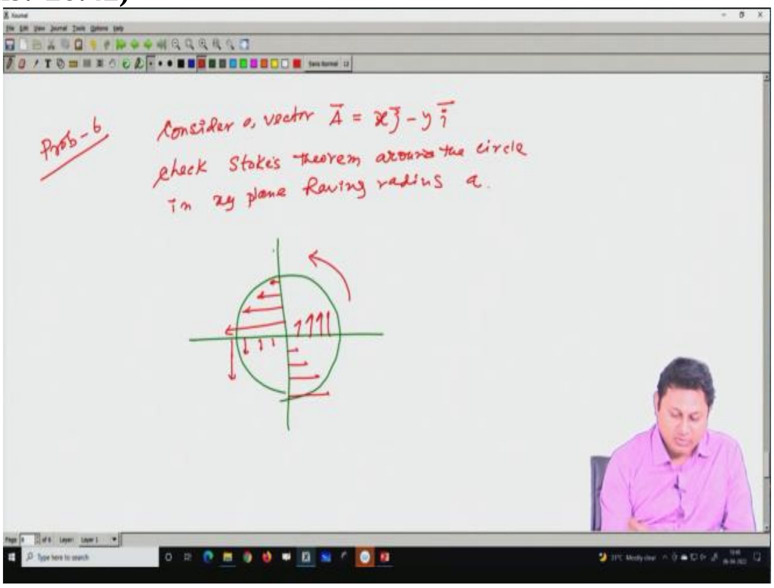

Problem 6 and problem 6 saying that consider a vector $\vec{A}$ vector function rather is x j - y i, this is the vector function. So, the problem is check Stokes theorem around the circle in xy plane having radius a. So, we need to you know verify the Stokes theorem for this vector field. So, if I now draw this surface first and then Stokes theorem means I need to do the close line integral.

So, it is a circle like this with radius a and if I now plot the vector function here then vector function if I plot it should be like this then it is a rotating kind of thing. So, here this is rotating the vector field is rotating like this and we need to calculate the Stokes theorem. So, let us first find out that in order to calculate.

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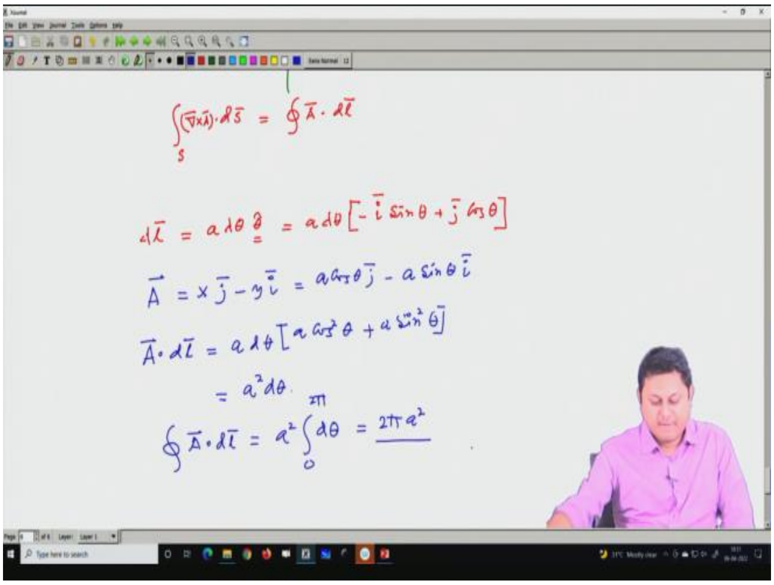

So, what is the Stokes theorem let me write, so, $\vec{A} \bullet d\vec{s}$ over surface integral should be the closed line integral sorry $(\vec{\nabla} \times \vec{A}) \bullet d\vec{s}$ I am making mistake here. So, the $\int(\vec{\nabla} \times \vec{A}) \bullet d\vec{s} = \oint \vec{A} \bullet d\vec{l}$. So, first what is $d\vec{l}$? Let us find out so $d\vec{l}$ is over this line. So, $d\vec{l}$ should be a dθ is constant dθ and the $\hat{\theta}$ because it is over this line. So, these things is nothing but a dθ in terms of i, j, k because my vector field is given in you know this Cartesian coordinate system.

So, I need to make a transformation how to write θ in terms of unit vector in terms of i, j, k we did it earlier. So, it should be i sin θ + j cos θ. So, now, $\vec{A}$ is xj – yi, which is a cos θ j - a sin θ i. Now $\vec{A} \bullet d\vec{l}$ if I calculate now, I write $\vec{A}$ in terms of θ and also this vector I can just make in Cartesian coordinate So, $\vec{A} \bullet d\vec{l}$ is a dθ and then this dot this if you calculate the this quantity dot this quantity.

So, the j • j will be $a \cos^2 \theta$ and then I have plus $a \sin^2 \theta$, which gives me something like a^2 dθ. Now, if I make it close line integral of this quantity $\vec{A} \bullet d\vec{l}$ it should be a^2 and close line integral, which is dθ ranging from 0 to 2π. So, my result should be simply $2\pi\, a^2$ so, that is the value I am getting here when we calculate the line integral close line integral of this given sphere given circle.

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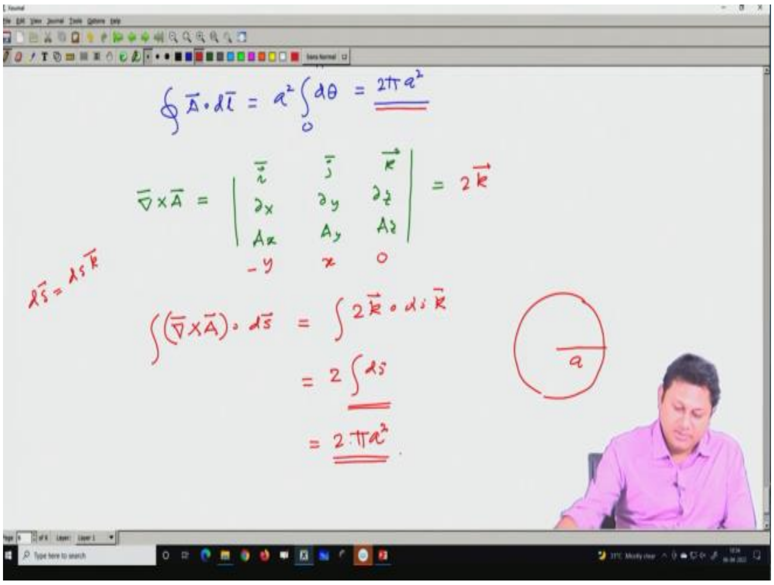

Now, the next part is I need to calculate $\vec{\nabla} \times \vec{A}$, $\vec{\nabla} \times \vec{A}$ is i j k and then the ∂_x, ∂_y, ∂_z then A_x A_y A_z and if I do the calculation because A_x A_y A_z what is A_x here? If I go back so, A_x is -y so, this value is –y. What is A_y? A_y is x, so, this is x and this is 0. So, this value if I calculate i component this is 0 and this is 0, j component, this is 0 and this is 0, k component we have 1.

So, 1 and this is 1 so, we have 2k this is the value one can get. So, now you calculate the surface integral of this, quantity dot $d\vec{s}$. Now, what is $d\vec{s}$ here? $d\vec{s}$ it is $d\vec{s}$ in the direction of k. So, that means I am having integration of $2\vec{k} \cdot ds\ \vec{k}$. So here we have $2\vec{k} \cdot \vec{k}$ 1 integration of ds. Now, ds is a total surface, and if I am having a circle of radius a, then the total surface simply gives me $2 \times \pi\ a^2$ this is the area of the surface. Now, you can see this is the same value I am getting here and we are getting the same value here.

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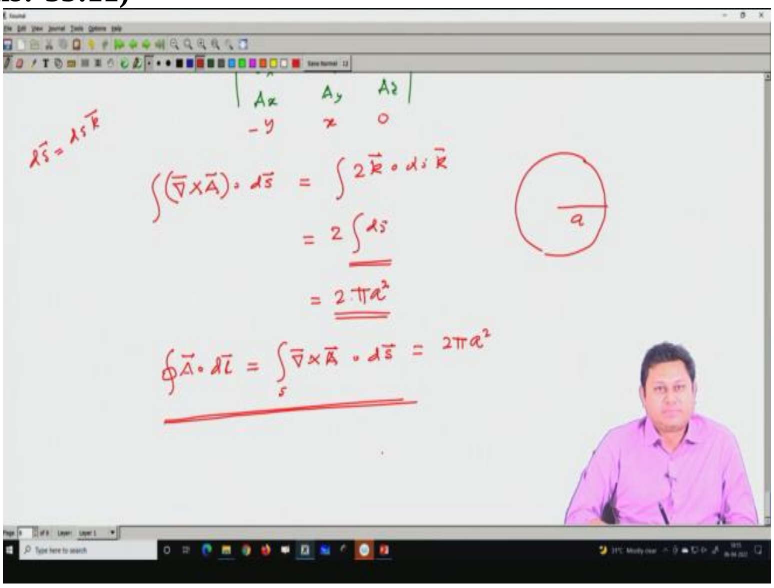

So, that means, I can write that the $\oint \vec{A} \bullet d\vec{l}$ for this given circle is equal to $\int(\vec{\nabla} \times \vec{A}) \bullet d\vec{s}$ that we calculate over the surface integral they are same both the cases the value is $2\pi\ a^2$ for this given problem. So, this is the verification of the Stokes theorem. So, we find that the Stokes theorem is valid here. So today we do not have much time. So, with this note, I will conclude here. And thank you for your attention and see you in the next class. So, where we start our module 2, thank you very much and see you in the next class.