Foundation of classical electrodynamics

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Lecture - 01

Vector Analysis, Scalar and Vector Fields, Vector Identities

Hello student. So, myself is professor Samudra Roy and our courses foundation of classical electrodynamics and I am from Department of Physics IIT Kharagpur. So, before starting the course, I like to you know, say the name of the textbooks that the students should go through and the first 3 you can see 1,2,3, it is written in the screen introduction to the electrodynamics by Griffiths is a very popular book among the students.

So, you can go through with these books and then you can also read electromagnetism by Pollack and Stump and then another very nice book that you can go through is modern electrodynamics by Zangwill. Apart from these three textbooks, we have two references, I like to suggest two reference books, one is classical electrodynamics by J D Jackson and another is Feynman's lecture volume 2.

So, these two books you can also read as reference reading. In today's class, we will mainly discuss about the vector analysis, scalar and vector fields, and vector identities, which should be in module 1 and that is mathematical preliminaries. So, in order to understand electrodynamics, you need to learn the initial stage few mathematical preliminaries, which are very important for this course.

Because later on we will do many problems, many treatments based on the electromagnetic theory, electrostatic and magnetostatic, and all these mathematical preliminaries will help you to understand the process that we you know going through step by step. Okay. So, with that note, let me start in today's class.

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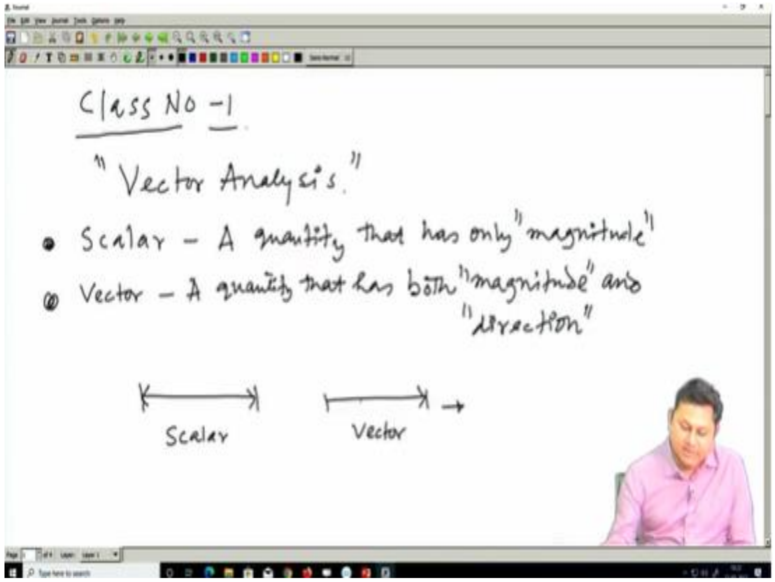

Class Number 1, Okay. So, as I mentioned in the first class, we will mainly discuss about the properties of the vectors and let us start the topic Vector Analysis. So, what is vector first and that we need to know but before that what is scalar that we need to understand. And Scalar is nothing but a quantity that has only magnitude, so a quantity which is only having a magnitude is called a scalar quantity, we all know that.

On the other hand, vector a quantity that should have both magnitude and another very important thing and that is direction. Magnitude and direction. So, that means, if I consider only length of certain quantity here if I want to you know understand this graphically, then this amount of length only the length is a scalar quantity. However, it is not only length you can mass and other quantities are also under scalar quantity.

But if I want to understand graphically or pictorially then a line is there and if I only measured the length, then that should be scalar quantity. On the other hand, if I have a line and not only the length but the direction if I also measure then that should be a vector quantity. So, here we are just concerned about the magnitude, here we also want to understand what is the direction.

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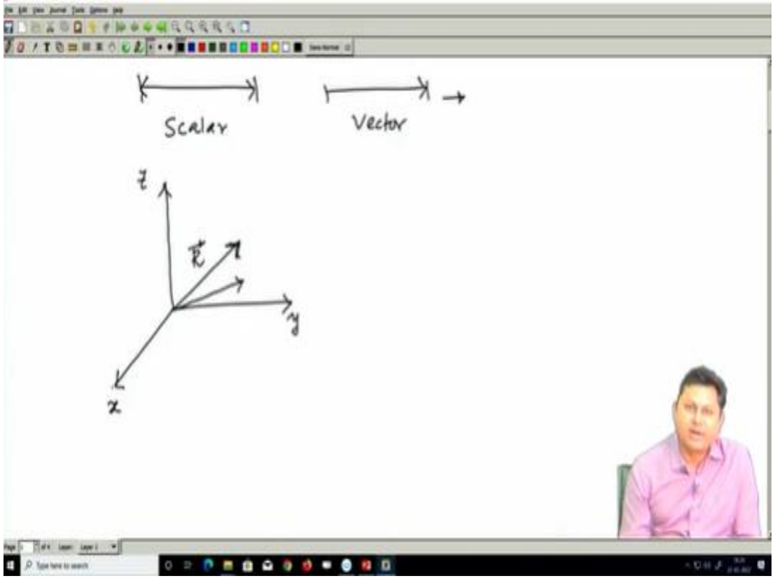

Since the directions are important, so, in vector quantity, when we define a vector quantity, we should have a coordinate system because, if you fix a coordinate system, then it will be easier for us to define a vector and then suppose this is a coordinate system, let us consider this as a Cartesian coordinate system having only x, y, z, and I am having a vector called, say, R vector and I always put this vector in this form like a straight line and showing that it is having a length not only that, it is having a distance also.

And it can change, this distance can change. There will be infinite time amount of change one can make as far as the distance is concerned, or as far as the direction is concerned. So, the direction is changing here. So, I am having a vector here the direction is changing. This is a vector, also this is a vector. So, vector is a quantity that is containing the magnitude as well as the direction.

And I can represent this vector to some coordinate system. I have a fixed axis, and on that axis, I can just put this quantity vector, which we called vector like this way. We will discuss later in this all these aspects of vector.

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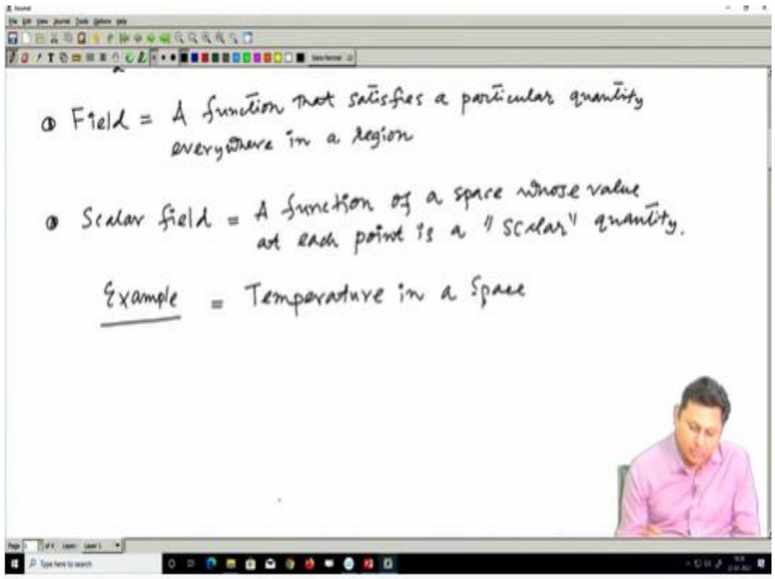

But before that let us understand the next thing, which is called the field. What is field? A function that satisfies a particular quantity everywhere in a region. So, I can have a region, and in this region, I can define a function, and this function can take whatever the value if possible. I can show you some examples then things will be clearer. So, field is eventually a function and that function can take different value in a given region.

At this moment, let us understand the field this way. Now, there are two kinds of field possible. One is scalar field and another is vector field. So, what is scalar field? A function of a space, say whose value at each point is a scalar quantity. So, I can define a field, which is a function that can take different values in a given region. And now I am specifying that a function in a space that is a function that depends on the different points in the space and the value of that particular function should be a scalar quantity. So, the example should be the temperature in the very famous example, temperature in a space.

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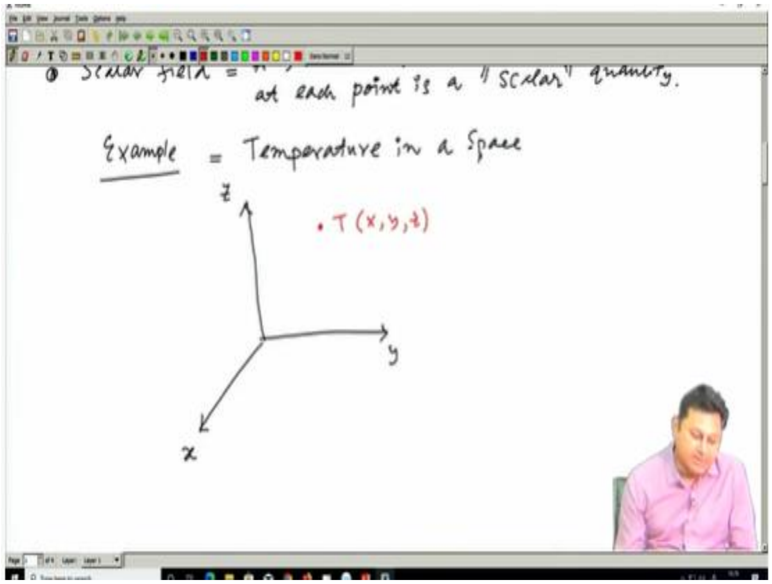

So, how it is become a scalar field let us try to understand quickly. So, I can have a coordinate system here x, y and z. And in this coordinate system, this is a space I am defining and different points I can define the temperature. So, for example, at some point here my temperature T, I can define and this is the value of x, this is a function of x, y, z, if I change my x, y, z, the value T the function will be going to change. But why it is a scalar field? Because temperature itself a scalar quantity. It should have a magnitude. There is nothing related to direction. So, that is why the temperature is an ideal example of a scalar field.

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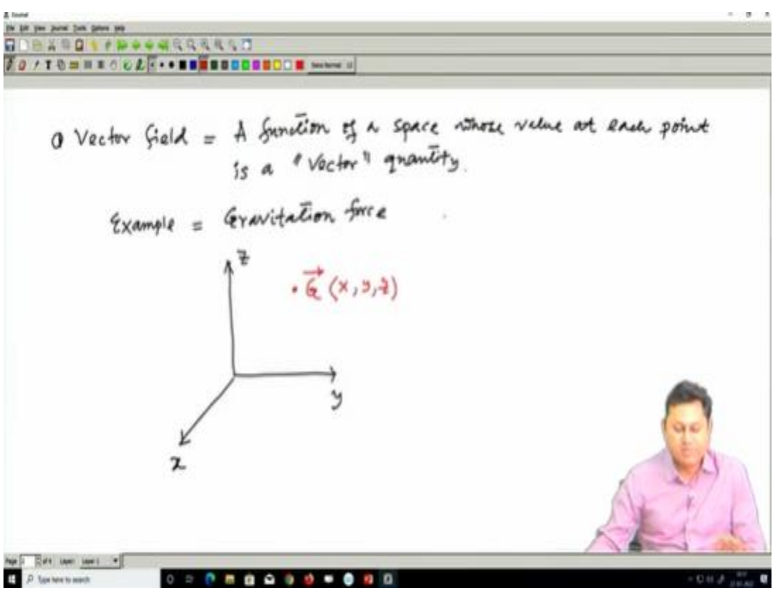

In the similar way, we can have vector field. What is vector field? A function of a space whose value at each point is a vector quantity. Example: gravitational force, very common example. So, again I can have a coordinate system defining a space say Cartesian coordinate system with x, y and z, and at any point I can have a gravitational force and this should be a vector quantity because it should have a direction as well.

So, it should be a function of x, y, z, but the only difference with the temperature is that it should have a value only. But here if I want to find out the gravitational force in this space, whatever the space I mentioned that this should be a vector quantity and that is why it becomes a vector field. So, scalar field and vector field are varying functions in one case the function should be a scalar function and in other case the function should be vector function. Let us try to understand these in this very simple way.

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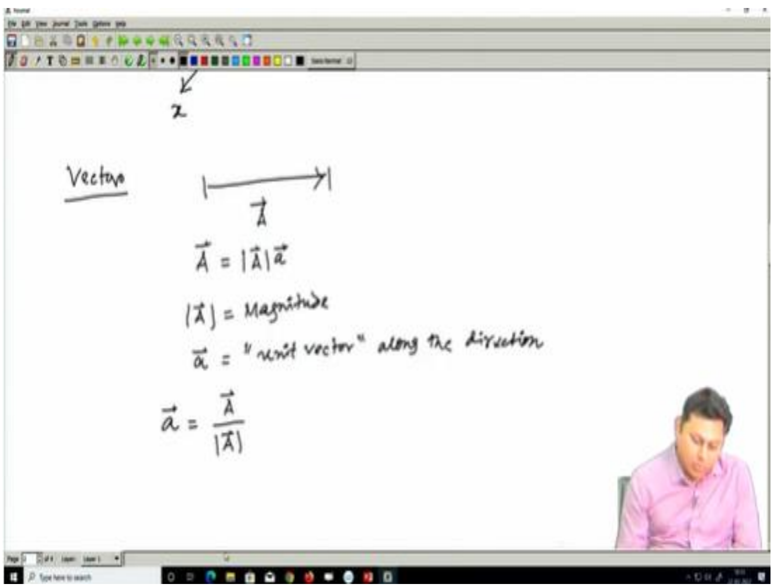

Well, then let us directly understand the details about the vectors the properties of the vectors. Already defined that vector is something, which should have a direction and magnitude. So, if I now represent these things in a space, then it should be like an arrow where the length of this arrow is the magnitude and direction of this arrow is the direction. These two quantities I am just supplying here and now I define this quantity the vector like A.

What is A? This vector A should contain the magnitude of these things I write is magnitude and also the direction I write a direction here. Now, this magnitude A, here A is the magnitude and a defining the direction and that should be a unit vector along the direction of the vector A defined. So, it should be unit vector along the direction. Okay. Fine. Now, if I want to find out what is this unit vector from this expression whatever the expression is written, I can simply write that unit vector is nothing whatever the vector is given to me then divided by the magnitude of that vector. So, that simply gives me the unit vector.

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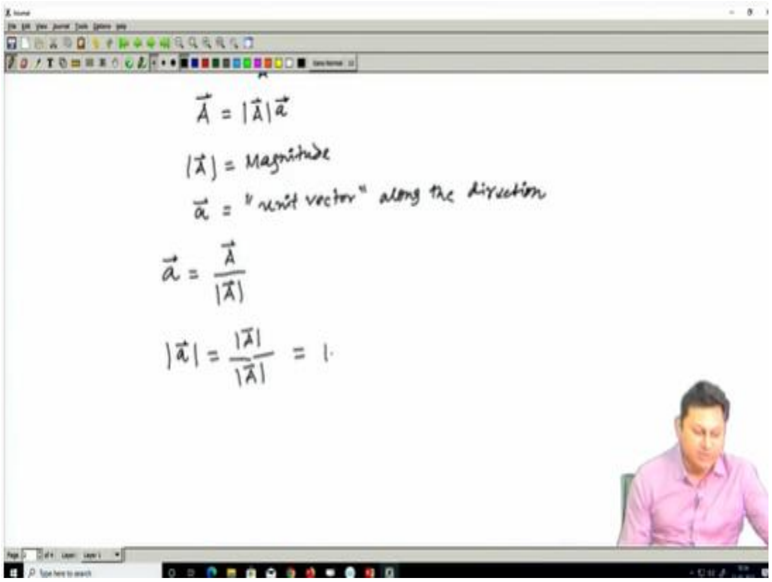

Now, what is the magnitude of the unit vector itself? So, the magnitude of the unit vector itself if I want to find out the magnitude, then I want to make the magnitude for both the side of this equation and gives me unity and that is quite obvious. Because since it is a unit vector, it should have a magnitude and this magnitude should be unit.

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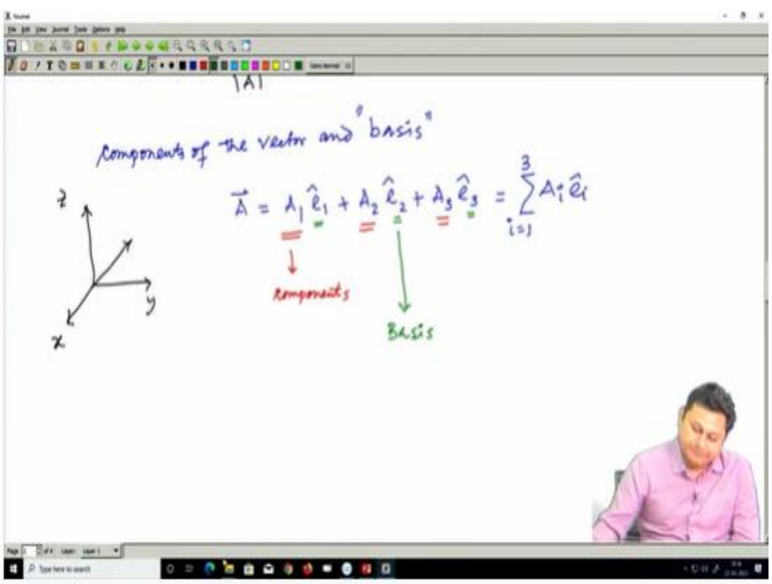

Next very important thing Next very important thing and that is components of the vector and basis, very important term basis. What do you mean by basis that I like to explain here, what is component of the vector and what is basis. So, I already mentioned that a vector one can define in coordinate system and if this is a coordinate system Cartesian coordinate system x, y, z, say x, y, z, I can define a vector like this.

Now, normally this is a three-dimensional system, but there is a possibility that I can also imagine the abstract space, where more dimensions are possible, still the vector is well defined there. So, let us now restrict ourselves into the three-dimension case. A vector is represented in this three-dimension coordinate system simply by the component with the combination of the component and basis. If I write it, it should be written in this way.

A_1, A_2, A_3, these are the components of the vector and e_1, e_2, e_3, these are the basis along these three directions x, y, z, as per as this figure. So, A_1, A_2, A_3, as I mentioned these are components. I can decompose a vector along these three directions and if I do, then I should have the components in these three directions we should call it as a projection. So, these are the components and these e_1, e_2, e_3, these are the basis. These are the basis. Fine.

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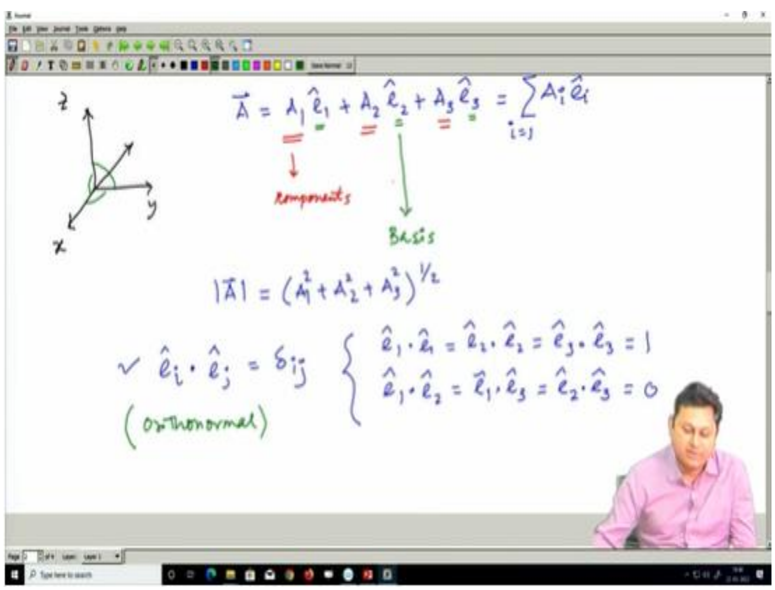

Now, if I want to find out the magnitude the way we define, then what should be the magnitude of this vector if it is given in this component form with basis. The magnitude should be simply the sum of all the component's square and root over of that. That should be the amount of the magnitude. Now, what are the properties of e_1, e_2, e_3 basis that is rather important here and that property is very interesting if we consider the coordinate system like this, this is a straightforward Cartesian coordinate system.

If we use the state for our Cartesian coordinate system, this basis should have certain relationship to follow and that is e_i dot $e_j = \delta_{ij}$. What is δ_{ij}? I will just discuss, but that means, if I now write in an expressive way it should be e_1 i j k can take the value 1 2 3, 1 2 3 means here x y z. So, e_1 dot e_1 that should be equal to e_2 dot e_2 that should be equal to e_3 dot e_3 these are the unit vectors so, their magnitudes are one that I mentioned that should be equal to 1.

And e_1 dot e_2 apart from one if I now make a dot product with any other component any other basis, e_1 dot e_3 or e_2 dot e_3, these are the possible three combinations one can have, then that should be 0. So, all this equation you can write at this stage as a simple equation using these δ function that e_i dot e_j is δ function. Now, this is true, this is not always true that all the basis should follow this. But this is true if the basis follows a property called orthonormal.

They are orthonormal that means they are perpendicular to each other like the way we have here in Cartesian coordinate system, they are perpendicular to each other that means this angle, this angle and this angle are 90 degrees. And the unit vector that we have the unit vectors here e_1, e_2 and they have the value 1. That is why it is called unit vector. So, this is called the orthonormal basis.

Orthonormal, So, orthonormal, this is the orthonormality condition. Normally we use this orthonormal condition always orthonormal basis always to you know to present a vector because it has certain advantages and we will discuss may be these advantages, but you should know that there may be some basis which are not orthonormal but, in that case, to represent a vector should be very very complicated. So, better later stick with this orthonormal basis where the e_i dot e_j δ_{ij} this condition is always followed.

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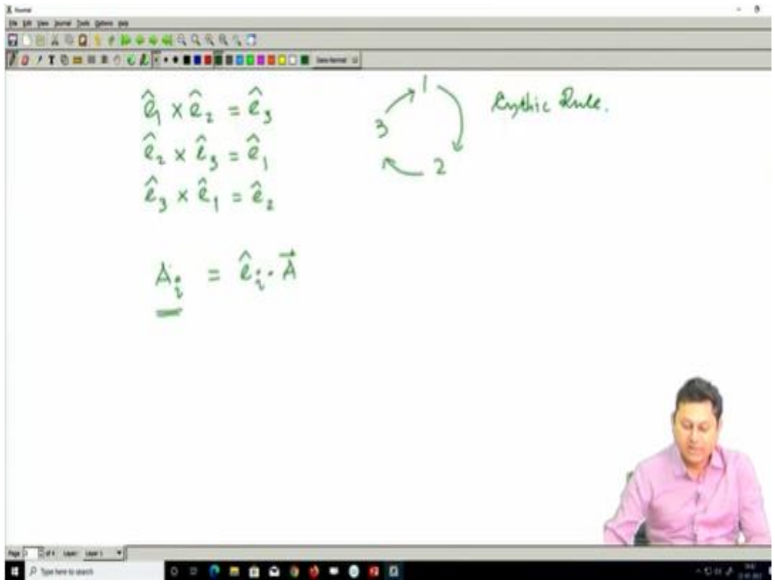

Now, not only that, we should have another properties of these. We have the dot product of 1 and 2 and 3, also we should have a relationship with the cross product and that relationship is called the cyclic relationship and that relationship says that if I have e_i and if I cross e_2 then it should be e_3, if I have e_2 cross e_3, it should be e1, and if I have e_3 cross e_1, it should be e_2. So, they are following a cyclic rule and that rule is like this, 1, 2, 3. So,1 cross 2, it will give you 3, 3 cross 1, it should give you 2, and 2 cross 3, it should give you 1. So, this is the cyclic rule. Okay.

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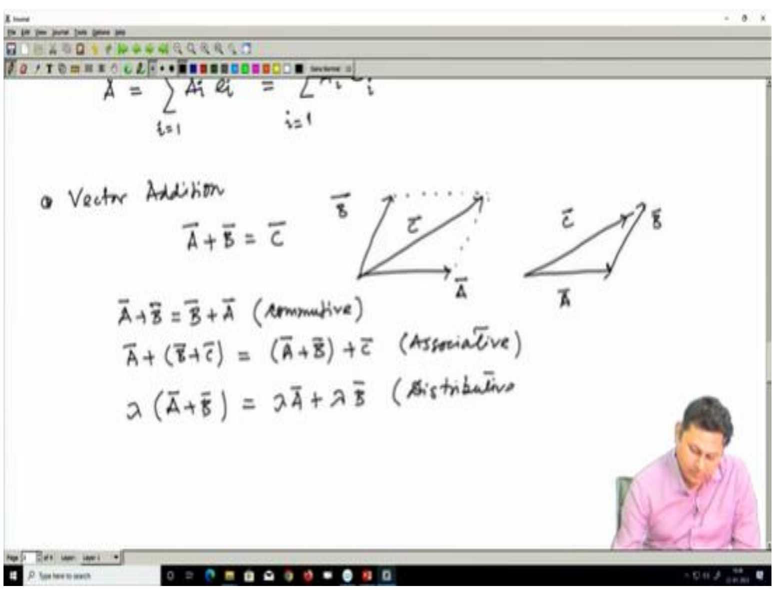

And also, I mentioned that if these bases are forming a orthonormal basis, then if I want to find out any component say ith component if I find, want to find, then I just need to have e dot A, if I make a dot product with the entire vector with the respective basis, then whatever we have that this component and it is to only when these bases are orthonormal. If it is not orthonormal then it will be very difficult to find this component. Okay.

So, also now, I like to mention that a given vector A can be decomposed in different basis. For example, here I am having the decomposition of A with one system, where the unit vectors are e_i but the same vector can be decomposed in the other basis also, let us consider this is a prime basis. So, obviously, there should be certain relationship between the e_1 and e' that we are going to understand, but before few more properties of the vector.

These are the well-known properties, but still, I think I should mention. So, 1 is the vector addition. So, vector addition suggests that A + B = C vector and we can add this vector with certain laws that we know that if this is my A vector and if this is my B vector, so, I can have this rule, this is a well-known rule I am not going to explain this here. So, it should be my resultant vector C.

Or I can have the A vector here, the triangle rule and this is my B vector. And if I joined from here to here, I will have my C vector. That is all. Apart from that vectors are also following certain very trivial rules, for example, A + B vector should be equal to B + A vector. These are the vector addition. This looks quite simply, but it is not always. So, these are called the commutative property.

Then also A + B + C whatever the vector you will get it should be A + B + C. You first add B, C and then add with the vector A, you will get the same result if you add A, B and then add C. So, this is called the associative. And if lambda is a scalar quantity and then you can have lambda multiplied by to a vector A + B then you will get the same result if I have this is called distributive. Okay,

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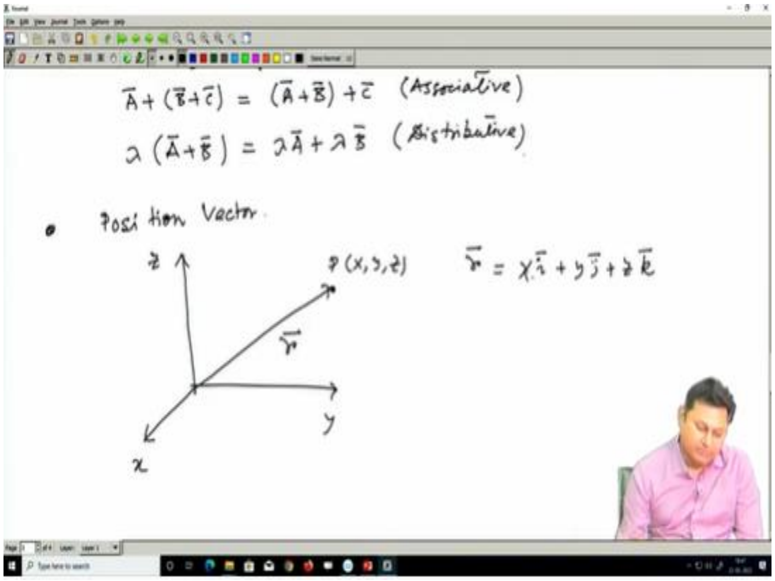

So, finally, today I like to just mention two very important vectors that we will discuss that will be you know useful, one is called the position vector. So, if in a coordinate system like x, y, z Cartesian coordinate system if I write any point at say p at x, y, z. This is defined with a vector called r and this r vector is called the position vector. Because any point here or here or here I can, I can define a r vector that basically gives me the position of that particular point.

And r vector is defined in this way xi + yj + zk, where i, j, k are the unit vectors along x, y, z. Normally, we write in i, j, k form. It can be also written e_1, e_2, e_3, but normally, we use for Cartesian coordinate system, we use i, j, k.

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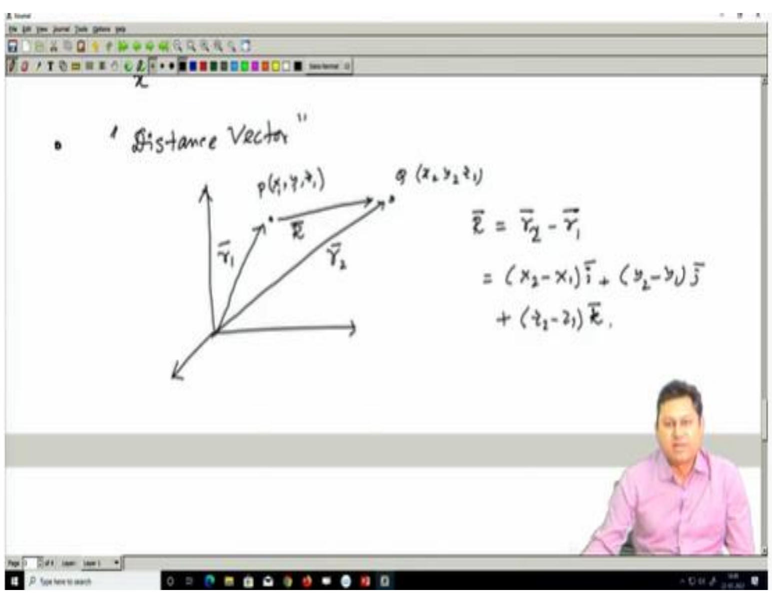

Well, another vector is also important, and that is called the distance vector, similar kind of vector but it gives me the distance between two points, the same coordinate system I had a coordinate system here and suppose I have a point here P and Q. P coordinate is x, y, z, say 1, 1, 1. x_1, y_1, z_1 and Q is x_2, y_2, z_2. Then the distance between these two points also gives us a vector. So, first I need to know what is the position vector? Say this is r_1. What is the position vector? This is r_2 for this point. So, what is the distance vector from here to here?

That is distance vector if I define this distance vector as say R, then R should be equal to $r_2 – r_1$, or in coordinate system should be $(x_2 – x_1)i + (y_2 – y_1)j + (z_2 - z_1)k$, so that should be the distance vector. So, with this note to it I do not have much time, so with this note I would like to conclude today's class. In the next class we will learn more about the vector operation and the different aspects of vector, so with that thank you for your attention and see you in the next class.