Thermal Physics
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Lecture - 57
Diffused Radiation and Kirchhoff's Law

Hello and welcome back to another lecture of this NPTEL course on thermal physics. Now in the last class we have, last lecture, we have started discussing about radiation, thermal radiation primarily, and we have already described few general features of radiation. That is every object above absolute zero which essentially means all the object that radiates some sort of thermal energy.

And that thermal energy is actually an electromagnetic wave. And we have also discussed the possible ranges of such wave that it will be of course not as high energy as a gamma ray, but definitely not as low energy as a microwave. So it will be somewhere covering the entire visible range near IR range and little bit of UV ranges as well. Of course, theoretically speaking, thermal radiation can have even larger range.

For example, it can go all the way up to far I mean, really short wavelength region, sorry yeah really short wavelength high frequency region all the way to microwave region. So that is why we choose integration limit of zero to infinity when we are integrating over all possible wavelengths. Also we talked about a concept of black body. Now what is black body?

Black body is a body which absorbs everything that falls on it and it will it is also an almost an ideal emitter. So it will have the highest emission power at any given wavelength or any given frequency at a given temperature of course. So we have discussed about all this and also how to realize black body in reality because black body is pretty much an idealized concept.

We have discussed that, there are different constructions and we will be primarily focusing on the Ferry’s black body which is actually a cavity, right. Now today let us continue with little more theory and then we will I mean we can develop a solid footing on which we will be able to understand many properties of this of thermal radiation, okay.

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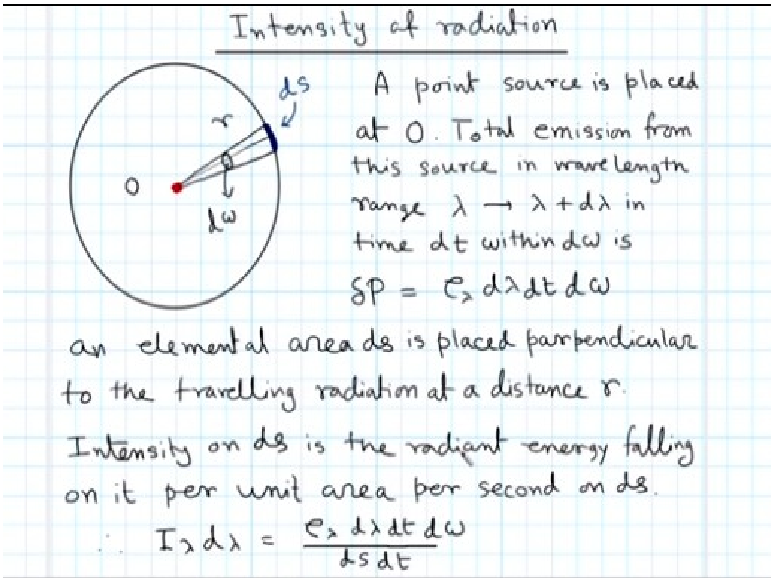

So let us look into the intensity of thermal radiation. Now what do you mean by intensity of radiation? Intensity is a point, let us assume that there is a point source at the middle, we call it O. This point source is a source of radiation. Now this source actually it emits in all the direction uniformly, which is also a nature of a black body radiation which we call a diffused radiation.

Now a point source by nature is a diffuse radiator. It will irradiate, it will radiate in all possible directions. That is to say it will radiate everywhere in this 4 pi solid angle, right. Now total emission from the source in the wavelength range lambda to lambda plus d lambda in time dt within a small solid angle d omega is dp or actually I should write yeah delta p is equal to e lambda d lambda dt times d omega.

So this is from the emission the definition of the emissivity or emission power of a object. Now an elemental area ds if we place this at a distance r on the surface of this sphere, and we have to place it such that it is in the I mean it is perpendicular to the radius that is joining this point O and the center of this elemental area ds.

So if we place such an element on the surface of this sphere, so the intensity on ds is defined as the radiant energy that is falling on that surface per unit area per unit time is called the intensity. And that intensity will be specific to this particular wavelength range lambda to lambda plus d lambda, right.

So we can write this as I lambda d lambda which is the intensity of radiation that is received by this elemental area ds at a distance r from I mean a distance r from the source and kept at a perpendicular position to the line joining the source and the center of this elemental area is I lambda d lambda is equal to e lambda d lambda dt d omega divided by dsdt.

Because, we already have, I mean we have to calculate number per second as well. And also this within this delta P there is a delta t term. So this delta t delta t nicely cancels out.

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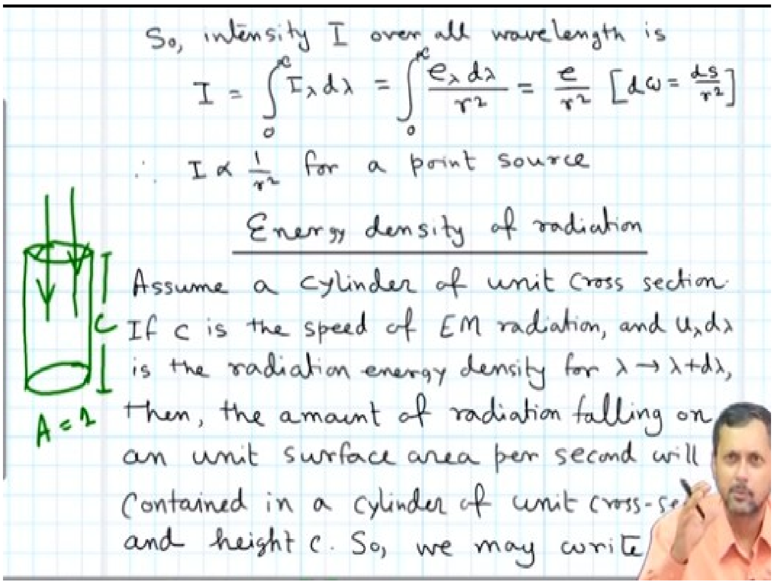

And also if we integrate for this entire solid angle, please we have to integrate over this entire solid angle or sorry not that. Remember the d omega is equal to ds divided by r square, right. So that is why if we just substitute d omega is equal to ds by r square and integrate over all possible wavelength range, then we get I the intensity at this point where the elemental area is placed.

Which is integration of 0 to infinity I lambda d lambda which is essentially e by r square e being the emissivity, total emission power of this object. So I is proportional to 1 over r square for a point source. And this is in general true for any spherical wave. I think we are already familiar with it.

If you have done an electromagnetism course, there also the same concept has been taught but in a maybe in a slightly different manner, right. The next concept is energy density of radiation. Now what is energy density of radiation? Assume a cylinder of unit cross section at this position once again. Now let us consider ds is equal to 1, right. Now by definition, energy density is the energy per unit volume.

Now what is the volume that the, so what is the amount of radiation energy that will be there inside or rather if c is the speed of the EM wave then and u lambda d lambda is the radiation energy density for this wavelength range lambda to lambda plus d lambda. Then the amount of radiation falling on an unit surface area per second will be contained in a cylinder of unit cross section and height c, okay.

So it is basically it is like I should not I do not need to put it at that elemental area, but let us say there is a uniform radiation coming in from any direction. Now please remember that assuming that r is sufficiently long, we can consider the radiation that is received at this element ds as uniform radiation, okay.

Now that is true because, for example sun is a spherical I mean it is all I mean from earth we can look at sun as if it is a point source. If not maybe an extended I mean solid extended source and we are really far away from it. So we can consider the sunlight that is coming to us as a uniform radiation and of course, it is not. Because we know that sunlight depending on which longitude, which latitude we live, depending on that the amount of radiation changes, right.

But for first approximation we can consider that as an uniform radiation, sunlight. So similarly, this point source, given that this point source is really far from ds, that means, r is considerably long, we can call it uniform radiation. Now any uniform radiation falling in and falling on an unit surface area that is a is equal to 1. And if I now draw a cylinder, imaginary cylinder that has a height c, c being the speed of light.

Now this height we know it is a really large number, okay. So that cylinder will contain a certain amount of radiation. Now if we want to calculate radiation energy density, if we know, I mean given that the wavelength of this radiation is known, and if we know the radiation energy density for this particular wavelength, then the relation of intensity and radiation will be straightforward.

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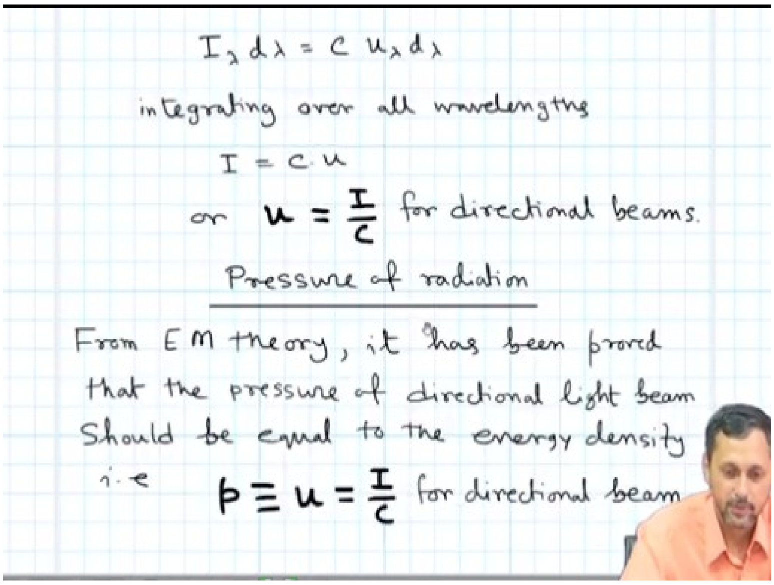

So let us have a look. If we have, sorry yeah so in one second, what is the amount of radiation falling in per unit area is the intensity. Now that is the energy density multiplied by c. That means the energy that is contained inside this cylinder, right. So we can write I lambda d lambda is equal to c times u lambda d lambda. And this is directly coming from the definition of electromagnetic wave energy density.

Because, you know because this energy density basically gives you the total amount of sorry amount of radiation per unit volume. And the volume of cylinder that can contain or rather the amount of radiation that will be falling on an unit surface area per second will be contained in a cylinder of height c and cross section area unity, if the radiation energy density is u lambda d lambda.

So that is how this relation comes. Now integrating over all wavelength, we get I is equal to c times u, which gives you u is equal to I by c for directional wave. Now please remember this relation is valid for directional beam only, okay. So if we have a diffused radiation, then we have an altogether different scenario, which we will be discussing later on.

Now from EM theory, the electromagnetic theory, Maxwell actually proved that for electromagnetic wave, the pressure of radiation has to be equal to its energy density.

So every radiation has a pressure. We will take it up in the next lecture, how to calculate the pressure of sunlight on earth. How to calculate you know for example, we can do that.

We can, if we know the intensity of a directional beam, we can very easily calculate the associated energy density and also, sorry we can calculate the pressure because of this relation that u is equivalent to p, right. So that is why Maxwell we are not going to prove that u is equal to p. So p is equal to u, sorry p is equal to I by c for direction wave, right.

So the pressure is equal to intensity divided by the velocity of light for a directional wave. And that gives you a very straightforward tool to compute radiation pressure under certain circumstances. But that is I mean that might that is not a general case because as we have already said, this is valid only for a directional beam and inside a cavity that we have discussed in the previous class, the black body radiation or the cavity radiation, this is a diffused radiation.

And other properties although these are diffused radiation, they are isotropic and homogeneous. But anyway isotropic and homogeneous means the energy density will be uniform at any point inside this radiation. But still we need to calculate the pressure for those type of radiation, okay.

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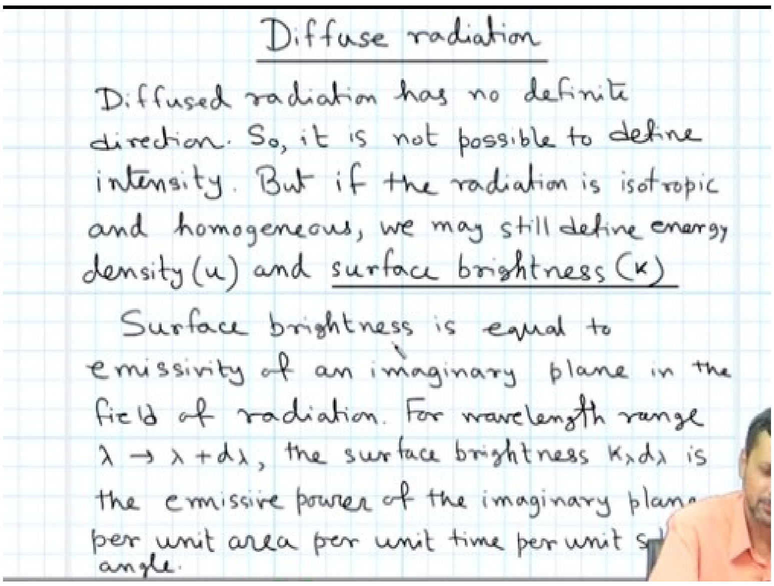

So let us focus on diffuse radiation. What is diffused radiation? This has no definite direction. So it is not possible to define intensity. Intensity is possible to define because we can place an object, sorry unit surface area and we can say that the energy that is incident on this particular surface on this particular surface area in unit time is the intensity of this radiation. But for diffused light what happens?

Radiation is coming from all possible directions and they are not directional at all, right? So we cannot really define this intensity, but we can define something called the surface brightness, which is somewhat equivalent to intensity. Now what is surface brightness? Surface brightness is equal to the emissivity of an imaginary plane in the field of radiation.

So we will take an example. We will or rather we will take the pictorial description of this very soon. But for now just understand that surface brightness is a substitute for intensity in case of diffused radiation. For wavelength range lambda to lambda plus d lambda, the surface brightness K lambda d lambda is the emissive power of an imaginary plane per unit area per unit time per unit solid angle.

And that is very important because this is diffused radiation. So the direction from which it is coming might also I mean or the rather the span it is through which it is coming that is defined by this solid angle, right.

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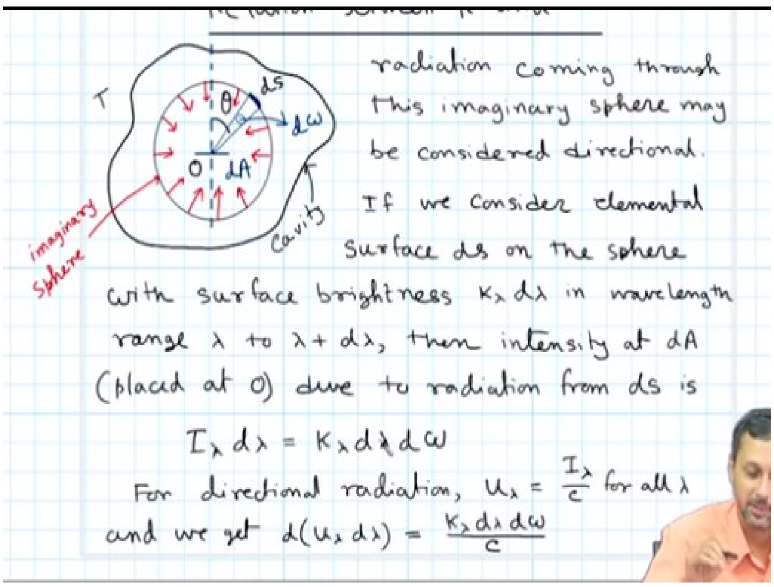

So now if we want to compute the relation between K and u, let us focus on this particular construction. So this is our cavity. Okay, so I think I will just use this, right. This is our cavity, this one, right. Now this cavity is at let us say some temperature T. So that is in thermal equilibrium. So what we do is we place a surface dA.

I mean, I am just drawing it as an open surface, but it could also be a closed surface, could be a small volume, some object here, which also I mean some volume. But still it will have some surface area, right? So it can be an open surface, it can be a closed surface. So just for general purpose, we are just calling it the elemental surface dA.

Only criteria is we consider dA to be small as compared to the area of this cavity. Now this cavity has radiation filled inside. I mean any cavity at any finite temperature will have a certain amount of radiation that is present inside it. Now in order to compute certain properties of this radiation we have to use, we will be using the same construction for the rest of the discussion for this class.

So please pay attention to it. So what we do is we assume that there is a imaginary surface, imaginary sphere that is close enough to this surface, elemental surface dA. Now let us assume that we have a diffuse radiation. Diffuse radiation means on one point it is coming from all possible directions. And there is for that fact we cannot even define the direction from which it is coming from.

Because it is so you know I mean basically it is everywhere. But if we assume, close to a point if we assume a very tiny volume right, close enough to the surface or enclosing this surface, enclosing this point if we assume a very tiny volume, then probably we can assume at least for this small really tiny area between the volume or between the sphere and the point that any radiation that is coming to that particular point is coming through the surface of the sphere.

And we can assume that there is an imaginary sphere which is the root cause of every radiation that is or rather the source itself is from this surface of this imaginary sphere. And any radiation that is coming through the source is actually on this sphere, right. So this is a I mean this is only an assumption. But this assumption works surprisingly well in order to simplify the calculation.

Because otherwise, if we do not make such assumption, we have no means to calculate what are the typical, I mean what are the parameters associated with this radiation. So basically the radiation that is falling on this surface, we are assuming that they are coming from, not from any not from the cavity, but it is coming only from the boundary or the wall of this sphere itself, okay.

So this is my assumption. And assuming that this distance is small as compared to the dimension of the cavity, the dimension of the sphere, this imaginary sphere is small enough, we can assume those beams to be directional in nature, right. So this is an imaginary object, okay. So it is an imaginary sphere and this red lines are the imaginary lines of radiation.

These are originating from the surface of this imaginary sphere. Now with this assumption, and assuming that the surface brightness of this sphere is K, which we will see later on that these are related to the somehow related to the emissivity of this wall itself, okay.

We can write, so what we can do is we can consider this elemental surface area ds on the surface of this imaginary sphere which subtend a solid angle d omega at the central point O of this surface, which has a area of dA. And this is at an angle theta with the normal direction of this surface okay. So this is the geometry of our choice.

Now once we can define this imaginary sphere now we are in a position to define intensity, although intensity in general cannot be defined for diffused radiation. But we assume that we are confining ourself within a small area around this elemental surface dA.

So that is why we can define the intensity and this intensity will be defined as I omega d omega is equal to K omega d omega times dw yeah, sorry I lambda d lambda is equal to K lambda d lambda times d omega. Why this d omega? d omega being, please remember that the surface brightness was defined per unit solid angle as well, okay.

So this is why it is important to multiply this with this elemental solid angle d omega so that you can compensate for the solid angle the for a I mean for every unit solid angle you have to multiply this with this the amount of solid angle it is subtending in the center of the sphere. Now for directional radiation we can write u lambda is equal to I lambda by c for all lambda.

That we have already proved, it is a general relation. And we can write for this small the radiation that is coming from this small area ds on the surface of this sphere, the energy density at dA, okay. So this is the energy density d of u lambda d lambda at dA due to this small surface ds is K lambda d lambda d omega by c, right.

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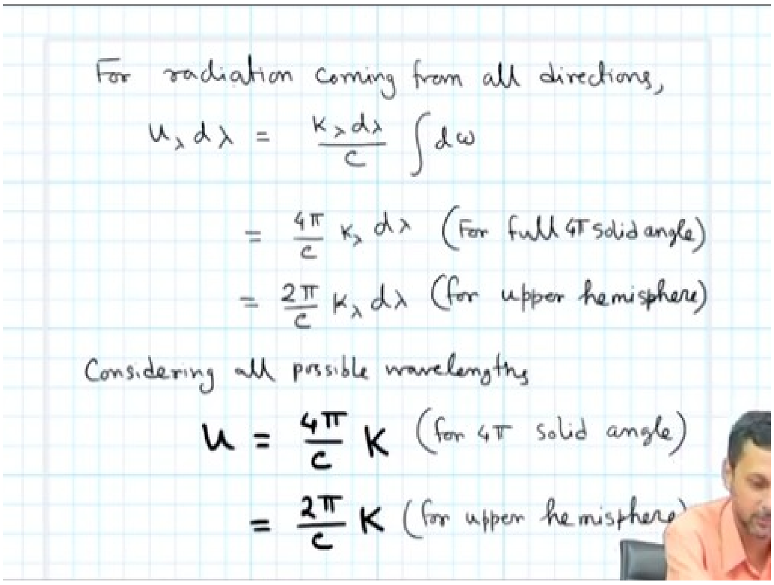

Next is for radiation coming from all directions we have to integrate it over the entire solid angle. Now for entire solid angle if we integrate, we should get either 4 pi if we consider this full 4 pi solid angle that means radiation coming from all sides.

Or in certain cases, we can also consider that okay this surface actually has only one I mean the radiation is coming from only one side and we can consider only the upper hemisphere, upper hemisphere of this imaginary sphere and this will be 2 pi by c K lambda d lambda. Once again we have to integrate it over all possible wavelength ranges as well and we can write u is equal to 4 pi by c times K.

That is for if we integrate it over this entire 4 pi solid angle. That means, radiation is coming from the top of the surface, from the bottom of the surface. Or what we can do is, for example if we just decide to take, okay so this is not exactly a open surface, but this is part of a closed object that is placed in the middle. If we consider that, then we have to consider maybe only the upper hemisphere for those type of calculation.

So in that case, we should get 2 pi by c times K. Either way, that is up to u, which how will you, how are you going to integrate this. But we can get those reference.

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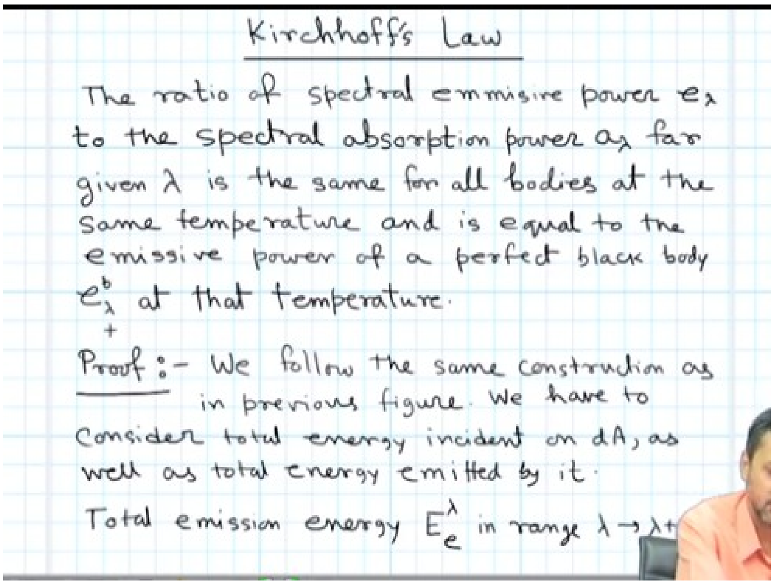

Next is we have to, we will learn a new law, probably not new to you, I mean you have already learned it during your plus two time, preparation for your joint IIT whatever you have learned this. The ratio of spectral emissive power e lambda to the spectral absorption power a lambda for a given lambda is the same for all bodies at the same temperature and that is equal to the emissive power of a perfect black body e lambda at that temperature.

Sorry, I should not write E lambda here because this will give you confusing, this one we should write e lambda b for black body, right. So we basically have to prove that for any object the ratio of e lambda by a lambda is a constant and then we have to prove that this constant is equal to e lambda or e lambda of a perfect black body, right. So we follow the same construction.

Basically what do we do, we basically follow the same construction once again. We assume that we are inside the inside a cavity. Our object in question this pA, this elemental surface is inside a cavity, which is filled with cavity radiation. And now what we need to do is we need to compute what is the amount of energy that has been absorbed by this elemental area.

And what is the amount of energy that has been emitted by this elemental area per unit time. And then we for thermal equilibrium we just have to compare or equate this to get a relation between the emissive power and the absorptivity, right. So we follow the yeah, so basically the total emission energy e lambda at, sorry E subscript e superscript lambda which gives you the total emissive power of this particular body in the wavelength range lambda to lambda plus d lambda.

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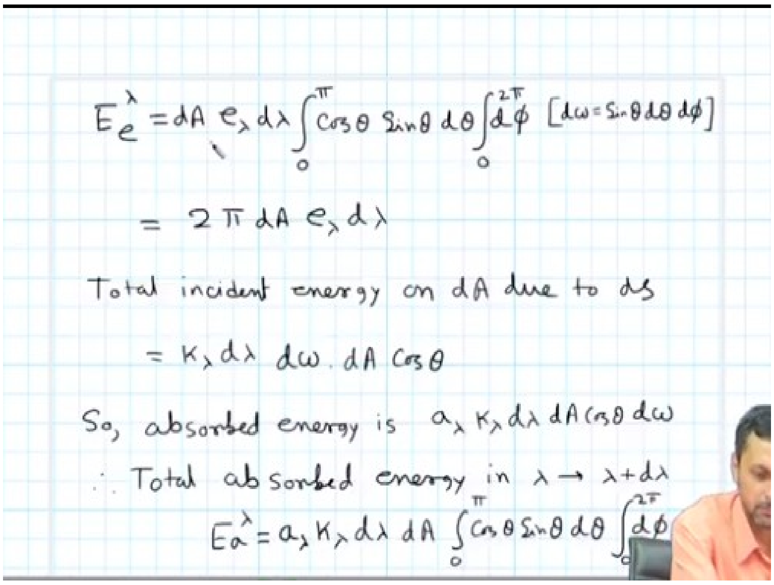

That is given by dA, dA being the total area of this body, e lambda d lambda that is the emissive power of that particular body, right? Then, we have to multiply this with a cos theta because this is already at an angle theta with this direction. So if we want to you know the emission that is being made by this body, okay. So it will go in the perpendicular direction, also it will diffuse in or it will go in other directions as well.

This is not a point object. So that is why we need to have a cos theta factor, which we will be giving you the angle perpendicular to the normal, sorry angle with respect to the normal of this particular surface. So for straight propagation theta will be equal to 0 and for oblique propagation we will have different values of theta. So instead of so basically we have to write dA cos theta and this cos theta goes inside this integral.

Remember that d omega is equal to sine theta d theta d pi. Because, if we simply integrate d omega over all the possible ranges of theta and phi, we should get 2 pi sorry 4 pi, right. So integrate sine theta d theta for in the range of 0 to pi. And integrate 2 pi, sorry d phi in the range of 0 to 2 pi, and you will see we will get 4 pi, right? So this is my elemental solid angle sine theta d theta d phi.

So we write, we multiply this with cos theta and integrate .So 0 to pi cos theta sine theta d theta and 0 to 2 pi d phi. Once again we are integrating over this entire surface assuming that radiation is coming, it is also radiating in all possible direction. So my assumption is not only it is receiving radiation in all possible from all possible direction, it is also radiating right, it is also radiating in all possible directions, okay.

I am just removing this one because I want to maintain this figure clean, right? Okay. So if this integration is performed, we get 2 pi dA e lambda d lambda. Now total incident energy on dA due to ds yeah? So once again we see how much energy is coming on to it and we have already calculated that will be K lambda, sorry we did not calculate that, but we have an idea that it will be K lambda d lambda d omega dA cos theta once again.

Why dA cos theta because the energy that is coming from this particular direction, it will experience I mean it will be falling on not on the surface, but it is or rather it will not be perpendicular to the surface, but it will be perpendicular, sorry it will fall at an angle theta with respect to the surface. So that is why there is a cost theta factor, right?

So once again the absorption energy will be this the incident energy and the absorption energy there is a factor of a lambda, right? So the energy that will be absorbed by this surface will be a lambda K lambda d lambda dA cos theta d omega. And total absorbed energy in this range lambda to lambda plus d lambda which is E subscript a superscript lambda will be a lambda K lambda d lambda dA cos theta sine theta d theta d phi integration will be on 0 to pi on theta and 0 to 2 pi on d theta.

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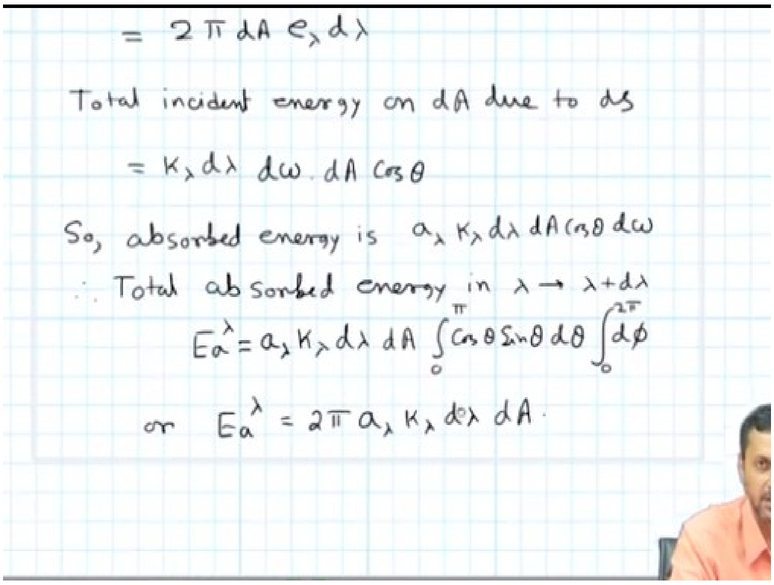

So if we compute this integration, we will see a lambda is equal to 2 pi dA, sorry 2 pi a lambda K lambda d lambda dA. Now in thermal equilibrium once again as I stated already this quantity here and this quantity here has to be equal. Now if we equate this we see dA cancels out 2 pi cancels out leaving behind e lambda d lambda is equal to a lambda K lambda d lambda. Also d lambda d lambda cancels out.

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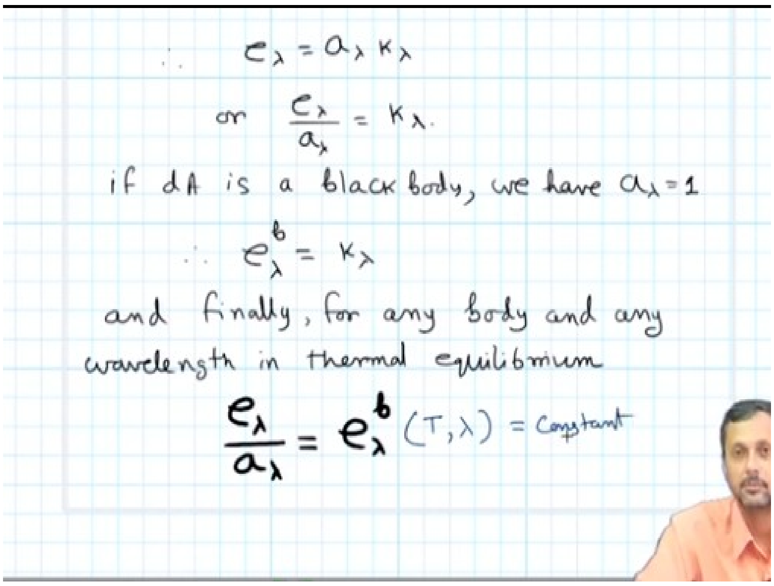

So just by equating we get e lambda is equal to a lambda K lambda or e lambda is equal or e lambda by a lambda which is equal to K lambda. Now what is K lambda? K lambda is the surface brightness of the imaginary surface in question. So now we assumed that this elemental area that we are considering, now let us assume that this is the black body.

Now if this is a black body we have a lambda is equal to 1 and K lambda will be equal to e lambda of b, right? So we can substitute K lambda for or e lambda of b which is the emissive power of a perfect black body for k lambda and we can write e lambda by a lambda is equal to e lambda superscript b. So this is the mathematical form of Kirchhoff's law, right.

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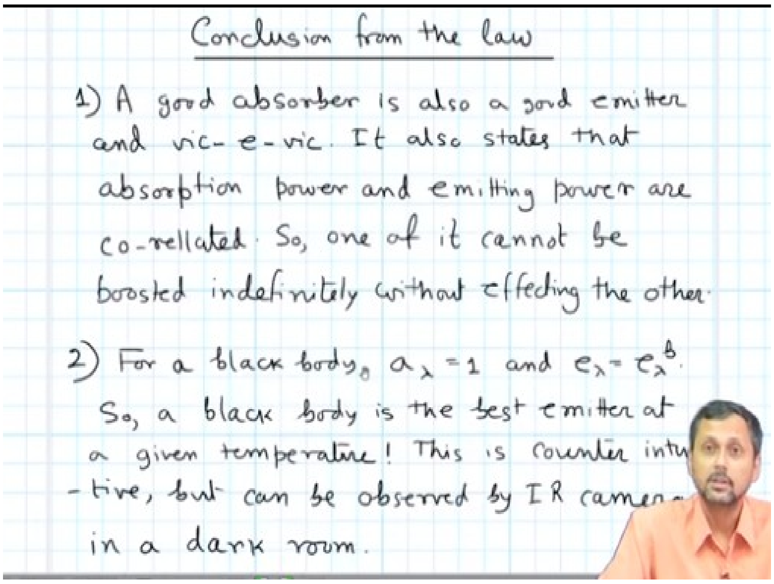

Now what are the implications, conclusion that we have from this law? I should not say conclusion, actually it is more of implication. So this is that says that a good absorber, and please remember the emissive power of a black body is a constant at a given temperature. Because all black bodies are identical. So if we take if, we specify a temperature and yeah black bodies they irradiate at all wavelength.

So e lambda b which is a function of specific temperature and wavelength is a constant. Please keep this in mind, okay. So this is not something that is very arbitrary. Because for all black bodies, these are identical. For all black body radiation patterns are also identical, we will see that later. So that is why this is a constant and e lambda by a lambda for any object is a constant.

Please remember in the general discussion for general purpose we do not consider the surface ds, sorry dA being a black body. So this is a normal object. Now, so the conclusion is a good absorber is also a good emitter and vice versa. So that means, if we have a material which absorbs it pretty well, that means it will also emit radiation, heat or radiation pretty well.

And that also means that we cannot really boost one of these properties without affecting the other. So if we boost the absorbing power of a surface, the emissive power will also be boosted automatically. Now for a black body also the second important outcome is for a black body a lambda is equal to 1 and e lambda is equal to e lambda b.

So a black body is the best emitter I mean so that means, from this relation we see if we put a lambda is equal to 1 right, so this means e lambda will be I mean maximum right? Because if we put any other quantity here, no actually that is not the good, not a good logic sorry. But that actually tells you, this equation actually tells you that the black body is the best possible emitter that you have in hand, right?

So at a given temperature of course. This is counterintuitive because black body is black. So we do not understand how it emits. But it can be proved by an infrared camera inside a dark room. So if we, inside a dark room nothing there is no other radiation coming in and if we have a black body placed inside the room, this will in a infrared camera this will have the maximum glow as compared to the background.

Or let us say there are other objects as well, but the black body will have the maximum glow in an IR camera because it radiates the best, emits the best, okay.

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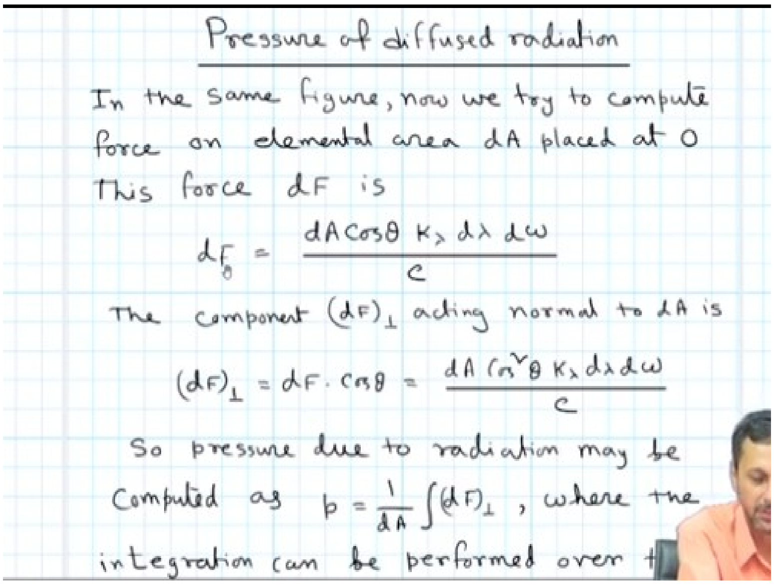

So the last topic of today, this is the pressure of diffused radiation. Now same construction, once again the same construction. We have this surface. We now want to compute the pressure on this particular area, particular surface area dA. And once again what we want to calculate the force of radiation that is coming from this small area ds and falling on this area dA and the force that is created by this radiation.

Please remember we can for this imaginary sphere being close to the surface, we can define intensity. If we can define intensity, then we can actually, we can actually write the force is equal to dA cos theta K lambda d lambda d omega by c, okay. So this is our, think about it, I am not going into the details of how to come to this particular expression. But this is very easy.

Now this force, this is this force is falling obliquely on the surface. So let us say this is the surface, so the force is at an angle, angle theta, right. So what we have to consider is the perpendicular component of this force. So if this is my surface. So the radiation is coming at an angle, so we have to only consider the perpendicular component. That means we have to multiply additionally with the additional cos theta factor here.

So dF perpendicular will be dA cos square theta K lambda d lambda d omega by c. So pressure due to radiation may be computed as p is equal to 1 over A integration over this dF perpendicular.

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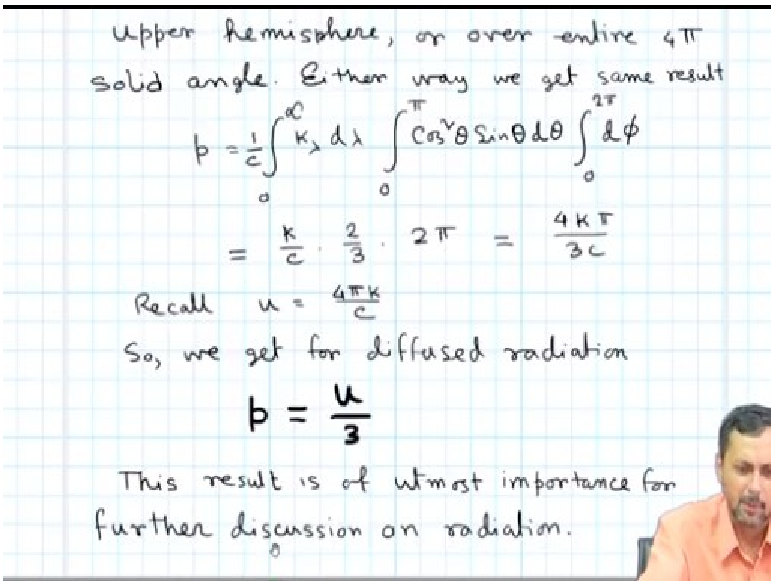

Where the integration can be performed over the upper hemisphere, or over the entire 4 pi solid angle once again, depending on what you are looking for. So either way, we get the same result. Because if we compute it over the entire solid angle, we get one relation here, or if we compute it over only the upper hemisphere, we get another relation here. But correspondingly, we will see.

So let us now what we did here, we just integrated it over the entire solid angle 4 pi solid angle, so the limit of theta is 0 to pi, this limit is 0 to 2 pi and integration limit over lambda is 0 to infinity. So we have K by c two third into 2 pi. So it is 4K pi divided by 2c. Now you will recall for a similar situation when we integrated over the entire 4 pi solid angle u was equal to 4 pi K divided by c.

And if we have integrated this one from 0 to pi by 2, we have to use u is equal to 2 pi K by c, okay. So both ways it will be compensated. Now just by comparison, u and expression for u and expression for p, sorry p and u, we see for diffused radiation, p is equal to u by 3. Now this result is a very important result and we will be keep using this result for the remaining discussion on radiation that will be coming in the next lecture onwards.

So we will stop here today. Next lecture, so far it has been all theory. Now we have one more theory to discuss that is, I should not say one more theory, but one more very important theorem to discuss or law to discuss that is Stefan-Boltzmann law.

Then we will try to treat this cavity radiation or black body radiation as assembly of ideal gas and we will see if we can calculate the quantities like entropy, free energy, and the nature of an isothermal or adiabatic change inside this ideal gas assembly. We will see that in the coming lectures. Thank you.