Thermal Physics
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Lecture-42
Topic-Efficiency of a Cycle From T-S Diagram

Hello and welcome back to another lecture on this NPTEL course on thermal physics. Now in the last class, we ended the lecture by talking about T-S diagram as equivalent of pv diagram

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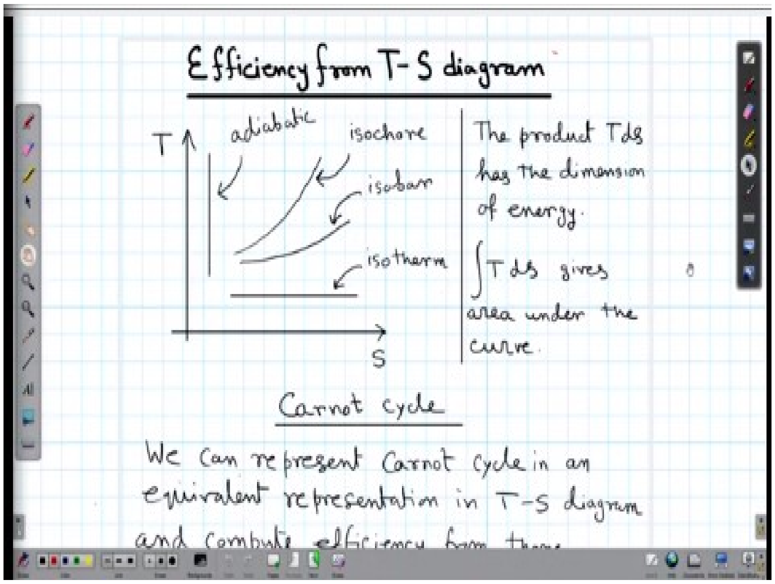

Efficiency from T-S diagram

The product Tds has the dimension of energy.

$\int T\,ds$ gives area under the curve.

Carnot cycle

We can represent Carnot cycle in an equivalent representation in T-S diagram and compute efficiency from there

So, and we have discussed like there are 4 types of main quasi static processes, how it occurs on a T-S diagram. Adiabatic process, because there is no exchange of heat, so the entropy change is literally 0, so that is why but the temperature changes, so this one has a 90 degree slope in this representation. Isothermal process, the temperature does not change but the entropy changes.

So, it has a horizontal as in 0 degree slope. So, this one is horizontal, this one is vertical. Isochore has or isobaric process has a slightly lower slope as compared to the isochoric process, because C p is greater than C v. Now and also please keep in mind that the product T ds has the dimension of energy. So, in this particular representation where we have entropy in the x axis and temperature in the y axis, if we integrate any of this curve from the start point to the end point, the area under the curve gives you the total heat exchange.

So, understandably the heat exchange under an adiabatic curve is 0 because there is no heat involved. Similarly, for an isotherm it will be simply a rectangle the area of the rectangle. Now if we want to represent Carnot cycle in this particular representation, where do we get?

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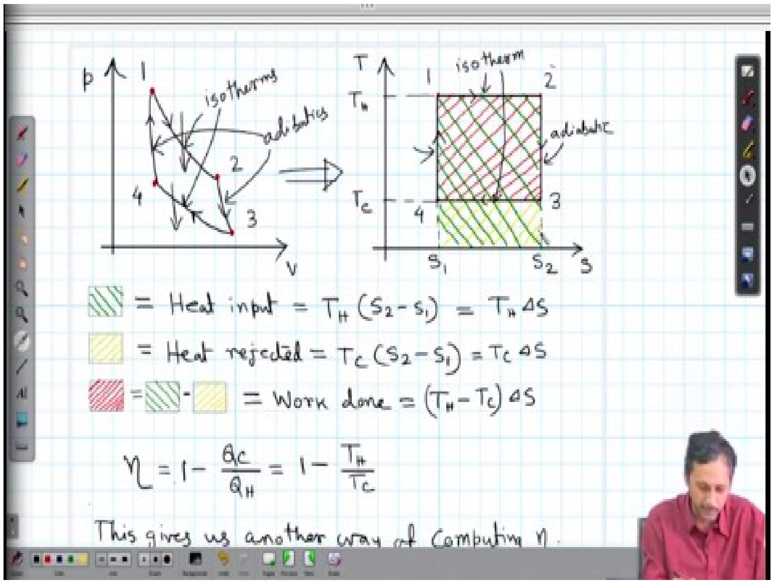

🟩 $= \text{Heat input} = T_H(S_2 - S_1) = T_H \Delta S$

🟨 $= \text{Heat rejected} = T_C(S_2 - S_1) = T_C \Delta S$

🟥 = 🟩 - 🟨 $= \text{Work done} = (T_H - T_C)\Delta S$

$$\eta = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_H}{T_C}$$

This gives us another way of computing η.

So, let us see without further delay, so to the left here we have the Carnot cycle in it is conventional representation where we have two isotherms, this one and this one and there are 2 adiabatics, this one and this one. So, we have the cycle, we can actually we should mark the direction of the cycle. So, we have 1 A was like 1 to 2, 2 to 3, 3 to 4 and 4 to 1. So, this is how the cycle goes, it first initially goes through a compression phase, 2 Step compression, then there are 2 steps expansion, heat comes in during the first isotherm and heat goes out during the last isotherm.

Now we have discussed that in details, we do not need to do it again. Here what we are going to do is we are going to represent the exact same diagram or exact same process into a entropy temperature diagram. Here as we have seen that there are two isothermals and two adiabatic processes and as we have discussed an adiabatic process will be a vertical line, whereas an isotherm will be a horizontal line.

So, the Carnot cycle will be a rectangular cycle in this particular case, because these 2 will be the two isotherm lines. Let us write it, so this one is one isotherm, this one is another isotherm, similarly this one is an adiabatic and this one is also an adiabatic. Now I have used different types of shading or different colour shading for different areas just to give you a better impression.

Here we can actually compute the exchange of heat for that isothermal process, adiabatic process from this diagram. But here it is even more straightforward. For example if we just look at the green shaded area, that is this large triangle, we can call it S 1, 1, 2, S 2. So, this larger triangle which represents area of this triangle actually represents the heat input. Now as we are operating between two fixed temperatures T H and TC represented by these isothermals are represented by 2 horizontal lines.

So, the heat input is actually this entire area which is simply T H times, so this is the height T H, the width will be 2 S 2 minus S 1. So, we do not know what is exactly the entropy change, we can definitely calculate that. Because the entropy changes are actually the entropy change due to this isothermal process, we can compute that. Actually you know this will be something like n R lon V 2 minus V 1, given that the volume is V 2 and p 1 but we do not need to do that.. So, we see heat input is simply T H times delta S. Now what is heat rejection? Heat rejected to the cold reservoir, this is given by this yellow shaded area.

So, it is S 1, 4, 3 S 2 the area of this rectangle, this is simply given by T C which is the height of this one times S 2 minus S 1. Now the work done in this case will be the separation between or the difference between the heat input and the heat rejected that means T H minus T C times delta S. And on this diagram we can represent this by this red shade area, so I have written just wanted to represent this pictorially.

So, I have written red is equal to green minus yellow, so this is the red area, so that is 1, 2, 3, 4 this is the work done which is given by T H minus T C times delta S. So, the efficiency calculation is straightforward, which will be eta is equal to 1 minus Q C by Q H. So, basically or rather it is even simpler, it will be simply W divided by Q, that is the heat input, either way we will get the familiar expression of 1 minus T H by T C. So, the representation of a cycle in a T-S diagram actually gives us an alternative way of computing the efficiency of a cyclic process.

Now also we have to keep in mind that we need to put the arrows in the proper direction. And a general convention is when a cycle goes in the clockwise direction then it becomes an engine. And when it goes along the anticlockwise direction, so in this case for example if it goes along this direction, so this will be the cycle of a Carnot refrigerator. So, Carnot engine or for the fact any engine goes into clockwise direction, whereas any engine or any refrigerator goes into the anticlockwise direction.

Also, and this is a very general property for any cycle in a T-S diagram that the area and inside the cycle whatever the cycle looks like, it could look like a proper circle, could be a rectangle, triangle, anything that is we cannot predict that. But for any arbitrary shape, the area enclosed inside this cycle will be the work done, whereas the area that is covered from the endpoints till the end. So, actually I should probably draw this that way will have a better feel for the thing.

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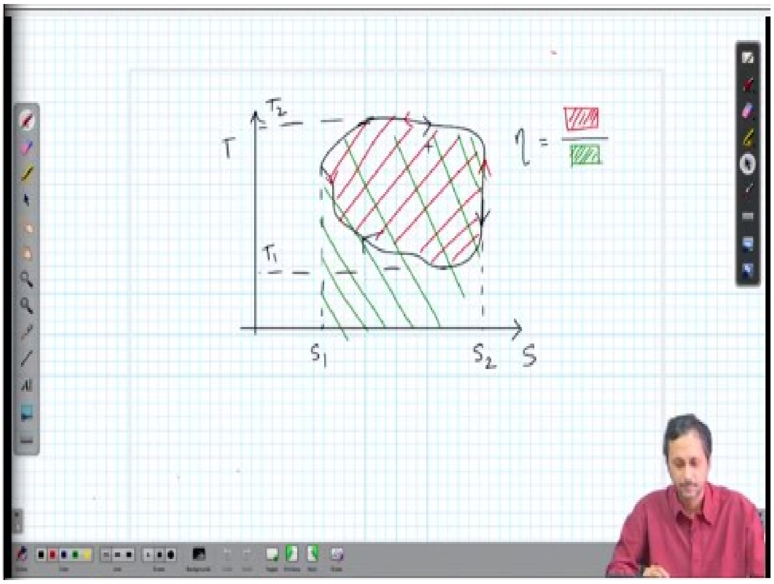

So, let us assume that this is my x axis, this is my y axis, so in this case, so this will be my S, T. So, we have any arbitrary cycle first of all if it is a engine then the cycle will go anticlockwise. So, the cycle will move in the anticlockwise direction. So, we have the two extreme points, whichever are the two extreme points, this will give us the maximum change in entropy and two extreme vertical points will give us the maximum change in temperature, let us call it T 1 or let us call it T 2 and let us call it T 1.

So, once again we have the same convention, the area under inside this circle this shaded area in red will be the work done. Whereas, the area which is enclosed by this S 1 this entire curve I will just once again shade it with green, this will give you the total heating. So, the efficiency eta will be what will be the efficiency? So, I will just do it in hand. So, we have a red box divided by green box, so we have this like this which essentially means the area that is shaded in green gives you the heat Q and the area that is shaded in red gives you the work done w.

So, efficiency is equal to W by Q. So, and this is general for any circle and if it is refrigerator then we can also compute the coefficient of refrigeration from this diagram once again. But in that case this will be in the opposite direction; it will go in the anticlockwise direction. So, clockwise is a in an engine clockwise rotation, anticlockwise rotation is a refrigerator, please keep that in mind. So, now what we are going to do is we are going to shift to classroom problems. So, I forgot to open the file, great. So, we have 1, 2, 3 and 4 problems, last problem is a T-S diagram.

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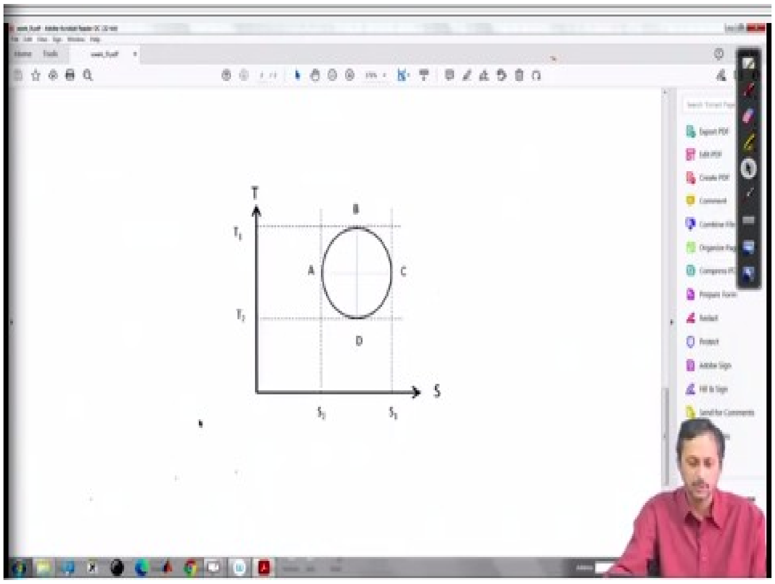

And the actual diagram has gone to the next page because of this space issue. But anyway what we need to do is we need to solve these 4 problems.

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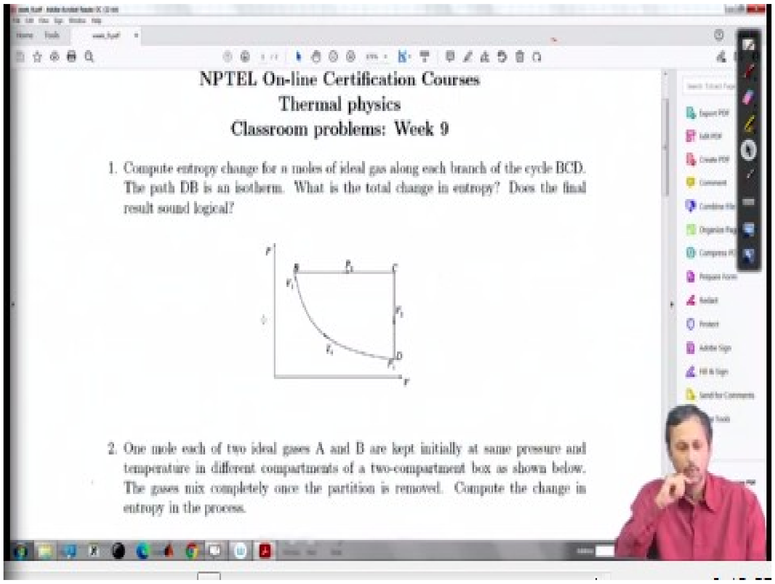

NPTEL On-line Certification Courses
Thermal physics
Classroom problems: Week 9

1. Compute entropy change for n moles of ideal gas along each branch of the cycle BCD. The path DB is an isotherm. What is the total change in entropy? Does the final result sound logical?

2. One mole each of two ideal gases A and B are kept initially at same pressure and temperature in different compartments of a two-compartment box as shown below. The gases mix completely once the partition is removed. Compute the change in entropy in the process.

The first problem we are given this particular diagram the cycle BCD, so what we have to do is we need to compute. So, this is for an n moles of an ideal gas and there are some markings like this is v 1, this is v 2, this is p 2, this is p 1, this is an isotherm, the path BD is actually an isotherm here, this path BD is an isotherm here. So, we have 1 isobar, 1 isochor and 1 isotherm, so it is a 3 stage cyclic process. We have to compute the entropy change along BC, entropy change along CD and entropy change along DB.

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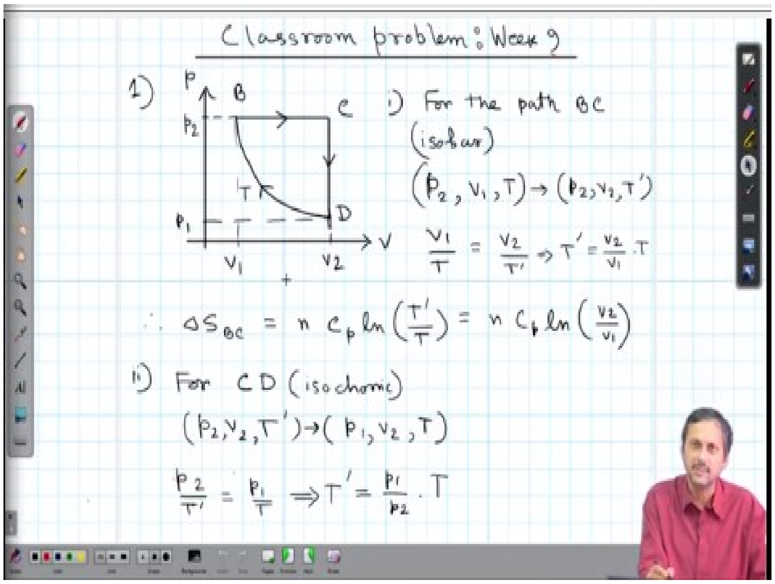

Classroom problem: Week 9

1)

i) For the path BC (isobar)

$(p_2, V_1, T) \rightarrow (p_2, V_2, T')$

$\frac{V_1}{T} = \frac{V_2}{T'} \Rightarrow T' = \frac{V_2}{V_1} \cdot T$

$\therefore \Delta S_{BC} = n\, c_p \ln\left(\frac{T'}{T}\right) = n\, C_p \ln\left(\frac{V_2}{V_1}\right)$

ii) For CD (isochoric)

$(p_2, V_2, T') \rightarrow (p_1, V_2, T)$

$\frac{p_2}{T'} = \frac{p_1}{T} \Rightarrow T' = \frac{p_1}{p_2} \cdot T$

So, let us try that. So, for path BC which is an isobar, we start from p 2, V 1, T and ended p 2, V 2, T prime. So, let us call the temperature, so temperature of B and temperature of D are on the same isotherm. So, the temperature has to be same as the point B and D are on the same isotherm. So, let us call this temperature T and D and let us call the temperature of this point as T prime.

So, we see it is a isobar, so because it is an ideal gas V 1 by T is equal to V 2 by T prime, which gives us T prime is equal to V 2 by V 1 times T. Now delta S BC which is n times C p ln T prime by T, please remember the formula we have derived in the last class. So, not last, last to last class I think that is where we have computed the entropy change for a general process. Now here we have to use C p because it is an isobaric process and the temperature change is from T to T prime, so it will be C p ln T prime by T the whole multiplied by n.

And given that T prime by T is equal to V 2 by V 1 we can write this as n C p ln V 2 by V 1. Now for CD this path it is an isochoric process. So, instead of C p we have to write C v and first let us compute that changes. So, we have p 2, V 2 and T 2, we will come down to from C to D, it will change to p 1, V 2 and T. So, p 2 by T 2 prime is equal to p 1 by T which gives you once again T prime = p 1 by p 2 times T.

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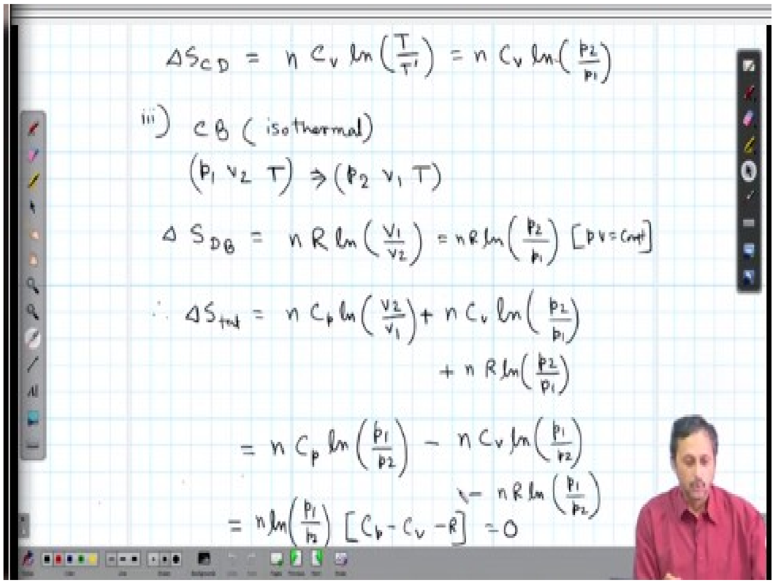

$$\Delta S_{CD} = n C_v \ln\left(\frac{T}{T'}\right) = n C_v \ln\left(\frac{p_2}{p_1}\right)$$

iii) CB (isothermal)

$(p_1\ V_2\ T) \Rightarrow (p_2\ V_1\ T)$

$$\Delta S_{DB} = n R \ln\left(\frac{V_1}{V_2}\right) = nR\ln\left(\frac{p_2}{p_1}\right)\ [pV = \text{Const}]$$

$$\therefore \Delta S_{tot} = n C_p \ln\left(\frac{V_2}{V_1}\right) + n C_v \ln\left(\frac{p_2}{p_1}\right) + n R \ln\left(\frac{p_2}{p_1}\right)$$

$$= n C_p \ln\left(\frac{p_1}{p_2}\right) - n C_v \ln\left(\frac{p_1}{p_2}\right) - n R \ln\left(\frac{p_1}{p_2}\right)$$

$$= n \ln\left(\frac{p_1}{p_2}\right)[C_p - C_v - R] = 0$$

So, delta S CD will be nC v ln T by T prime which will be equal to nC v ln p 2 by p1. And lastly this is an isotherm, so we will go from p 1, V 2, T to p 2, V 1, T, so delta S DB is equal to nR ln V 1 by V 2. Because the final volume is V 1, initial volume is V 2, so we have nR ln V 1 by V 2.

Once again this will be equal to nR ln p 2 by p 1 because p times V is a constant, we know it is an isotherm.

So, V 1 by V 2 is equal to p 2 by p 1. So, all we have to do is, now we are looking at this total entropy change. So, we have nC p ln V 2 by V 1 plus nC v ln p 2 by p 1 plus nR ln p 2 by p 1, all we have to do is we have to turn everything in terms of either p 1 by p 2 or V 1 by V 2. So, let us just compute, so instead of V 2 by V 1 if we write p 1 by p 2, then it becomes easier. So, we have nC p ln p 1 by p 2 plus now it will be minus because it is p 2 by p 1, so we write minus n C v ln p 1 by p 2.

Once again this is also minus; there will be a minus sign here because it is nR ln p 2 by p 1. If we write p 1 by p 2 it will come with a minus sign. Next we take n lon p 1 by p 2 common, so we have C p minus C v minus R. Now, what is C p minus C v? C p minus C v is R and it will be nothing but R minus R which is equal to 0. So, the final answer the total entropy change is equal to 0 now. So, let us go back to the problem, does the final result sound logical?

So, the total entropy change going from B, C, D all the way back to B is 0 and it is absolutely logical. Because entropy is a state function going through any cyclic process means the entropy change has to be equal to 0. So, this is what we expect, but we just showed that by calculating separate individual segments of the cycle and we have found 0 as expected.

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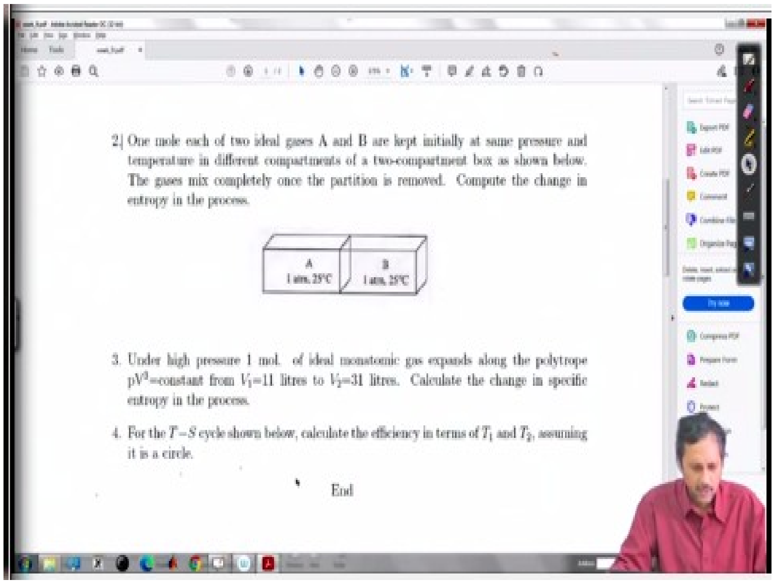

2. One mole each of two ideal gases A and B are kept initially at same pressure and temperature in different compartments of a two-compartment box as shown below. The gases mix completely once the partition is removed. Compute the change in entropy in the process.

3. Under high pressure 1 mol. of ideal monatomic gas expands along the polytrope pV^3=constant from V_1=11 litres to V_2=31 litres. Calculate the change in specific entropy in the process.

4. For the $T-S$ cycle shown below, calculate the efficiency in terms of T_1 and T_2, assuming it is a circle.

End

Now, problem number 2, in this problem we have 1 mole each of 2 ideal gases A and B are kept initially at same pressure and temperature. So, pressure and temperature it is 1 atmosphere and 25 degree Centigrades. Now, there is a partition between those and then what do we do? At some time we just remove the partition and we allow the gases to mix completely. We have to compute the change in entropy in this process. Now, what is the relation or what is the formula we have derived in the last class for change in entropy?

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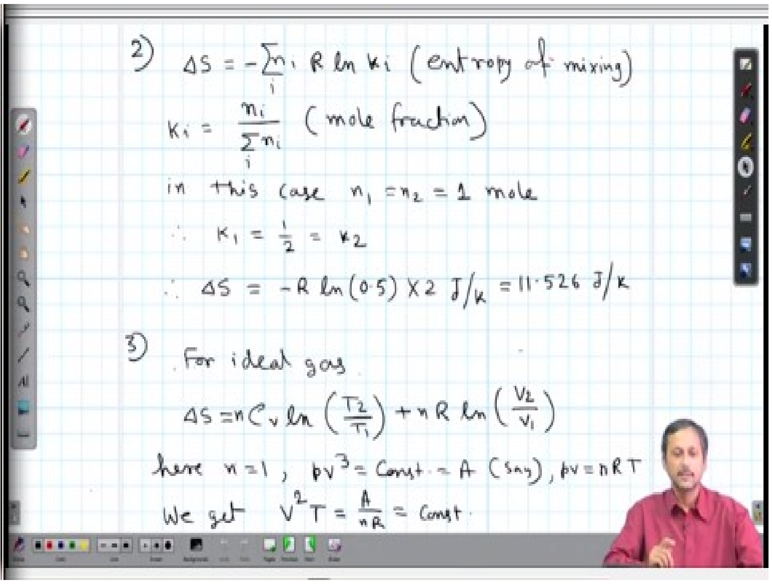

2) $\Delta S = -\sum_i n_i R \ln k_i$ (entropy of mixing)

$k_i = \frac{n_i}{\sum_i n_i}$ (mole fraction)

in this case $n_1 = n_2 = 1$ mole

$\therefore k_1 = \frac{1}{2} = k_2$

$\therefore \Delta S = -R \ln(0.5) \times 2 \; J/K = 11.526 \; J/K$

3) For ideal gas

$\Delta S = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right)$

here $n=1$, $pV^3 = \text{Const} = A$ (say), $pV = nRT$

We get $V^2 T = \frac{A}{nR} = \text{Const}$

delta S is equal to minus sum over i n i R ln k i, where k i is actually the mole fraction which is n i divided by sum over n i. So, in this case we have 1 mole of each of these cases, so n 1 = n 2 = 1 mole, so k is equal to half is equal to k 1 is equal to half is equal to k 2. So, delta S is simply minus R ln 0.5 into 2 Joules per Kelvin. So, for 2 moles of gas mixed together, we have the final entropy of mixing as 11.526 Joules per Kelvin, actually we should write this as mole.

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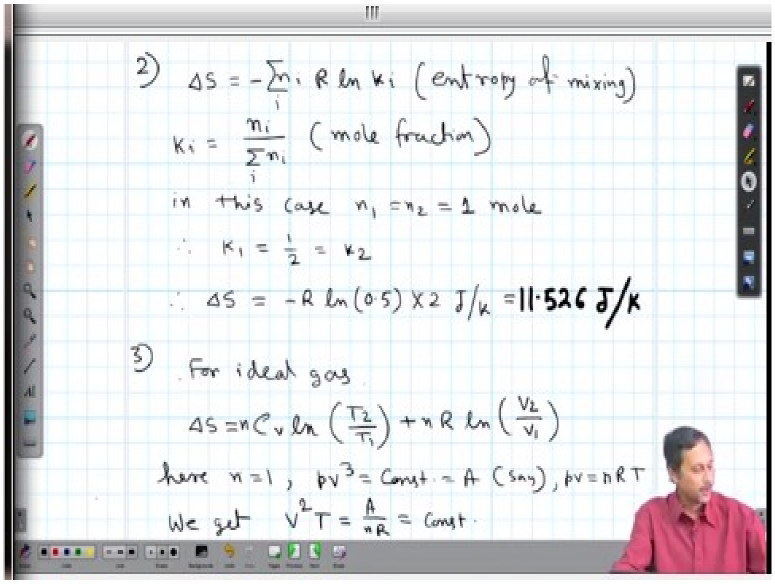

2) $\Delta S = -\sum_i n_i R \ln x_i$ (entropy of mixing)

$x_i = \frac{n_i}{\sum_i n_i}$ (mole fraction)

in this case $n_1 = n_2 = 1$ mole

$\therefore x_1 = \frac{1}{2} = x_2$

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3) For ideal gas

$\Delta S = nC_v \ln\left(\frac{T_2}{T_1}\right) + nR \ln\left(\frac{V_2}{V_1}\right)$

here $n = 1$, $pV^3 = \text{Const.} = A$ (say), $pV = nRT$

We get $V^2 T = \frac{A}{nR} = \text{Const.}$

So, we should write 11.526 Joules per Kelvin, so this is our final result. Problem number 3, under high pressure 1 mole of ideal monoatomic gas expands along the polytrope p V cubed is equal to constant from V 1 is equal to 11 liters to V 2 is equal to 31 liters. Calculate the change in specific entropy in this process. Now when we are considering 1 mole of ideal gas specific entropy is typically specified per unit mole or unit volume or unit mass, whatever is expected.

Now here nothing has been said about the molecular weight, so we cannot really compute it in terms of masses or volume. So, and the volume is also changing, so specific entropy per unit volume is not the idea. So, all we have to do is we have to compute the total entropy change and that change is for 1 mole of ideal gas, so that is the specific entropy change per mole. Now in order to do so, we remember that for ideal gas the relation that we have derived in the last class is delta S is equal to nC v ln T 2 by T 1 plus nR non V 2 by V 1. Now here n = 1 and the relation that we must follow is p V cubed is equal to constant.

And also remember, because it is an ideal gas we have and we just call it A this constant. And we also do not forget that pv = nRT. Now we have 2 relations one is p v cubed is equal to A and we have pv = nRT, from there we can get V square T. So, if I divide for example this relation with this relation, we get see p, p cancels out and then it will be 1 over V square and there will be a T in the numerator. If we rearrange we get V square T is equal to A by nR which is once again a constant. So, that means T 1 by T 2 this or whatever we have here we have T 2 by T 1 will be V 1 by V 2 whole square.

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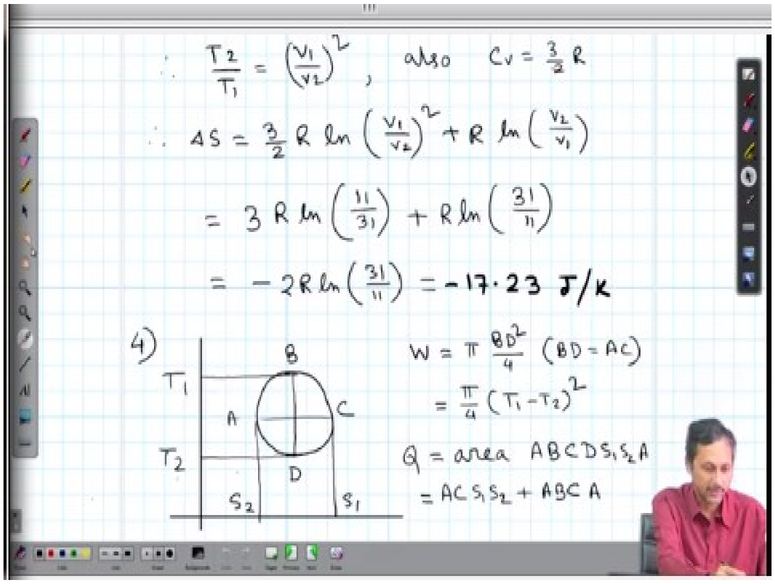

$$\therefore \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^2, \quad \text{also} \quad C_v = \frac{3}{2}R$$

$$\therefore \Delta S = \frac{3}{2}R \ln\left(\frac{V_1}{V_2}\right)^2 + R \ln\left(\frac{V_2}{V_1}\right)$$

$$= 3R\ln\left(\frac{11}{31}\right) + R\ln\left(\frac{31}{11}\right)$$

$$= -2R\ln\left(\frac{31}{11}\right) = -17.23 \text{ J/K}$$

4)

$$W = \pi \frac{BD^2}{4} \quad (BD = AC)$$

$$= \frac{\pi}{4}(T_1 - T_2)^2$$

$$Q = \text{area } ABCDS_1S_2A$$

$$= ACS_1S_2 + ABCA$$

So, that is exactly we have computed here. So, T 2 by T 1 is equal to V 1 by V 2 whole square, also we have to keep in mind that C v is equal to 3 half R because it is an ideal monatomic gas. So, delta S, so we put the values in this formula we have n is equal to 1, C v is equal to 3 half R, this one and this one both will be given in terms of the volume V 2 and V 1. So, we write 3 half R ln V 1 by V 2 whole square plus R ln V 2 by V 1.

So, we have 3 R ln 11 by 31 which is V 1 by V 2 and please remember there is a 2 here, square. So, these 2 will go in front will get cancelled here and we have 3, this term and then we have R ln 31 by 11. So, 31 by 11 will be positive, whereas ln 11 by 31 will be negative. So, what we do is we simply put a minus sign here, write this as minus 3 ln 31 by 11 whole multiplied by R and this will be R ln 31 by 11, so it will be minus 2R ln 31 by 11 which gives you a value of minus 17.23 Joules per Kelvin.

Now this negative sign implies that during the process heat is rejected by the gas. So, that means the entropy of the gas assembly when expanding from the initial volume of 11 liters to final volume of 31 liters under this particular condition, it is actually rejecting a certain amount, so that the entropy is decreasing. So, now we come to the 4th and the last problem of today's lecture. So, for the T-S cycle below, calculate the efficiency in terms of T 1 and T 2, assuming it is a circle. So, let us look at this diagram.

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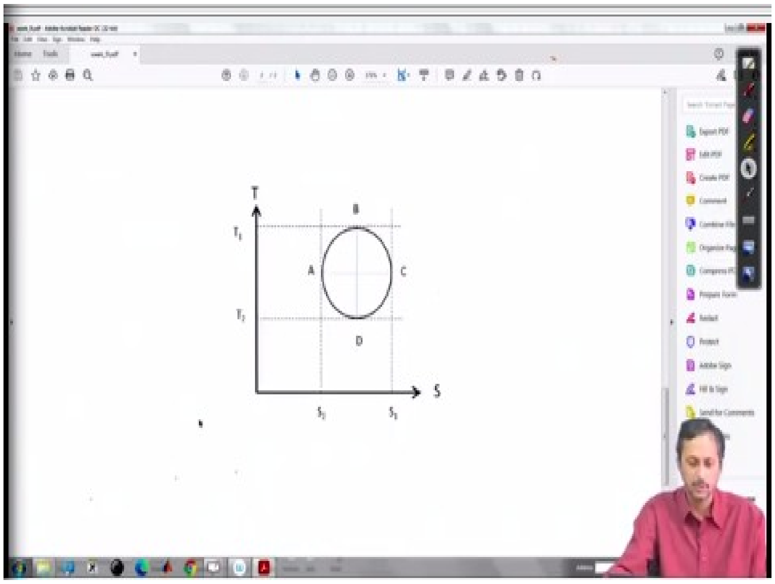

So, we have a circle ABCD, so we need to calculate the efficiency. Now as it is already said that it is a circle, so that means the length is AC is equal to the length BD. So, what we need to do is, we need to compute the area of this path S 2, A, B, C, S 1. The area under this particular curve and then we had to calculate the area of this circle both in terms of T 1 and T 2. And then we just have to take the division of these 2 numbers which will give us the efficiency.

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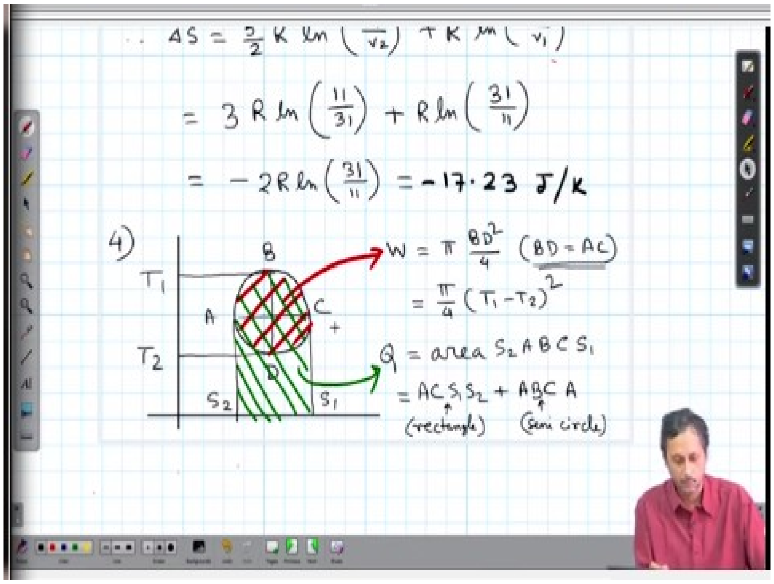

$$= 3R\ln\left(\frac{11}{31}\right) + R\ln\left(\frac{31}{11}\right)$$

$$= -2R\ln\left(\frac{31}{11}\right) = -17.23\ \text{J/K}$$

$$W = \pi\frac{BD^2}{4} \quad (BD = AC)$$

$$= \frac{\pi}{4}(T_1 - T_2)^2$$

$$Q = \text{area } S_2ABCS_1$$

$$= \underset{\substack{\uparrow\\(\text{rectangle})}}{ACS_1S_2} + \underset{\substack{\uparrow\\(\text{semi circle})}}{ABCA}$$

So, once again let me tell you this will be my work done W, so this is W. And this area is my heat Q, W straightforward it is BD square by 4 because the BD is equal to AC, we can either take this length or this length. So, S 2 minus S 1 and T 2 minus T 1 has the same magnitude, so we write pi by 4 T 1 minus T 2, so this W is equal to pi by 4 T 1 minus T 2 whole square. Q is the area ABCDS 1 S 2 A, so not B, my mistake ABC it should be actually S 2 ABCS 1.

Now this is a combination of a rectangular area which is ACS 1 S 2. So, basically this rectangle over here plus the semicircle ABCA. So, this is the semicircle ABCA, so this one I should actually write rectangle and this is semi circle. So, now I hope it is clear to you, so we need to compute the area of these 2 individuals, add them together, so that we get the total heat input.

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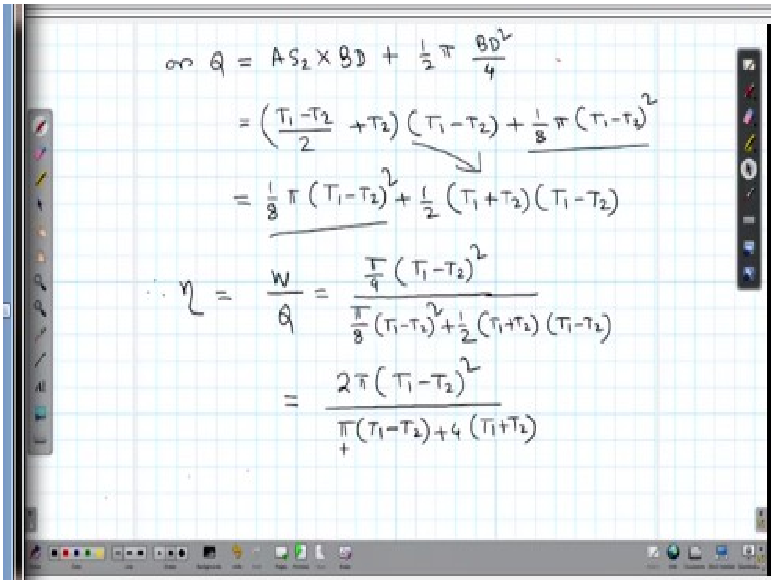

$$\text{or } Q = AS_2 \times BD + \frac{1}{2}\pi \frac{BD^2}{4}$$

$$= \left(\frac{T_1 - T_2}{2} + T_2\right)(T_1 - T_2) + \underline{\frac{1}{8}\pi(T_1 - T_2)^2}$$

$$= \underline{\frac{1}{8}\pi(T_1 - T_2)^2} + \frac{1}{2}(T_1 + T_2)(T_1 - T_2)$$

$$\therefore \eta = \frac{W}{Q} = \frac{\frac{\pi}{4}(T_1 - T_2)^2}{\frac{\pi}{8}(T_1 - T_2)^2 + \frac{1}{2}(T_1 + T_2)(T_1 - T_2)}$$

$$= \frac{2\pi(T_1 - T_2)^2}{\underset{+}{\pi}(T_1 - T_2) + 4(T_1 + T_2)}$$

Now this area the first we need to calculate this for the rectangle, so it is AS 2 times BD, BD is simply 2 minus T 1, there is no problem with this. So, this is AS 2 times AC, AC is equal to BD this we keep this in mind, this is a very important relation here AC is equal to BD. So, this is nothing but T 2 minus T 1 times AS 2. Now what is AS 2? A is basically the midpoint or the central point of this circle here.

So, the **y** y coordinate of this is exactly the same as the central point. So, we can write this as T 1 minus T 2 by 2 plus T 2. Now what is T 1 minus T 2? T 1 minus T 2 by 2 will give you the midpoint of this one T 1 minus T 2 by 2, this position will be given by T 1 minus T 2 by 2 plus the length T 2. So, this area is T 1 or this length is T 1 minus T 2 by 2. So, what we need to do is, we need to add this length plus this length. So, this and these are the same actually, so we have to add T 1 minus T 2 by 2 plus T 2. So, this gives you the area this one, whereas the second part is simply half of pi BD square by 4.

So, which we have already computed it is pi by 4 T 1 minus T 2 square, so that is the total area. So, half of this will be pi by 8 T 1 minus T 2 square, so we have pi by 8 T 1 minus T 2 square. After simplification this one remains the same. So, this one is actually this one, whereas this one is this one we can write this as half T 1 plus T 2 times T 1 minus T 2. So, the efficiency eta is equal to W by Q which is pi by 4 T 1 minus T 2 square divided by pi by 8 T 1 minus T 2 square plus half T 1 plus T 2 times T 1 minus T 2.

Some simplification and we get the final answer which is eta is equal to 2 pi T 1 minus T 2 square divided by pi T 1 minus T 2 plus 4 into T 1 plus T 2. So, it looks slightly complicated but looks at the simplicity of this, I mean although we have started off with a very arbitrary circle where only these 2 things are specified. Entropy is a specified S 1 and S 2 but it turns out that we do not need to use the values at all. Only using this temperature values we can compute the efficiency of the circle.

So, this actually gives us another alternative way of computing the efficiency of a circular process, which we will be keep using for future cases also. So, today we stop here. For today's lecture, we have discussed mostly problems, some concepts of course where we have discussed the T-S diagram of the circular process.

Now I would suggest that I have given you some examples of entropy change, some calculations related to entropy change but this is nothing. There are so many different cases possible and in each case most of the time there are simple ways of computing the entropy change. I suggest take any standard textbook where some examples are given. For example, Garg Bansal and Ghosh or Gupta and Roy. Go through the problems, example problems that is given there before you try attempting the assignment that we have provided.

Of course if you cannot solve any particular problem, you can always post in the forum, we will try to solve it for you, we will try to answer, we will try to discuss. So, that is what my suggestion remains in terms of entropy change calculations. As many calculations as you do it will help you improving your concept. Now today we stop here, next class we will start a very interesting topic which is related to the real life engine operation. That is the operation of the internal combustion engines, till then good bye.