Thermal Physics
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Lecture-38
Topic-Refrigerator and Carnot Theorem

Hello and welcome back to another lecture of this week 8 NPTEL course on thermal physics. Now in the last class we talked about Carnot engine, we talked about we solved several problems on Carnot engine and there are two more problems that we wanted to do in the last class but could not finish. So, let us quickly start with those and then we will continue.

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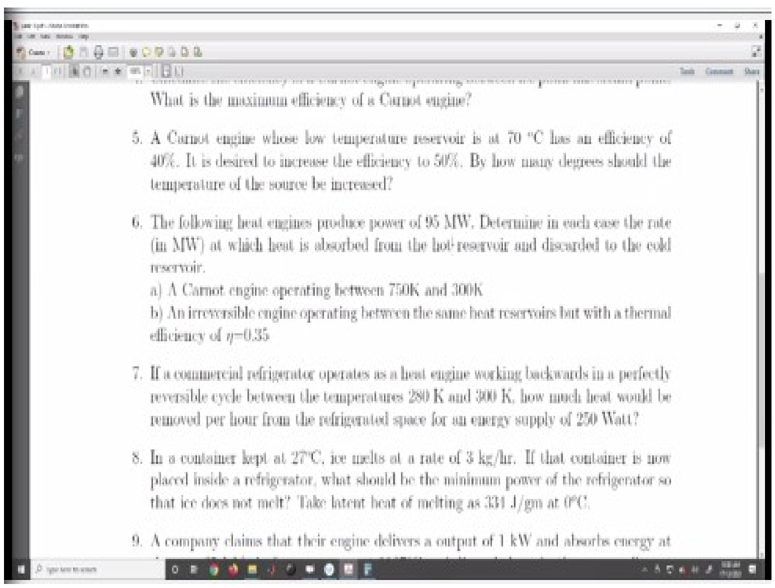
What is the maximum efficiency of a Carnot engine?

5. A Carnot engine whose low temperature reservoir is at 70 °C has an efficiency of 40%. It is desired to increase the efficiency to 50%. By how many degrees should the temperature of the source be increased?

6. The following heat engines produce power of 95 MW. Determine in each case the rate (in MW) at which heat is absorbed from the hot reservoir and discarded to the cold reservoir.
a) A Carnot engine operating between 750K and 300K
b) An irreversible engine operating between the same heat reservoirs but with a thermal efficiency of η=0.35

7. If a commercial refrigerator operates as a heat engine working backwards in a perfectly reversible cycle between the temperatures 280 K and 300 K, how much heat would be removed per hour from the refrigerated space for an energy supply of 250 Watt?

8. In a container kept at 27°C, ice melts at a rate of 3 kg/hr. If that container is now placed inside a refrigerator, what should be the minimum power of the refrigerator so that ice does not melt? Take latent heat of melting as 334 J/gm at 0°C.

9. A company claims that their engine delivers a output of 1 kW and absorbs energy at

So, let us start with problem number 5 and 6 for today's lecture. And first problem is that Carnot engine whose low temperature reservoir is at 70 degree Centigrade has an efficiency of 40%. So, it is desired to increase the efficiency to 50%, by how much? So, now we have to increase it from 40% to 50%. By how many degrees should the temperature of the source be increased?

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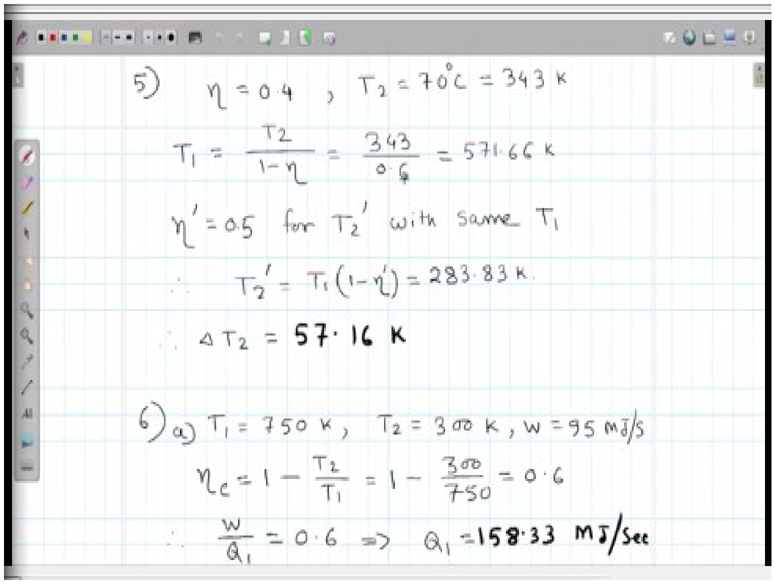

So, let us first examine this let us go back quickly to this last week's note to begin with.

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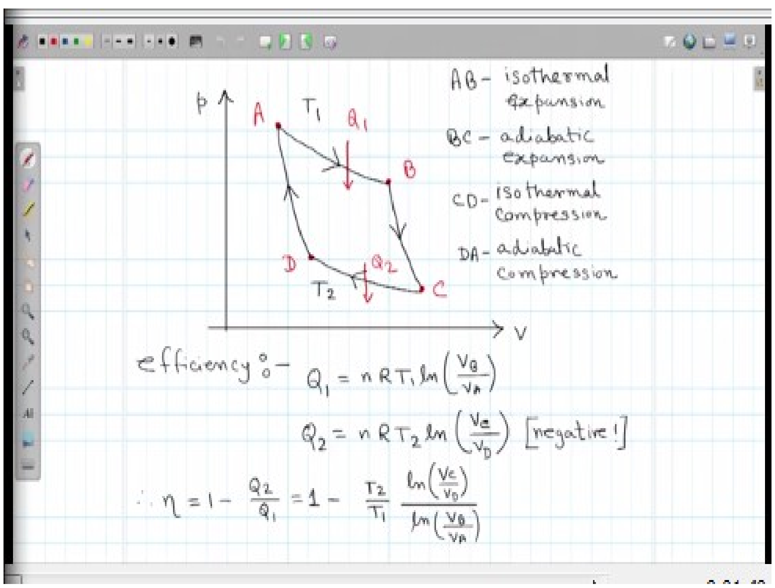

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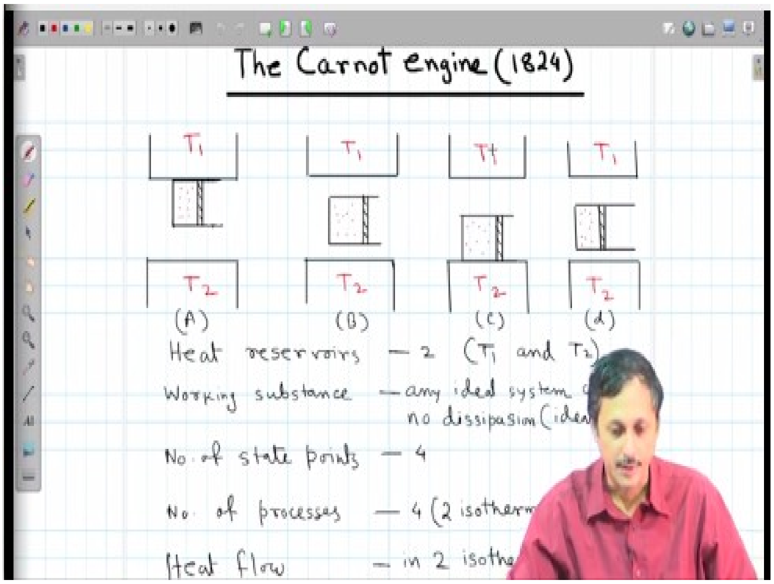

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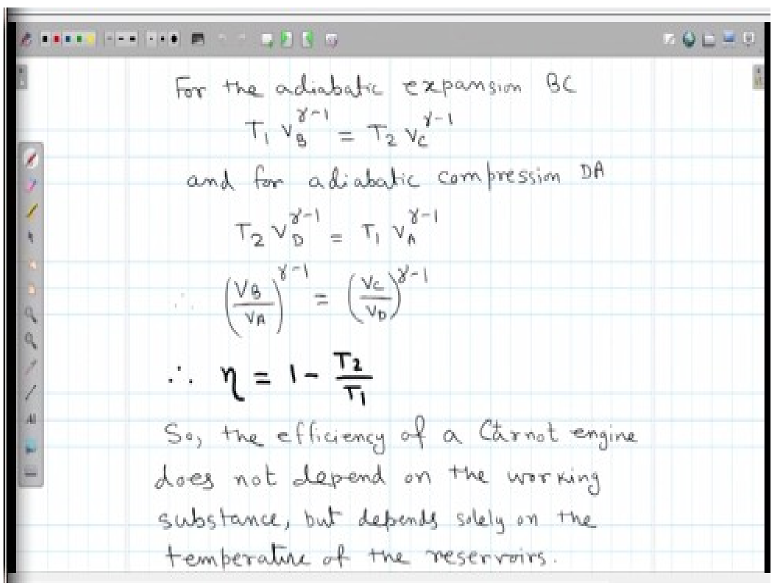

So, we see this efficiency expression for Carnot cycle or Carnot engine is eta is equal to 1 minus T 2 by T 1. Now as we have seen in the last problem that the limiting case when T 2 goes to 0 we have eta is equal to 1 but once again there is an idealized situation because T 2 can never be is equal to 0 as it is by the third law of thermodynamics, we will come back to that that towards the end of this course. But in general if I decrease T 2 the efficiency will increase or if I increase T 1 the efficiency will also increase.

So, there are 2 ways of doing it in order to increase the efficiency of a given Carnot engine which is where the engine is fixed whether we have to increase the temperature of the source or we have to reduce the temperature of the sink. So, there is another limit when T 1 goes to infinity the efficiency also goes to 1 which is the maximum possible value for an engine. So, either we have to have a very high like infinite temperature of the source or 0 temperature of the sink both of these are not possible theoretically.

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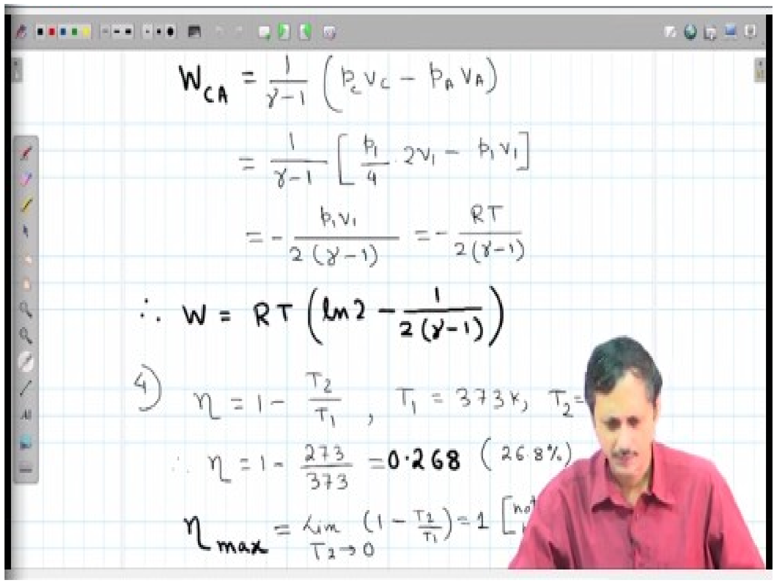

So, let us go back to this today's notes. So, this is the given data sorry n = 0.4 and T 2 is equal to 70 degree centigrade which is 343 Kelvin. So, from there we can immediately compute the value of T 1, T 1 will be equal to T 2 divided by 1 minus eta which is 3. So, please remember this is 40%, 40% means in scale of 0 to 1 it is 0.4, 50% means in the scale of 0 to 1 it is 0.5. So, either we can represent it as a fraction or we multiply it with 100 and represent it as percentage.

So, T 1 will be equal to T 2 1 minus eta, so which is 343 divided by 0.6, 571.66 Kelvin. Now we need, so that is the initial temperature of the source. Now when we have eta prime is equal to 0.5 for I think I just made a mistake here. By how many degrees should the temperature of the source be increased? So, yeah, that is a mistake actually, what I did here is actually by how many degrees the temperature of the sink be decreased.

But that is okay, I mean you can do it for what I did is not exactly what is asked in the problem, so that is a mistake from my end. So, what I did is I calculated the new temperature of the sink which is 283.83 Kelvin which is 10.83 degree Centigrades which gives an efficiency of 0.5. So, sink temperature has to be decreased by, this should be not 57.16 it is a mistake actually, just a minute. But I think you got the idea, so what can be done? So, it is 343 minus 283.88, so it is 59.17, not 47, so 59.17 Kelvin.

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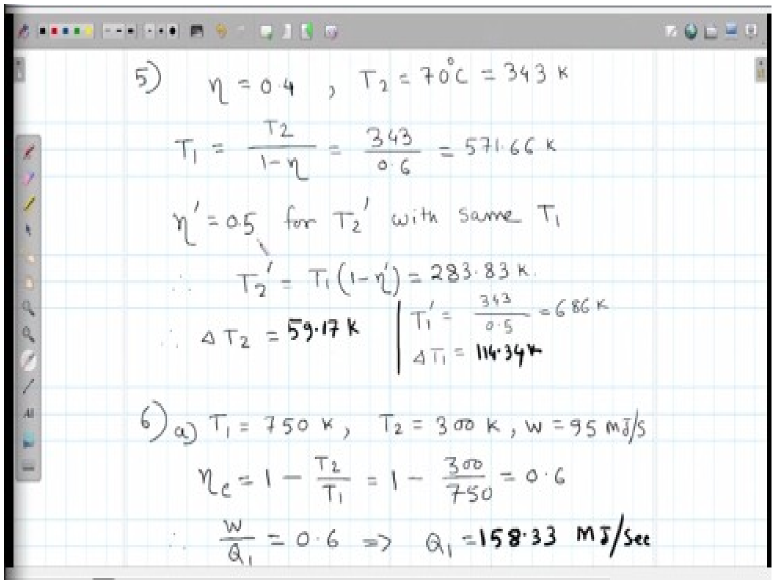

So, this is the right answer but once again what I did is I have calculated the temperature of the source that has to be decreased. But if I want to increase the temperature of the sink that is decreased by almost 60 degrees but if I want to increase the temperature of the source that is also possible. So, if I just now calculate T 1 prime sorry, I will use this one actually. So, T 1 prime will be equal to T 2 which once again is 343 divided by 1 minus eta which is 0.5.

So, basically this is 343 into 2, so 343 into 2 which is 686 Kelvin. So, initially the temperature is 571 Kelvin, so delta T 1 prime is equal to or delta T 1 I should not call it prime, delta T 1 is equal to 686 minus 571.66 which is 114.34. So, either we have to increase the temperature, just a minute 114.34. So, what we see is either we have to increase the temperature of the source by 114.3 Kelvin or we have to reduce the temperature of the sink by 60 Kelvin in order to boost the efficiency from 0.4 to 0.5.

So, both ways this problem is solvable. Now problem number 6 it is the following, so it is also another problem on heat engine. So, it says the following heat engines produces power of 95 milliwatt. Determine in each case the rate in milliwatt, milliwatt means watt is joules per second.

So, 95 megawatt sorry it is M, so it is megawatt. So, 95 megawatt means 95 mega joules per second. Determine in case the rate at which heat is absorbed from the hot reservoir and discarded to the cold reservoir. So, both cases the work output W is 95 megawatt, now a Carnot engine operating between 750 Kelvin and 300 Kelvin.

So, in this case we can easily calculate the efficiency, so let us do it. So, the efficiency should be, so T 1 is given as 570, T 2 is given as 300 and W is 95 mega joules per second. So, the efficiency is 1 minus T 2 by T 1 which is equal to 0.6. Now if the efficiency is 0.6 but by definition efficiency is W by Q 1 which is equal to 0.6, W is given as 95. So, Q 1 will be 95 divided by 0.6 which is 158.33 mega joules per second or megawatt, so it is the same units actually. So, 158.3 megawatt should be the power or heat input to that engine in order to get a power output of 95 megawatt.

Now there is a second part to this problem, that says an irreversible engine operating between the same heat reservoir but with a thermal efficiency of eta is equal to 0.35. Now, in this case we do not need the temperature of the source and the sink because the efficiency is already given. All we have to do is we have to compute Q 1. So, there are 2 things actually heat is absorbed from the hot reservoir and discarded to the coal reservoir.

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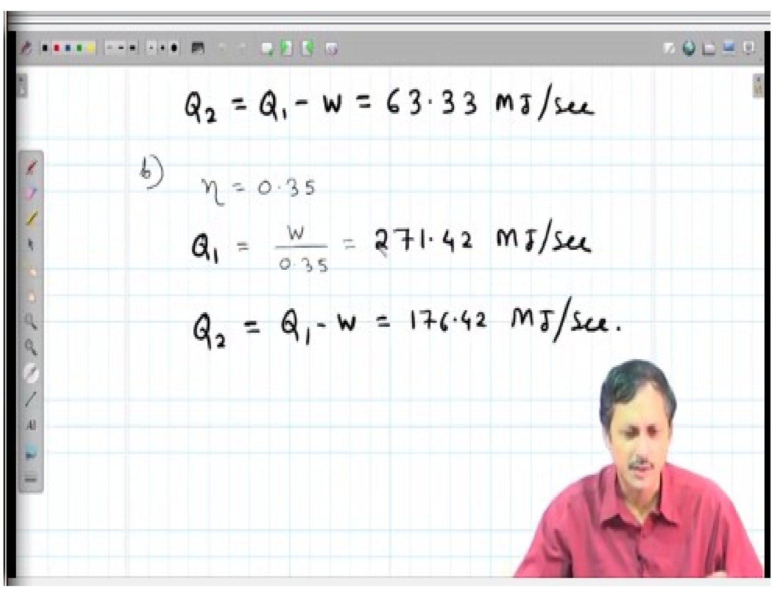

Now if this is the rate of heat absorbed then Q 2 is simply Q 1 minus W which is 63.33 mega joules per second. So, that is the first case here, now in the second case when the efficiency is 0.35 we can simply use Q 1 is equal to W by 0.35 which is 271.42 mega joules per second, so Q 2 will be Q 1 minus W which is 176.42 mega joules per second. Of course you see the Carnot engine has a much higher efficiency, so it can not only takes less amount of heat to produce same work.

But also it rejects less amount of heat, whereas the other engine which has a much lower efficiency takes a large amount of heat and also rejects a large amount of heat in order to get the same amount of work. So, it is not only work efficient, so boosting the efficiency of an engine not only good for power consumption but also this amount of heat which is rejected. And most of the time this heat is directly rejected to the environment whether to the air or to the water which eventually ends up in the river or in the ocean.

So, this leads to heat pollution and every one of us we are aware of about this global warming nowadays, what is happening around us. So, this heat pollution is one of the major sources of pollution apart from let us say air pollution and sound pollution. The temperature, of course we know due to global warming the temperature is in general increasing. But also the engine which is less efficient produces heat in the local environment which is not good for that environment.

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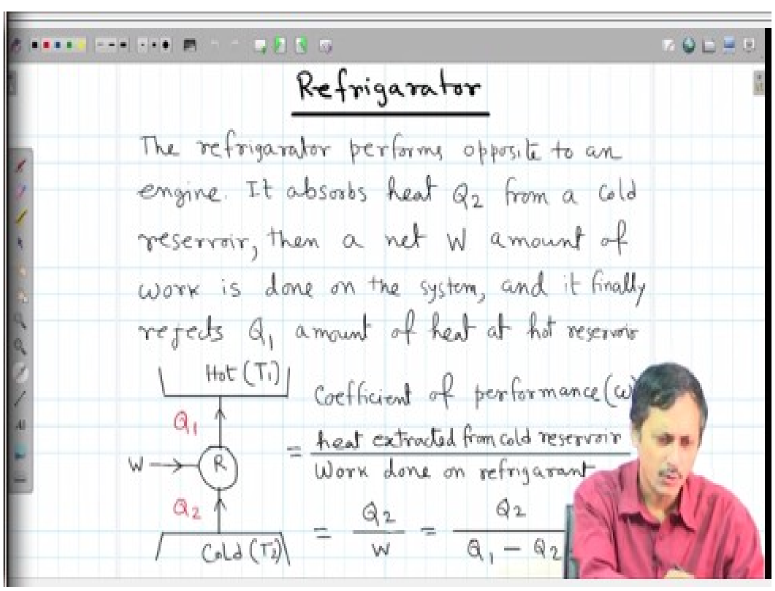

So, with this we change the topic and talk about refrigerator. Now what is a refrigerator? We are all familiar with refrigerator almost every one of us has a fridge in the household. This is one sort of refrigerator, if we go to a shop to buy an ice cream or a cold drinks typically those are kept inside refrigerator. So, we are all familiar with this thing, familiar with the word refrigerator. Now what it does actually it takes out heat from a given closed environment whatever it is kept inside, it will take out heat from there and reject it once again in the environment or similarly we are all familiar with air conditioner.

So, air conditioner what it does it actually takes heat from inside the room, takes out heat and rejects it outside in the environment. So, the sink which we talk about in case of an engine or in a refrigerator for example. The sink is actually the environment, especially in case of refrigerator. Now what is the refrigerator? If we go look at it schematically it takes out heat from a cold object, let us say it takes Q 2 amount of heat and then W amount of work has to be done on the machine and then it will reject Q 1 amount of heat of course Q 1 is equal to W plus Q 2.

The law of conservation of energy holds as always and so it takes lesser amount of heat from the substance it needs to cool or the area which it needs to cool. And then some mechanical or electrical work has to be performed on the machine, typically it is mechanical work at the end but the main power source is electrical. So, we will discuss it in a little elaborate manner and then it rejects that heat into the sink. In this case the sink temperature is more than the source temperature. So, this cold here is the source, hot this is the sink.

Now typically what happens is like we can define efficiency of an engine, we can define coefficient of performance of a refrigerator which is the ratio of heat that is extracted from the cold reservoir and the work that is done on the machine, on the refrigerator. So, if it takes Q 2 amount of heat from the cold reservoir or the system of interest and W amount of heat has to be produced on it. Then my coefficient of performance omega is equal to Q 2 by W which is Q 2 divided by Q 1 minus Q 2.

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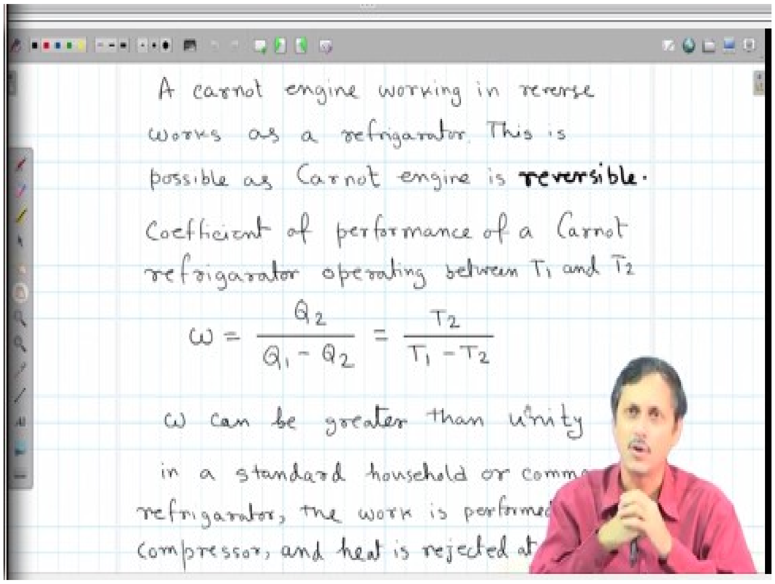

Now Carnot engine is a reversible engine as in all the processes are reversible. So, if we instead of using it as an engine if we just reverse the process, so that means it will absorb initially it will absorb heat from the cold reservoir and then isothermal compression will take place then it will have an adiabatic compression, finally it will reject heat to the hot reservoir.

So, basically if we make every process, if we invert every step in a Carnot cycle it works as a Carnot refrigerator which once again is a refrigerator with highest coefficient of performance. And in case of a Carnot refrigerator reversible refrigerator Q 2 by Q 1 minus Q 2 can be written as T 2 divided by T 1 minus T 2. So, I am not proving it explicitly, if you want you can try it yourself, it is not very difficult to prove.

So, all you have to do is, you have to revert all the steps when actually in a Carnot engine heat is absorbed from the hot reservoir and rejected at the cold reservoir, here it has to be opposite. Heat has to be absorbed from the cold reservoir and rejected to the hot reservoir and work has to be done on the system. So, net work has to be done on the system then you will get back Q 2 by Q 1 minus Q 2 is equal to T 2 by T 1 minus T 2.

And in this case or in general for a refrigerator omega can be greater than unity, like in case of an engine the efficiency cannot be greater than unity, in case of omega there is no such limitation. You see Q 1 minus Q 2 is always typically less than Q 2 and it is in general greater than 1. Now in a standard or household or commercial refrigerator the work that we talk about is performed at the compressor.

So, what does the compressor do? Compressor actually compresses the cooling gas whatever the working substance compresses the cooling gas and then it becomes liquid under very high pressure. And we have talked about vapour liquid conversion when we were discussing Andrew's experiment on carbon dioxide. So, you can see if it is below critical point by applying pressure you can actually make a gas into a liquid.

And then what happens is, there will be a place called the condenser in where it will sorry, not the condenser it should be the evaporator. So, I am not going to discuss it in details but typically what happens that liquid goes to the evaporator part where it becomes it gathers the energy from the chamber which has to be cooled, then it evaporates that is why it is called the evaporator. And then it once again goes to this compressor and in the compressor or sorry, first it goes to this condenser system where the excess heat is rejected in the environment, then it goes to the compressor where it becomes liquid again and once again it goes into the evaporator.

So, actually there are three major parts I have not mentioned the evaporator here, so there is a compressor, there is an evaporator and there is a condenser. So, there are these three main components of a refrigerator. So, we are not going into the details of that maybe later on when we will be discussing about real life engines we will be revisiting refrigerator briefly.

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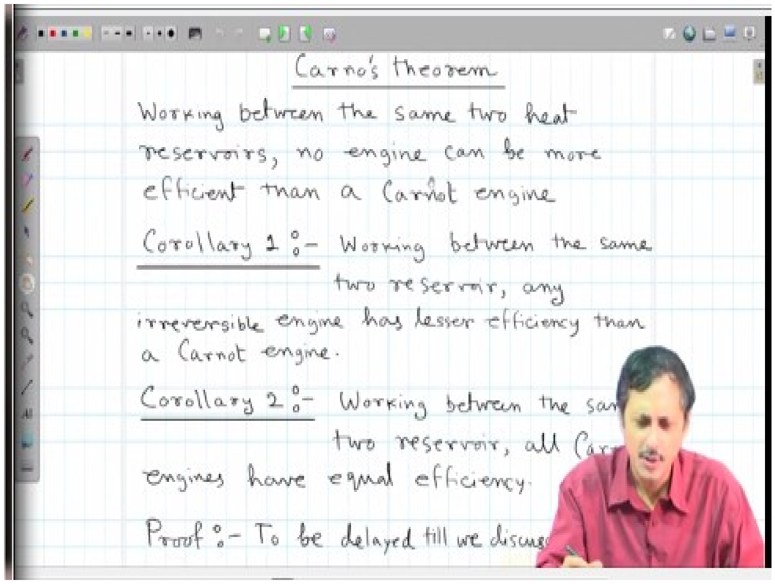

But let us go into something called the Carnot theorem. Once again this talks about heat engine, it says working between the same two heat reservoirs, no engine can be more efficient than a Carnot engine. So, this is a theorem, once again it can be proved given that we accept second law of thermodynamics we will come back to that maybe in the next class.

But it was purely observational; I cannot call it observational also because it is more of a theoretical concept to begin with because as we have already discussed a Carnot engine cannot be realized in real life. Now if we accept this for now, we will prove it later on. So, there are two additional conclusions that comes out of Carnot theorem, these are corollary 1 and corollary 2.

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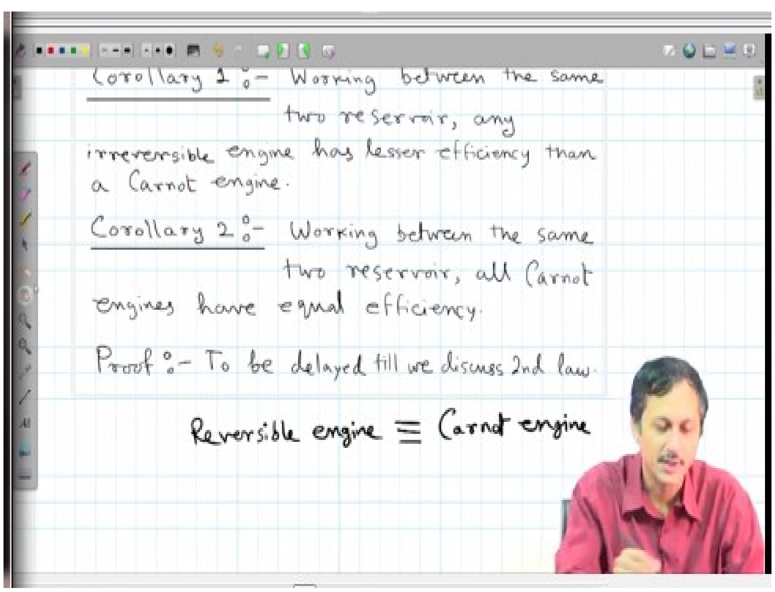

Corollary 1 says working between the same two reservoirs any irreversible engine has lesser efficiency than a Carnot engine. Please remember, we keep using the word Carnot engine and a reversible engine interchangeably. Whenever an engine is reversible we assume that this is Carnot. So, reversible engine and vice versa. So, when we say Carnot reversible engine we mean Carnot engine and vice versa.

So, please keep this in mind, for the remaining discussion on thermodynamics whenever we talk about a reversible engine it means a Carnot engine and the reverse is also true. Now there is a second corollary, working between the same 2 reservoirs all Carnot engine should have equal efficiency. So, we will prove it but till we have discussed the second law of thermodynamics.

But this Carnot theorem has implications, like if any experiment produces an efficiency mode given that you can define the source and the sink and if it produces an efficiency value which is more than the Carnot engine we can blindly discard that result. Because once again Carnot theorem is based on second law of thermodynamics and we so far if we accept that Carnot theorem is not valid that means we are not accepting the second law of thermodynamics.

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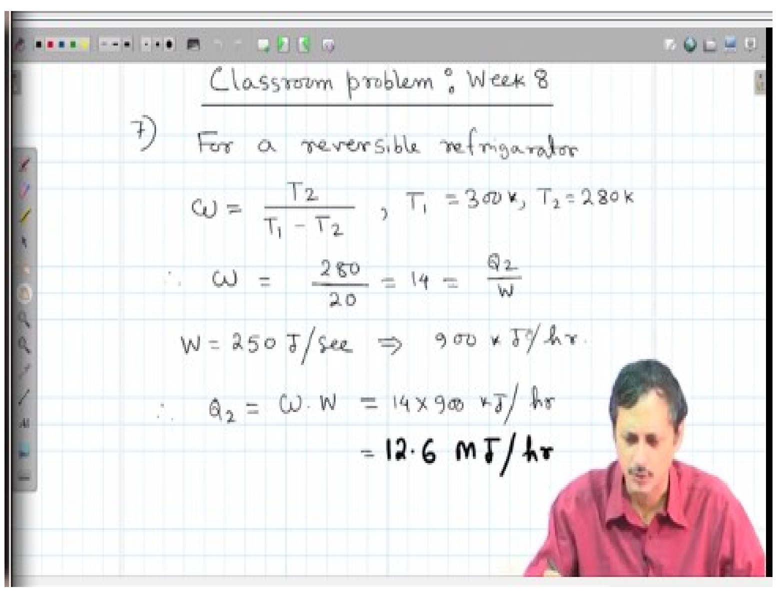

So, without further delay let us talk about. So, there are few there are few classroom problems before we go to the second law of thermodynamics.

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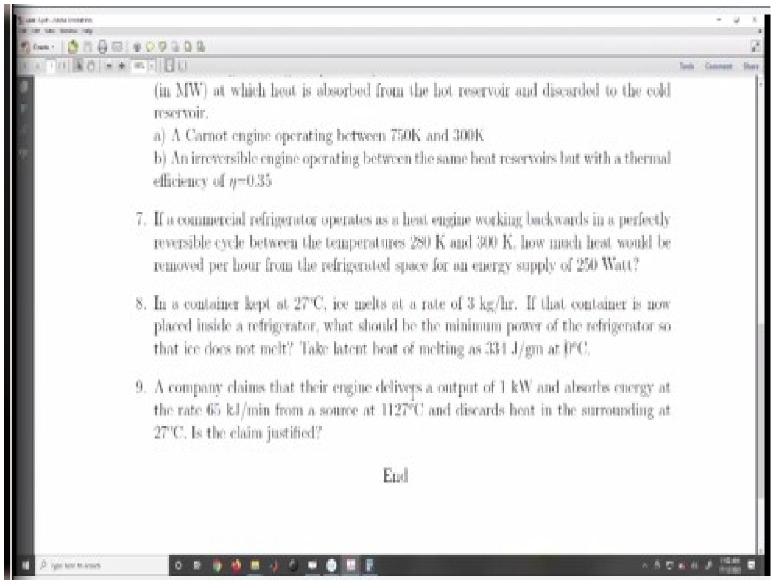
(in MW) at which heat is absorbed from the hot reservoir and discarded to the cold reservoir.

a) A Carnot engine operating between 750K and 300K

b) An irreversible engine operating between the same heat reservoirs but with a thermal efficiency of η=0.35

7. If a commercial refrigerator operates as a heat engine working backwards in a perfectly reversible cycle between the temperatures 280 K and 300 K, how much heat would be removed per hour from the refrigerated space for an energy supply of 250 Watt?

8. In a container kept at 27°C, ice melts at a rate of 3 kg/hr. If that container is now placed inside a refrigerator, what should be the minimum power of the refrigerator so that ice does not melt? Take latent heat of melting as 334 J/gm at 0°C.

9. A company claims that their engine delivers a output of 1 kW and absorbs energy at the rate 65 kJ/min from a source at 1127°C and discards heat in the surrounding at 27°C. Is the claim justified?

End

So, problem number 7, 8 and 9. So, in a commercial refrigerator operates as a heat engine working backward in a perfectly reversible cycle between temperature 280 Kelvin and 300 Kelvin. How much heat would be removed per hour from the refrigerated space for an energy supply of 250 Watts? So, basically here the source and the sink temperature is given. So, the source temperature in this case is 280 Kelvin, please remember for a refrigerator the source temperature is lower than the sink temperature.

So, heat is taken out from this area of 280 Kelvin and it is rejected at a temperature of 300 Kelvin which is the room temperature and 280 Kelvin is what? 280 Kelvin is roughly 7 degrees. So, typically that is the temperature maintained inside a household refrigerator 7 to 8 degrees. And inside the ice compartment the temperature is minus 2 degrees typically or may be minus 4 degrees not less than that.

Now of course there are modern day refrigerator which can be set at -20 degrees also but I am just talking about the standard normal single door, the ordinary refrigerator. So, this is what we have to do is how much heat would be removed per hour. So, that we need to calculate. So, first of all it given as a reversible refrigerator.

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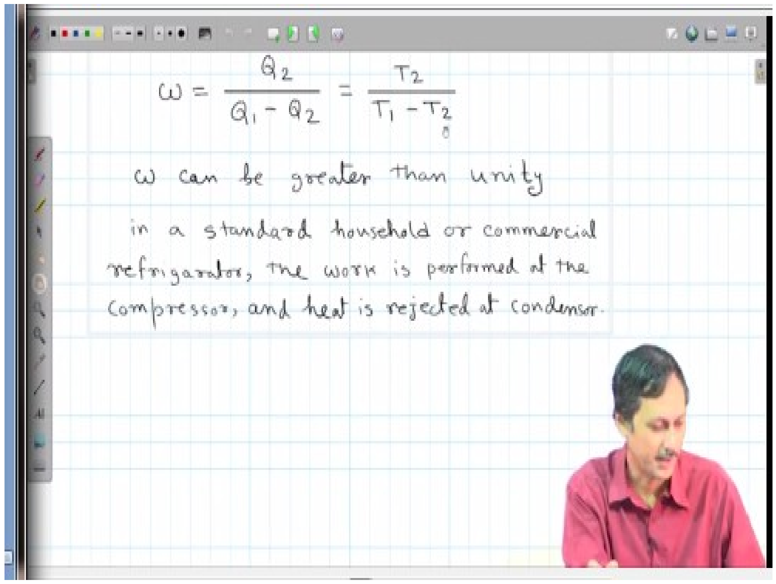

An reversible refrigerator we have just seen that the efficiency is written as or the coefficient of performance omega is T 2 by T 1 minus T 2. So, T 1 is 300, T 2 is 280, so we have omega is equal to 280 by 20 which is equal to 14. So, that means Q 2 by W. So, omega is the heat that is absorbed from the cold reservoir or cold object and the work done on the refrigerator. Now W is given as 250 joules per second, watts is once again 250 Joules per second, so it is 900 kilojoules per hour.

So, we need to answer this in hour, so how much heat would be removed per hour? So, 250 joules per second means we have to multiply this with 3600 to get 900 kilojoules per hour. So, Q 2 is equal to omega times W, which is 14 into 900 kilojoules per hour which is 12.6 mega joules per hour which is 12.6 megawatt. So, if a normal refrigerator the work is done at the compressor, so if the compressor power is 250 watts, you know so you can kind of approximately.

And if you know the temperature outside 300 Kelvin is the temperature of the surrounding and 250 Kelvin or 280 Kelvin is the temperature inside. What you can do is? If you take a normal lab thermometer you can measure the temperature of your household refrigerator, measure the temperature of the environment. Look at the compressor rating that should be written on the compressor, back of the fridge you can look at the level.

There should be some rating, if not you can search by the model number and you can get a rating. Be careful do not electrify yourself, be careful about it. But you can kind of calculate how much heat your refrigerator is giving away per unit time, let it be per second, let it be per hour to the environment. So, that is a very once again we are assuming that it is an ideal refrigerator, so with this equation holds.

But even if you make this calculation you will not make very big mistakes. So, typically modern day machines are kind of efficient and especially the electric ones they are kind of efficient and you will get an idea that is what I am trying to tell you. So, question number 8, in a container kept at 27 degree centigrades ice melts at a rate of 3 kg per hour. So, when I say kept at 27 degree centigrade, I mean the outside the container the environmental temperature is 27 degrees.

If that container is now placed inside a refrigerator what should be the minimum power of the refrigerator? So, the ice does not melt, take latent heat of melting as 334 joules per gram at 0 degree centigrade. So, the question is basically we have a refrigerator where the outside temperature is 27 degree centigrade which is once again 300 Kelvin and inside we have a box of ice which melts at a rate of 3 kg per hour.

So, if I just keep that box as it is in the open environment every hour 3 kg of ice will melt. Now we have to maintain that box at 0 degree centigrade, so that the ice does not melt. So, of course if we maintain it at minus 2 it will be even better. But as it is required to compute the minimum amount of heat, minimum power we will consider that temperature of the source, in this case the low temperature part is 0 degree Centigrades.

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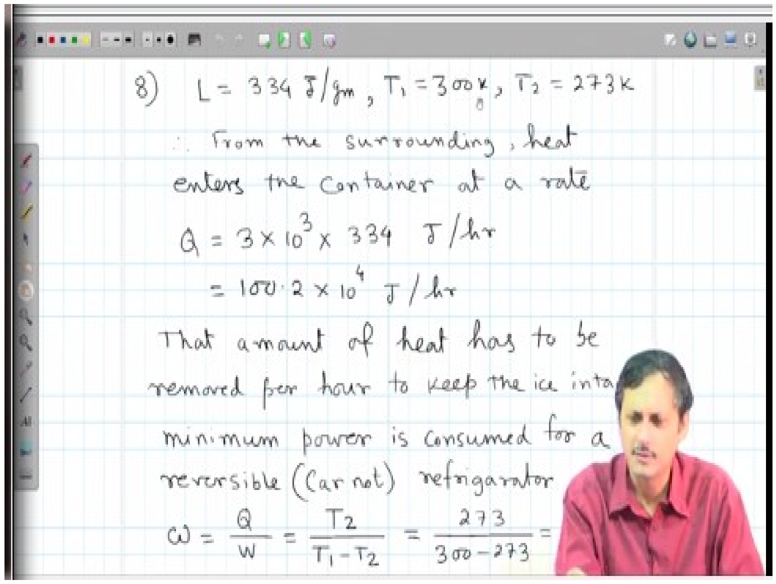

So, we have T 1 is equal to 300 Kelvin, T 2 is equal to 273 Kelvin. So, from the surrounding heat enters the container at a rate of 3 into 10 to the power 3 into, so latent heat is given. So, that means the total heat that enters because the ice melts at a rate of 3 kgs per hour. So, that means 3 into 10 to the power 3 grams per hour, so the total heat is 3 into 10 to the power 3 into 334 joules per hour, which is 100.2 into 10 to the power 4 joules per hour.

Now that amount of heat has to be taken away from that container, so that the temperature is maintained at 0 degree Centigrades. So, for a reversible refrigerator, and once again we are computing the minimum power. Minimum power is obtained when we are using a reversible refrigerator with the coefficient of performance W is equal to T 2 by T 1minus T 2 which is 273 by 27. So, just I have not calculated, it remains as 273 by 27, so this is Q by W, so Q is given Q is 100.2 into 10 to the power 4 joules per hour, so this amount of heat has to be removed.

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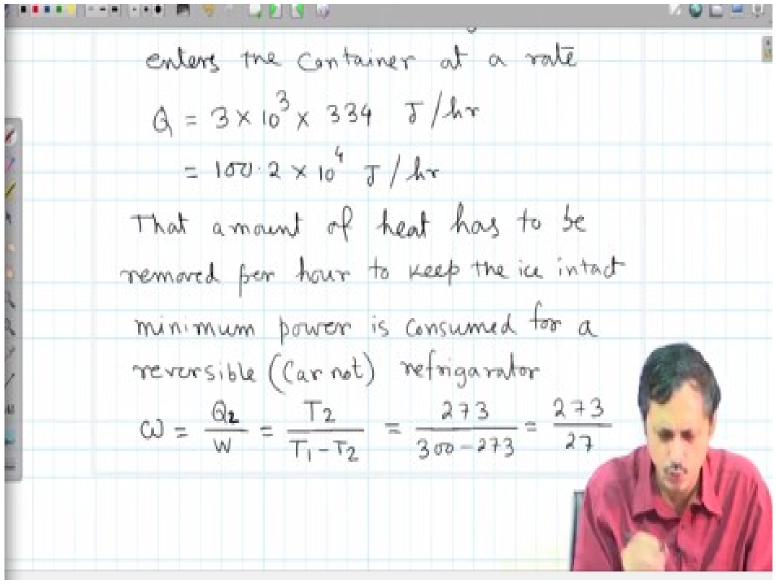

So, actually I should write Q 2 here then it will be easy for you, Q 2 by W. Now what we have to do is, we have to compute W which is Q 2 into 27 divided by 273.

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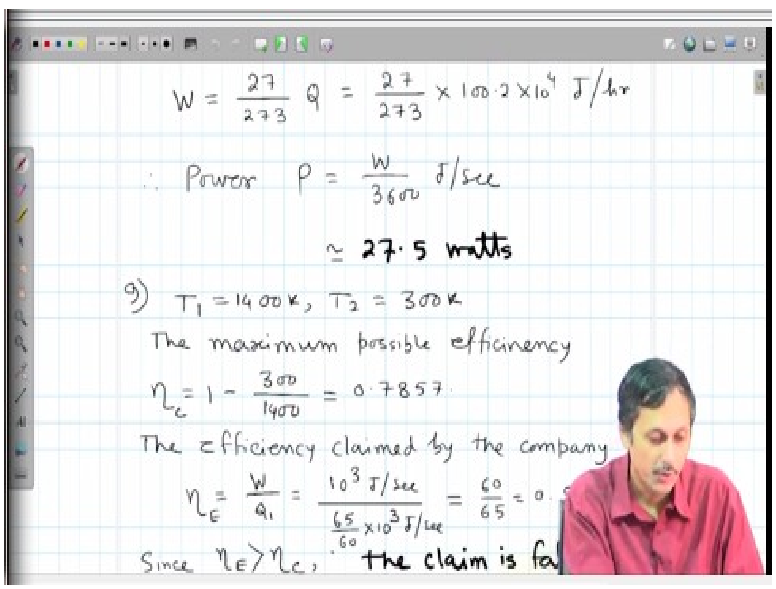

So, if I put that this is my work done per hour. Now what do we need? We need to compute, so we want to give it in a more convenient units. So, what we do is, we just divide it by 3600 and convert it to joules per second instead of joules per hour and we get 27.5 watt. So, something is not right here, I think it should be 275 watts ok, I just need to check this once again. Because the previous problem we got 250 watts and this is 27.5, I think I have missed a factor of 10, let us do it so 27 into 100.2 into 10 to the power 4 divided by 273 divided by 3600, no, 27.5.

Any way I look into this calculation maybe I made a mistake somewhere, maybe in this number itself there is a mistake but I will look into it. But I think you got the idea, so how to solve this problem? If there is a mistake I will correct it in the final class note version otherwise it stays as it is. So, you got the idea, so all we have to do is we need to first compute the amount of heat that is being taken out by this relation.

Then we compute the efficiency given that the refrigerator is completely reversible and from there we compute the amount of work done W by the simple relation and finally we convert it into power. So, we come to the last problem of this lecture, we have a company claims that their engine delivers an output of 1 kilowatt and absorbs energy at a rate of 65 kilojoules per minute from a source at 1127 degree Centigrades. And discard the heat in the surrounding at 27 degree centigrade. Is the claim justified? So, this is the question. So, basically what is given is work output, the heat input is given here, source temperature and the sink temperature is given here.

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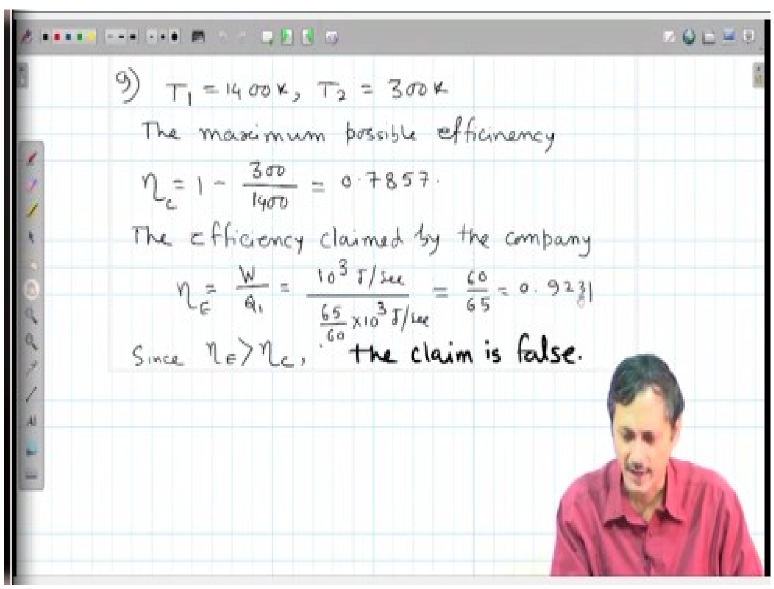

So, all we need to do is, we need to compute the efficiency T 1, T 2 is given, the maximum possible efficiency that is for a Carnot engine is eta c is equal to 0.7857, so this is the maximum efficiency the efficiency claimed by the company is for their engine which we call eta E is equal to W by Q 1 which is 10 to the power 3 joules per second divided by 65 into 10 to the power 3 joules per second divided by 60 because it is given as 65 kilojoules per minute.

So, we just have to divide it by 60 once again and we get an efficiency of 0.9231. So, the engine efficiency claimed by the company is more than the efficiency of a Carnot engine operating between the same two heat reservoirs. Now according to Carnot theorem that is not possible, so the claim is false.

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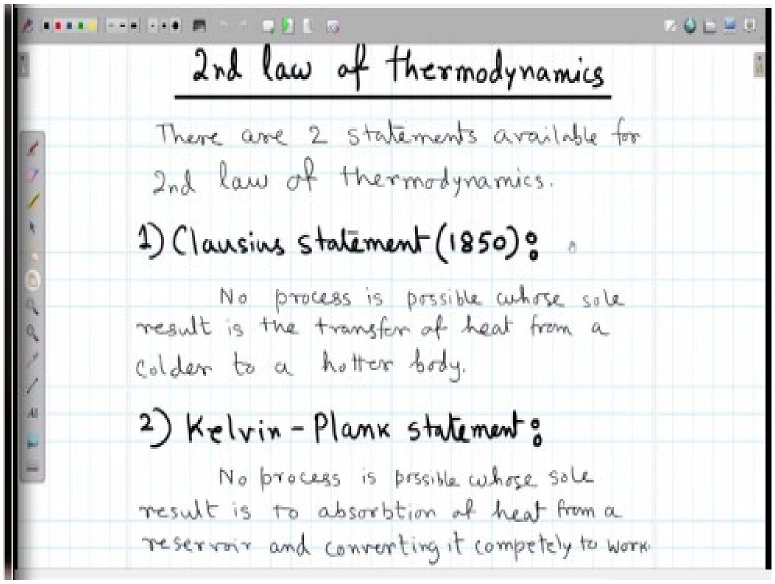

Now with this we come to the last part of our discussion today which is 2nd law of thermodynamics. Now once again the 2nd law as just like the 1st law it cannot be proved. It comes out from a series of observation and theoretical discussion, theoretical brainstorming. It cannot be proved but in case if we accept 2nd law of thermodynamics we can explain so much, so many experimental observation in a correct manner that so far there has been no doubt about the 2nd law of thermodynamics.

So, same goes for the 1st law, these laws cannot be proved, we need to accept it but if we accept those all the experimental results can be justified by the logic of those laws. So, there are two statements available for the second law of thermodynamics and in the next class we will show you that these two statements are actually equivalent. So, for today's lecture we will simply state these two statements and call it a day.

So, the Clausius statement which was given in 1850 says, no process is possible whose sole result is the transfer of heat from a colder to a hotter body. What I mean by this is in Clausius statement, it actually discards the concept of an ideal refrigerator. Ideal refrigerator needs no work, it takes, so going back here an ideal refrigerator the amount of coefficient of performance is infinity.

So, it does not need any W, it just takes Q 2 amount of heat and discards the same Q 2 amount of heat to the hot reservoir without any external work. So, it is kind of a perpetual machine. So, the Clausius statements discuss the possibility of a ideal refrigerator. And then Kelvin-Planck statement actually the Kelvin and Planck separately made two statements which were later combined into one single statement.

That says, no process is possible whose sole result is to absorption of heat from a reservoir and convert it completely to work. So, the Kelvin-Planck statement actually discards the concept of an ideal heat engine. So, that means if in an ideal heat engine the heat output to the sink is equal to 0 and the efficiency 1 minus Q 2 by Q 1. And if Q 2 is equal to 0 then the efficiency becomes 1, so Kelvin-Planck statement not only the third law of thermodynamics but also Kelvin-Planck statement discards the concept of an ideal heat engine.

So, Clausius statement discards the concept of an ideal refrigerator and Kelvin-Planck statements discuss the concept of ideal heat engine. Now if you look into any standard textbook you might find these statements in a much flowery language. Basically the same thing, what I did here, I just have breached those statements in a very simple language tried to put it in a simple language without altering it is essential message and put it here. Anyway, so in the next class what we are going to do is we are going to talk about the equivalence of this Kelvin-Planck and Clausius statement.

And then if we accept these laws which are the two statements of 2nd law of thermodynamics, we will see that we can actually prove Carnot theorem given that these laws are valid. So, then Carnot theorem will be proved and we will take it one step forward into something called the Clausius inequality. And from there we are going to discuss or going to define a quantity called the entropy of a system, so till then good bye.