Thermal Physics
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Lecture-37
Topic-The Reversible Heat Engine: Carnot Cycle

Hello and welcome back to another lecture of week 8 on this NPTEL course on thermal physics. Now in the last lecture we have discussed about cyclic process and we have discussed about how to calculate the heat exchange and work output in a cyclic process. Now today we are going to discuss something called the heat engine. Now what is a heat engine? Heat engine is a device that can systematically convert heat into work.

Now you must have learned that if you take a piston, if you take a gas enclosed inside a piston and start heating it up then it will expand and the piston will move out. So, that means some work is produced by the piston. So, and you might ask why it is not an engine? What is the difference between this system and an engine? Virtually there is no difference, only thing is engine must work in a cycle, so it should come back to it is original position after 1 complete cycle and then it can do the thing once again.

Because if we just do not bring it back to it is original position after some point the piston must stop or whether it will when the pressure inside will be sufficient to balance the atmospheric pressure it will stop or if the temperature is or rather if the piston reaches it is upper limit of course there is a finite stroke length when it reaches the upper portion of the cylinder it must stop.

So, that is where the limitation of this particular device is. Now in an engine it has to work in a cycle, so that means it has to be compressed back again to it is original volume and the same procedure has to be repeated over and over again. That is the primary difference between a single piston arrangement for example and the engine which must be working in a cycle.

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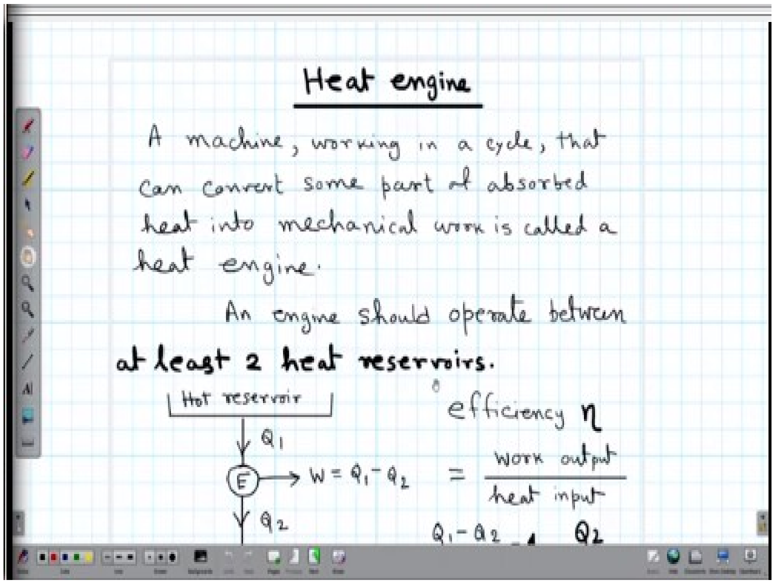

Now, so as I have said every machine cannot be an engine, the piston we just discussed is actually a machine and not an engine unless it is working in a cycle. So, heat engine by definition is a machine working in a cycle that can convert some part of the absorbed heat into mechanical work is called a heat engine.

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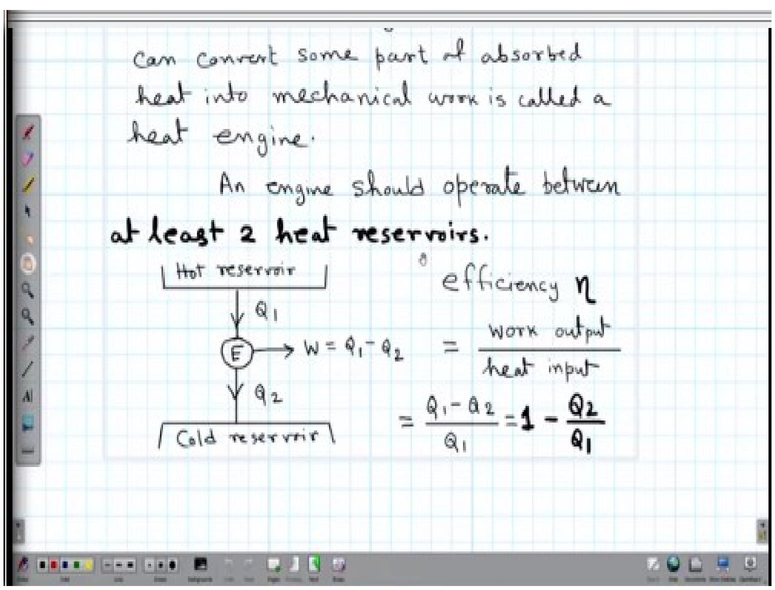

So, if we want to describe this in a schematic and please remember, for example again going back to this piston example. If the piston has to be compressed back it has to be cooled, there has to be a place where it has to discard the heat because when under compression that system will discard some heat. So, it need to have a place or need to have one reservoir from which it will absorb the heat and need to have one place where it will discard the heat.

So, in a proper engine the machine has to have at least two heat reservoirs to work with. So, we can say that a proper machine which can work between at least two heat reservoirs is called an engine. So, as I have said the criteria is there has to be at least two heat reservoir, it can work with more than that, I mean there is no upper bound two number of reservoirs it needs but it needs at least two heat reservoir.

So, if I try to represent it schematically we need to have a hot reservoir from which it will absorb Q 1 amount of heat and there should be a cold reservoir in which the engine E will reject Q 2 amount of heat and it will convert equivalent of Q 1 minus Q 2 amount of hit into work. And this is the schematic per each cycle. So, in the next cycle the same thing will happen again. If we consider only a machine which works in one direction then this might happen but only in one cycle some part, so it absorbs heat some part becomes goes into internal energy or let us say some part goes into work done and finally it might reject some part as well.

But unless and until it keeps doing it in every cycle we cannot call it an engine, so this is a basic difference. Now we can define the efficiency eta of a heat engine which is work output divided by heat input. So, in this case it performs W amount of work which is given by Q 1 minus Q 2 and it takes Q 1 amount of heat as input. So, the efficiency eta is given by W by Q 1 which is Q 1, W is equal to Q 1 minus Q 2, so it is Q 1 minus Q 2 by Q 1 which is equal to 1 minus Q 2 by Q 1. So, if we know this relation of the efficiency we can compute the efficiency of any cyclic process.

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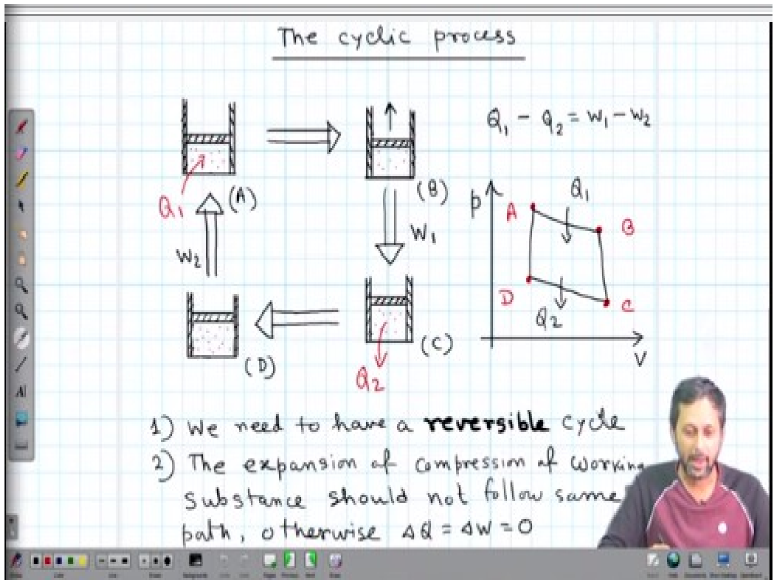

So, let us look back, so basically what I mean by cyclic process is we have to have let us say if we represent that in a pv indicator diagram just to represent this I have taken 4 points A, B, C and D. So, let us say this piston and cylinder arrangement which has some enclosed gas; it goes through this four step cycle. So, in step 1 it absorbs a certain amount of heat Q 1 while going from A to B then it does during going from B to C it performs W 1 amount of work.

Then it rejects Q 2 amount of heat and when it is compressed it goes to C to D it rejects Q 2 amount of heat and while going from D to A W 2 amount of work has to be done on the system, so that it goes back to it is original state. So, we can represent that cycle in kind of a representative diagram here. These are arbitrary we do not consider this to be isothermal, isobaric, adiabatic nothing like that, I have just given a 4 step cyclic process.

And only criteria here is for an engine that this path has to be reversible which once again we know in reality is not possible to achieve some time. But at present we are discussing only ideal engine, so forget about the real engines for now. So, let us consider we will discuss that in maybe probably end of this week or maybe next week we will be discussing that. But for ideal engine we need to have a reversible cycle, what I mean by this is all this path has to be reversible.

And also one more thing is the expansion and the compression work should not follow the same path. So, if it does if it follows exactly the same path then we have discussed it in the last class then the net work output will be equal to 0. But in this case I have drawn it in such a manner that the expansion path and the compression paths are different, so we have a net work output which is W 1 minus W 2 which is equal to Q 1 minus Q 2. So, this has to happen because in a reversible cycle if we go through 1 cycle this relation has to hold but I have not represented any specific type of cycle or anything like this, it is just a general idea.

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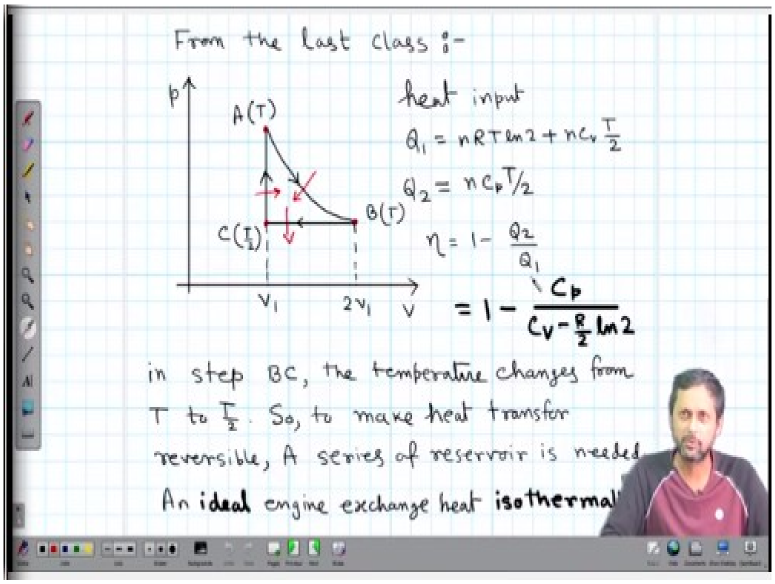

So, let us go back to the example from last class we have taken this ABC this cycle, this path is an isotherm, this path is an isobaric and this path is an isochoric process. So, we have already calculated the heat input heat is coming in during AB and CA and heat is being rejected during the process BC. So, the input heat which is a combination of AB plus CA, please go back to your previous last week's note, so that you can find it in more details.

It is given by n R T ln 2 and this is for AB and for this one it is n C v T by 2 because it is a constant volume path, isochoric path. Q 2 on the other hand is the heat that is rejected during this process B to C which is given by n C p T by 2. Please remember we ideally if I am writing Q 2 in the usual convention there has to be a negative sign because heat that is being rejected by the system.

But as we are already considering Q 2 amount of heat is going out of the system we just ignore the negative sign here. And we write Q 2 is equal to n C p minus T by 2. So, eta which is equal to 1 minus Q 2 by Q 1 is 1 minus C p divided by, so what happens is n T by 2 n T cancels out and finally we have C p divided by C v minus R by 2 ln 2. So, this is the expression for efficiency of this particular cycle. Now we can use this particular cycle and we have calculated the efficiency but is it a good engine?

Theoretically speaking, at this point everything is theoretical because all the paths are taken as reversible, all the processes are considered ideal which is not the case in general. But if we consider the path B to C in which the temperature changes from T to T by 2 and the path C to A where the temperature once again changes from T y 2 to T. Now just consider this path BC, the temperature change if this path has to be a reversible path in reality or in a scenario where the heat has to go to a heat reservoir in a reversible manner then what do we need actually?

We need to have an infinite many numbers of reservoir with temperature ranging from T to T by 2 to be in contact with the cylinder in which the working substance that is in this case ideal gas actually is n moles of ideal gas is contained, so try to think of it. We need to have we start from a temperature T; we have an isobaric process, isobaric volume change, so that means the temperature has to decrease.

Theoretically speaking it can be achieved, so we can have many different temperature reservoirs with temperature ranging from T to T by 2 and each time we place this cylinder on 1 after it equilibrates we place it on the second one, then we place it on the third one. I mean forget about infinite number but even if we consider this process to be quasi static, so we need to have at least sufficient number of points closely space point, so that we can still consider this process a quasi static process.

So, in this case it is difficult to construct this engine in reality because we need to have a good number of reservoirs ranging from T to T by 2 and similarly here also it is going back from T by 2 to 2, so the reverse process. So, each time we have to place it to a reservoir of slightly higher temperature, let it equilibrate, move it to the next reservoir of slightly higher temperature, let it equilibrate so on and so forth. So, even if this process is quasi-static this process BC and CA we need to have many reservoirs present.

But consider this process AB, it is an isothermal process the temperature does not change but it takes an amount of heat Q which is let us call it Q 1 prime or whatever, it does not matter. But the name does not matter but isothermally when the system is absorbing heat what is happening? We need to have only one reservoir, system is nicely placed on that reservoir, it is absorbing heat, we are done. So, even in the idealized case, even in the idealized world where reversible processes do exist an ideal engine should exchange heat isothermally. So, whenever it is exchanging heat the process should be isothermal.

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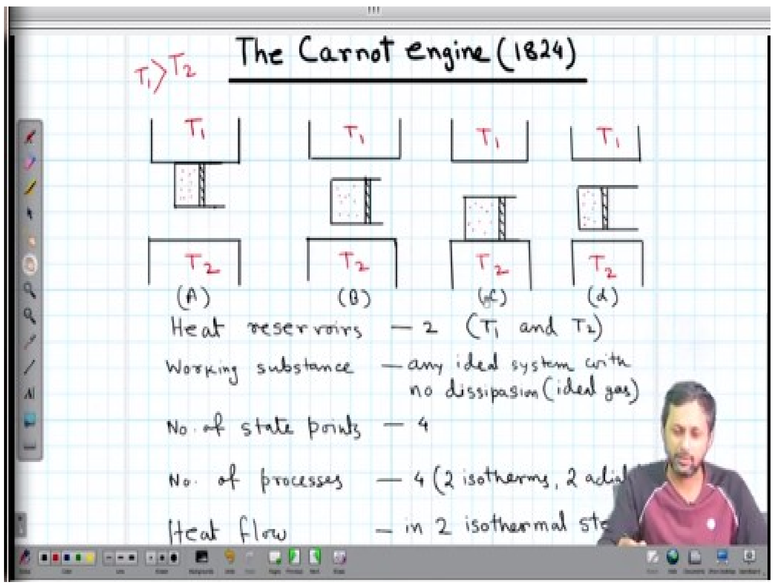

So, keeping that in mind in the year 1824 Carnot came up with a construction. Once again it is a hypothetical construction to begin with of an engine, which is a reversible engine operating between 2 fixed reservoirs. So, the criteria of an engine is to have at least 2 reservoirs, so he considered 2 reservoirs one at a temperature T 1, another at a temperature T 2 with T 1, I have not written it some anywhere.

But T 1 is greater than T 2, so T 1 is the hot reservoir T 2 is the cold reservoir. And the working substance, later on it was found out that the working substance of a Carnot engine could be anything. But the simplest possible engine that we can or working substance we can think of is actually ideal gas assembly inside a piston. So, that is probably the simplest system which we keep using in the classical thermodynamics for demonstrating our ideas.

So, let us assume, so it is a 4 step process, so the cycle he has considered has 4 steps A, B, C and D. So, in the step A what happens? So, let us see, so there are 4 state points in the cycle and number of processes that connects these 4 state points are again 4. So, there are out of that 4, 2 are isothermal and 2 are adiabatic path and the heat flow is only during the isothermal steps as adiabatic step means there is no heat exchange.

So, let us consider these steps A, B, C and D carefully and even if you go through my explanation very carefully I would suggest that please pick up any standard textbook and read the chapter where the description or working principle of a Carnot cycle is described very carefully because nothing can replace a good book when it comes to this classical thought experiments.

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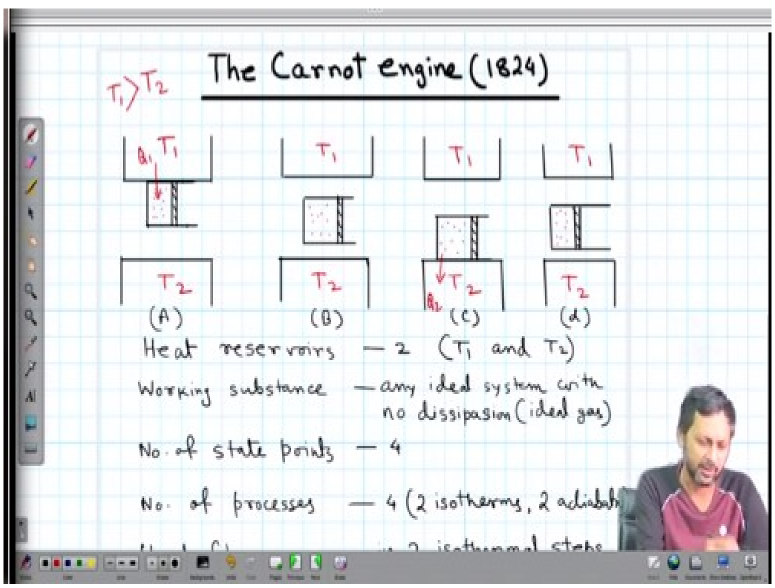

So, **so** initially we have the working substance which is the cylinder in this case is in contact with the reservoir T 1. Now during this process it will absorb an amount of heat Q 1, so this process as I have said is an isothermal process, the heat absorption is totally isothermal. Then what was done was? Please remember this was once again I am keep telling this, this is all hypothetical but Carnot engine is a very important aspect of classical thermodynamics for many reasons, we will come back to that.

So, then the contact is, it is disconnected from the heat reservoir but still it is allowed to expand. Now this expansion will be in the expanse of internal energy as we have discussed already, there will be a temperature drop, there will be a pressure change, there will be a volume change, everything will change but this change will be adiabatic. So, for an ideal gas system in general it should be a polytroph but when we have heat exchange is equal to 0 this change is an adiabatic exchange.

Then it will reach it is maximum allowed volume and lowest allowed pressure, then at that point it will be in contact with, I should not say maximum because what will happen is? After a certain amount of expansion it will be in touch with this cold reservoir where it will discard Q 2 amount of heat and not only that it will be compressed isothermally. So, let me go through it again. This is an isothermal absorption of heat then and there will be of course isothermal expansion during the process, there will be an end of adiabatic expansion, it will be placed on a cold reservoir and it will be compressed.

Now as compressed isothermally, as during the compression there will be heat generated because during the compression the work will be converted to heat. And as the temperature remains intact the internal energy will not change and this heat Q 2 will be rejected to the cold reservoir. And then it will be once again the thermal contact will be lost and the last part of the compression will be in adiabatic manner; so that it will come back to it is original volume. So, this is a 4 step cycle isothermal expansion, adiabatic expansion, isothermal compression here and adiabatic compression and finally this and this will be the same.

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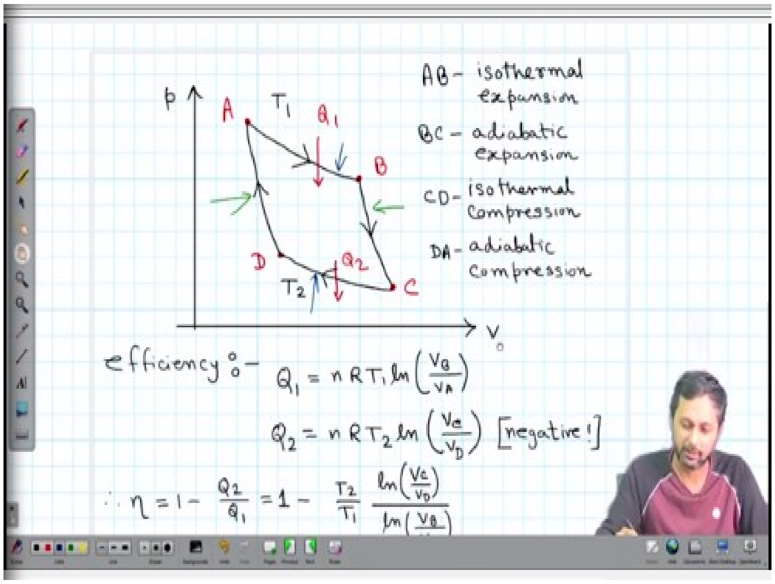

So, if I represent this in a pv indicator diagram it should look like this. So, we have points A, B, C and D, so these 2 will be my isotherms marked by red arrows, these are my isotherms. And if I just use green for here this will be my adiabatic. So, let us say I will use blue maybe, blue for isotherm and green for adiabatic. Now if we need to calculate the efficiency of this Carnot cycle which is once again is a very important result in a classical thermodynamics.

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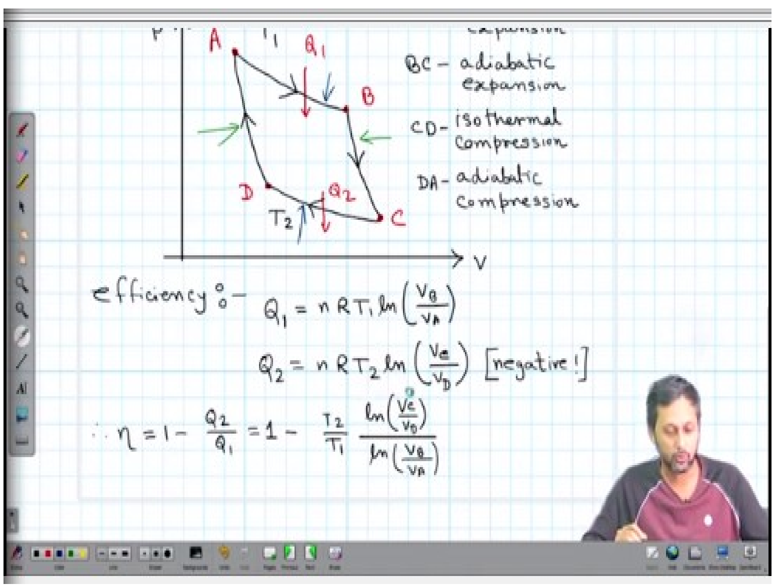

So, what we have to do is? So, in a cycle we know that in a cyclic process if we just calculate the amount of heat that is exchanged between the system and the reservoir then from there we can always calculate compute the work done, we do not need to compute the work done separately.

So, keeping that in mind, of course we have the tools, we know the relations in order to compute the work done in each of these cycles, compute the heat flow.

But we do not need to do that in order to compute the efficiency. So, we have a Q 1 which is the isothermal heat intake which is given by n R T 1, T 1 being the temperature and n being the number of moles ln V B by V A, so it is the final divided by the initial. Similarly for Q 2 the heat exchange which will be equal to once again the work done. But please remember in this case work is done on the system, so the work done will be negative, so we have n R T 2.

Once again T 2 is the temperature of the reservoir ln V C by V D, so this is not final by initial but initial by final because there is when we write final by initial we can do that but it will come with a negative sign because heat is being extracted from the system. So, considering that into account if we consider the sign of the Q 2 to be the heat coming out of the system already. We can simply absorb that negative sign and we can write nRT 2 ln V C by V D.

So, the efficiency eta will be 1 minus Q 2 by Q 1 which is 1 minus T 2 by T 1 ln V C by V D divided by ln V A by V B. Now what we can do is, we can use the equations of this 2 adiabatic path BC and DA in order to compute a relation between this ratios here and compute the final efficiency, so let us do that.

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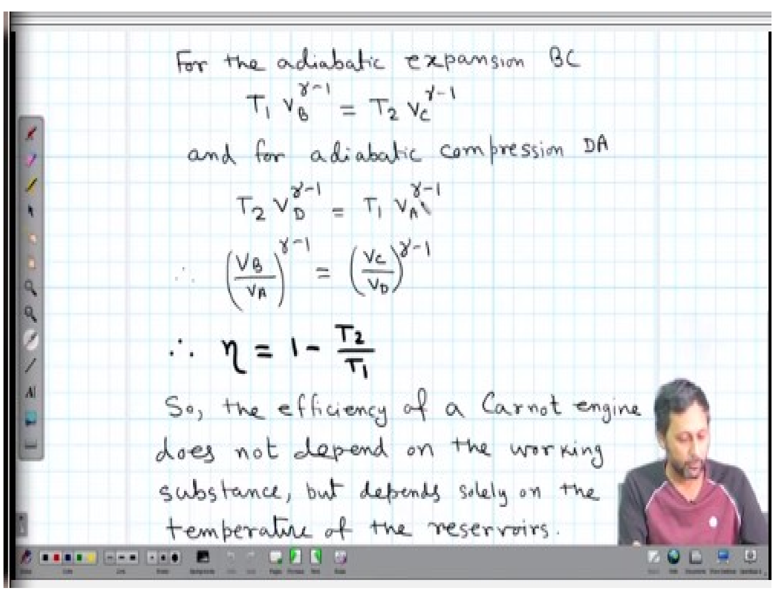

Now for the adiabatic expression BC the temperature and volume is related by T V to the power gamma minus 1 is equal to constant. Now as BC is the process in which the temperature at this point is T 1 and temperature at point C is T 2 and the volume let us say is V B and V C. So, we write T 1 V B times gamma minus 1 is equal to T 2 V C times gamma – 1. And for the adiabatic compression here D and A, so here the temperature is T 2 volume is V D, temperature is T 1 and volume is V A.

So, we write T 2 V D to the power gamma minus 1 is equal to T 1 V A to the power gamma minus 1. Now from these two relations if we just divide them. Let us say basically we should invert this relation and divide, so that this term is divided by this term and this term is divided by this term, I hope you understand. So, it will be V B by V A whole to the power gamma minus 1 is equal to V C by V D whole to the power gamma minus 1.

So, you see V C by V D is equal to V B by V A. So, I mean we can get rid of this gamma minus 1 is very easily because these are the same power we can get rid of it. So, V B by V A is equal to V C by V D. Now if we go back here, so we see this lawn of this quantity is equal to ln of this quantity, so efficiency eta is essentially 1 minus T 2 by T 1. So, the efficiency of a Carnot engine most importantly does not depend on the working substance.

I mean we have taken ideal gas, we could have very well be taking Van der Waals gas then these relations will be slightly different but let me tell you this same equation would have come out of it, same relation would have come out. So, there are many other variations just take a standard book like for example Zymansky which is a very classical textbook of thermodynamics. You will find at least 3, 4 different types of working substance Carnot cycle described using different working cycles.

For example a vapour and liquid mixture stretched sorry a surface film, a paramagnetic system. In all those cases different proper Carnot cycle can be defined and in every case it can be proved that eta is equal to 1 miuns T 2 by T 1. So, it does not matter what is the working substance that we have taken here in order to describe a Carnot cycle. We always get the exact same efficiency; it depends only on the temperature of the reservoir.

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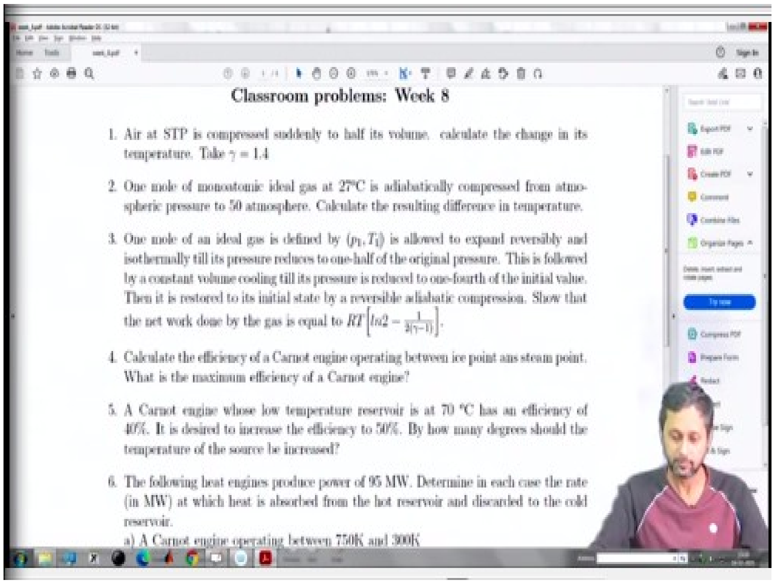

So, let us move to the classroom problems we have as promised I have added some more problems which will be done over next few lectures. So, the first problem is 1 mole of an ideal gas is defined by p1, T 1 is allowed to expand reversibly and isothermally till it is pressure reduces to 1 half of the original pressure. This is followed by a constant volume cooling till it is pressure is reduced to one fourth of the initial value then it is restored to it is initial stage by a reversible adiabatic compression.

Show that the net work done by the gas is equal to RT ln 2 minus 1 over 2 by gamma minus 1. So, we start from this point p1, T 1 and first is an isothermal expansion till the volume reduces to half or the pressure reduces to half. And then a constant volume cooling till it is pressure is reduced to one fourth of it is initial value.

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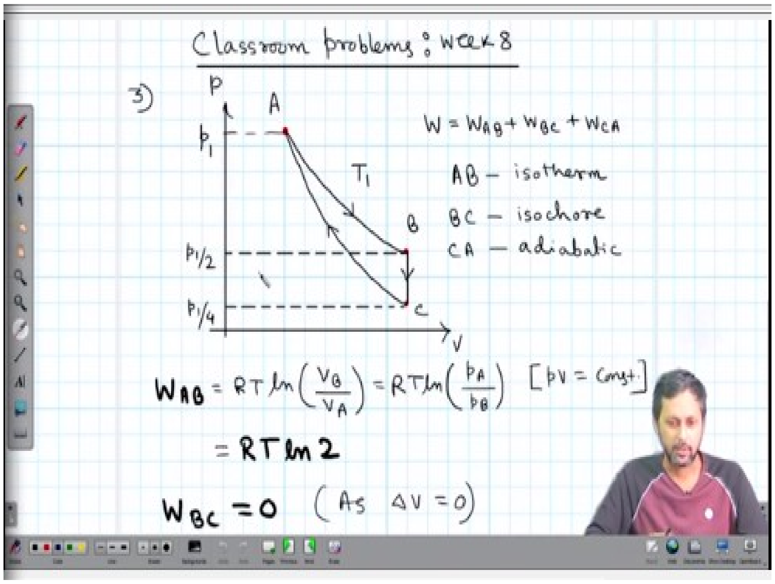

Now in order to represent that in a pv indicator diagram, we have this isotherm here this AB which is an isotherm. Then we have a BC which is an isochore and the corresponding pressure is p 1, p 1 by 2, p 1 by 4 and then finally we have an adiabatic that takes us all the way to the initial point A. So, it is also a cyclic process and what we need to do is? We need to compute the final work done in this system.

So, and this is an ideal guess, so remember that pv = RT that relation is valid and pv = constant for the path A to B. So, if we keep that in mind W AB which is RT ln V B by V A which is the work done by the gas which is positive, so it is RT lon p A by p B so because pv = constant and what is pA by pB? pA by pB is nothing but a factor of 2 because pA is equal to p 1, pB is equal to p 1 by 2. So, finally we get a ln 2 here and our relation is RT ln 2.

Now, what is the work done in path BC which has to be equal to 0 because it is a constant volume isochoric process? And finally the work done along this path CA which is an adiabatic. Now for an adiabatic work done we have already computed the expression which is W CA = 1 by gamma minus 1 p c v c minus p A V A. So, initial minus final 1 by gamma minus 1 initial minus final. Now initial pressure is p 1 divided by 4, final pressure is p 1, final volume is V 1, what is the initial volume?

Initial volume; please remember this is a constant volume process, so the volume here should be the volume here, now what is the volume here? The pressure has reduced to half as pv is equal to constant the volume has to be doubled.

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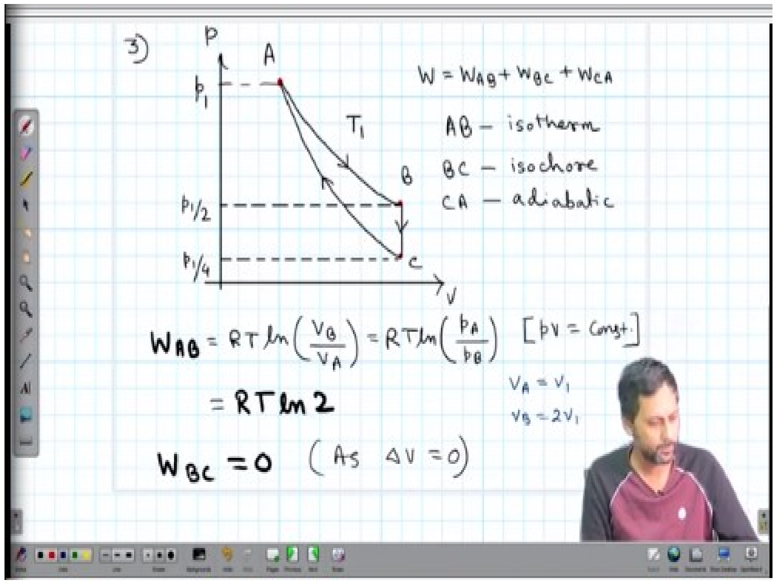

So, the V A is equal to V 1 let us call it V 1, so V B is equal to 2V 1.

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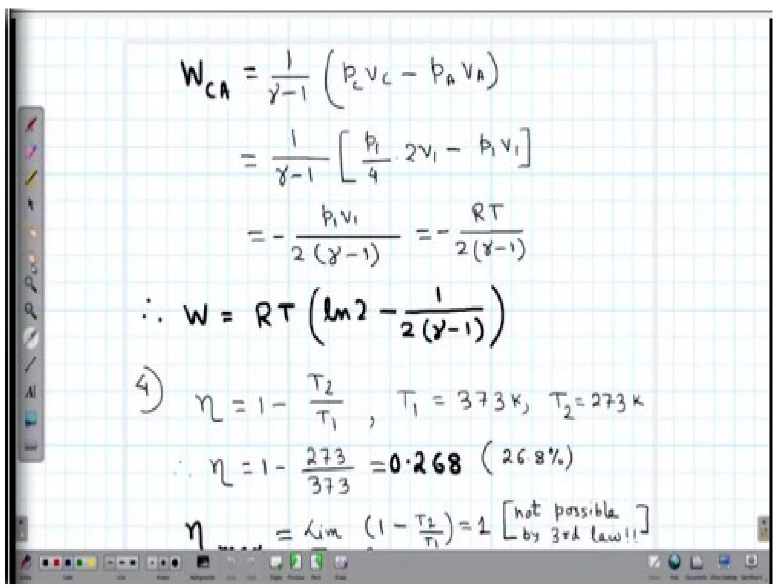

So, if we simply put 2V 1 p 1 by 4 and this is p 1 V 1 here, so we get 1 minus, so there is the overall minus sign, ok, so because this will be p 1 V 1 by 2 minus p 1 v 1 and we have minus p 1 v 1 by 2 gamma minus 1 and p 1 v 1 is once again is equal to RT because this is the ideal gas equation. So, we have minus of RT by 2 gamma minus 1. So, the total work done is a combination of this plus this plus this second segment is 0. So, we have RT ln 2 minus 1 by 2 gamma minus 1. So, next problem calculate the efficiency of a Carnot engine operating between ice point and steam point, what is the maximum efficiency of a Carnot engine?

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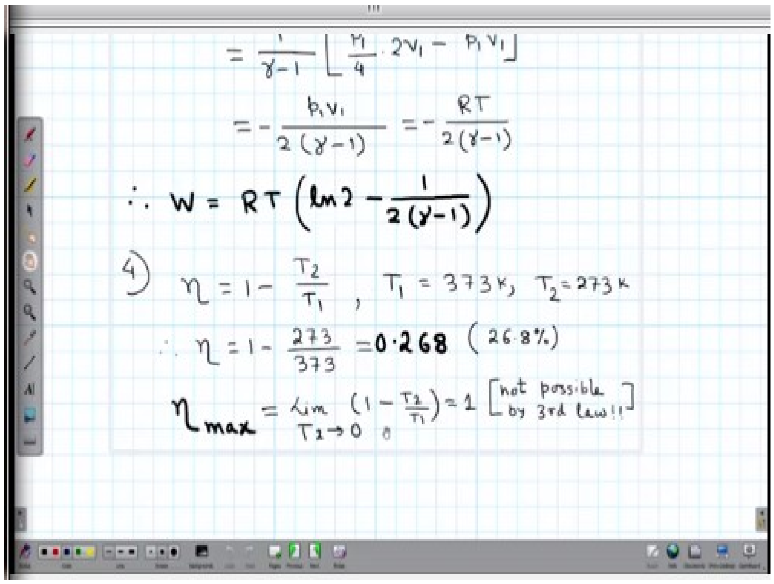

So, we have eta is equal to 1 minus T 2 by T 1, T 1 the steam point is 373 Kelvin, T 2 the ice point is 273 Kelvin, so eta is 1 minus 273 divided by 373 is equal to 0.268 which is once again we can just express the efficiency in percentage which is this multiplied by 100 which will give you approximately 26.8%. Now for the maximum efficiency we get the maximum efficiency when in this expression T 2 goes to 0.

So, that means the cold reservoir is at absolute 0, so eta max is limit T 2 going to 0 1 minus T 2 by T 1 which is equal to 1. But once again this is not possible there is a third law, I mean we have not yet discussed second law of thermodynamics. And let me tell you there is a third law that actually prohibits any system any physical system to go to absolute 0. So, we can never have an efficiency of Carnot engine minus 1 because absolute 0 can never be reached.

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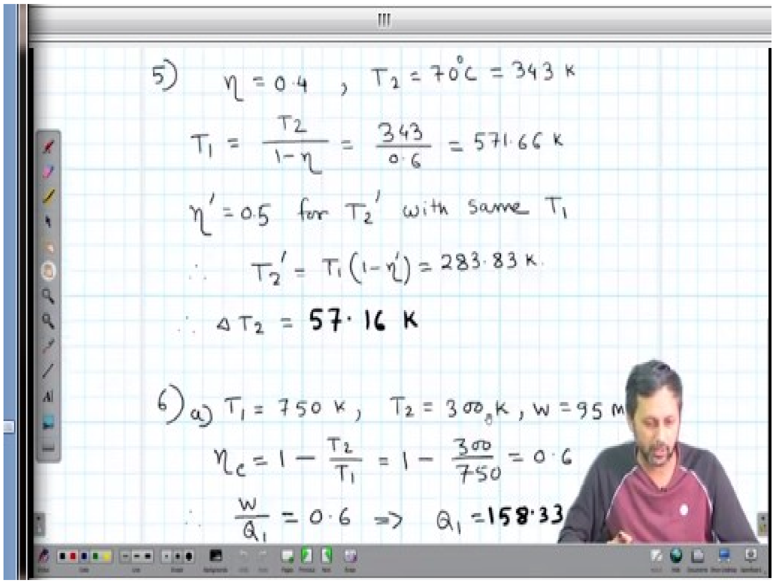

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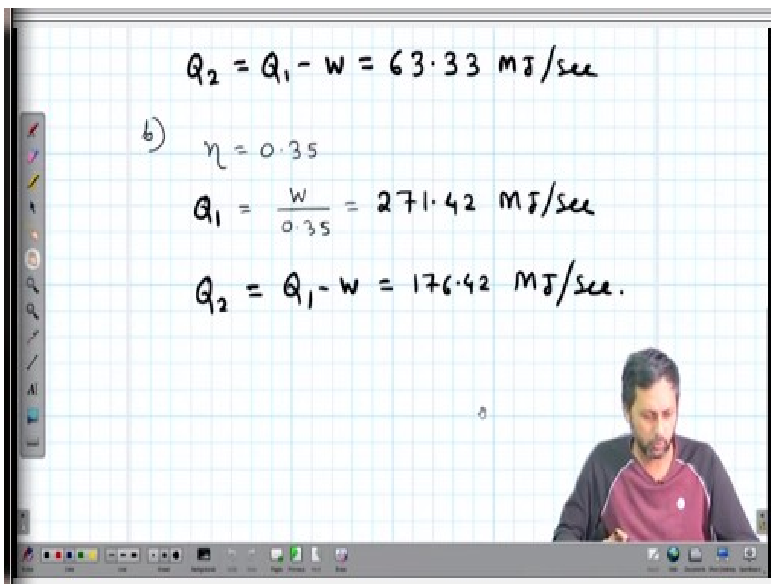

So, there are 2 more problems which we will probably, so time is up, so probably what we will do is we will just move this start the next class with these 2 problems. And then in the next lecture we will be discussing something called a refrigerator which is just the opposite of an engine and also we will be discussing Carnot theorem. And finally from there we will go to the second law of thermodynamics, so till then goodbye.