Thermal Physics
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Lecture-36
Topic-Cyclic Processes

Hello and welcome back to week 8 lecture series of this NPTEL course on thermal physics. Now this week we will be primarily discussing heat engines and related thermodynamic processes. So, before we go into the cyclic process where which is very important for this week's lecture. Let me start once again by reviewing or revisiting adiabatic process.

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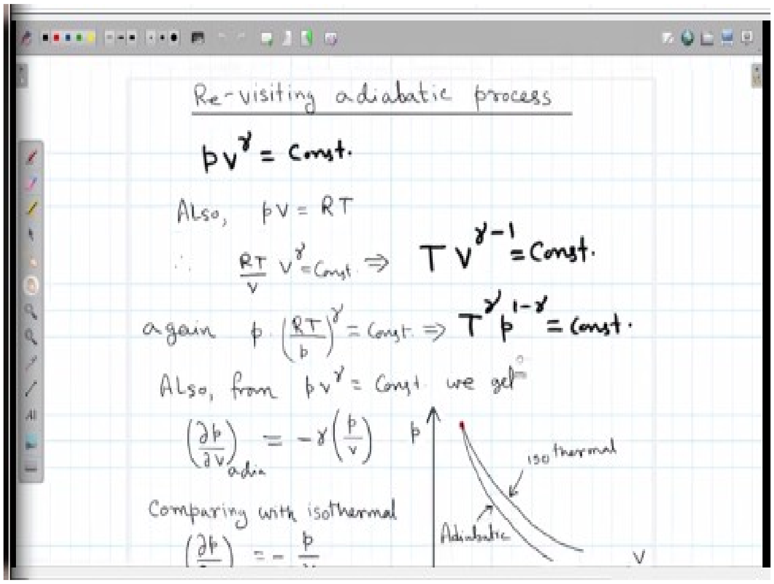

Re-visiting adiabatic process

$$pv^{\gamma} = \text{Const.}$$

Also, $pv = RT$

$\therefore \quad \frac{RT}{v} v^{\gamma} = \text{Const} \Rightarrow T v^{\gamma - 1} = \text{Const.}$

again $p \cdot \left(\frac{RT}{p}\right)^{\gamma} = \text{Const.} \Rightarrow T^{\gamma} p^{1-\gamma} = \text{Const.}$

Also, from $pv^{\gamma} = \text{Const.}$ we get

$$\left(\frac{\partial p}{\partial v}\right)_{adia} = -\gamma\left(\frac{p}{v}\right)$$

Comparing with isothermal

$\left(\frac{\partial p}{\ }\right) = -\frac{p}{\ }$

Now for an ideal gas the adiabatic process is described by pv to the power gamma is equal to constant which is a special case of a polytropic process with delta q is equal to 0. Also when we consider ideal gas 1 mole of ideal gas we have pv is equal to RT. So, we can rearrange this equation, so let me just quickly do this go through these steps here. So, we have pv to the power gamma is equal to constant but once again pv is equal to RT.

So, rearranging this equation we can either eliminate we can once eliminate pressure in terms of over replace pressure in terms of temperature or we can replace volume in terms of temperature. And we get to these two new relations one is T v to the power gamma minus 1 is equal to constant and the other one is T to the power gamma p to the power 1 minus gamma is equal to constant.

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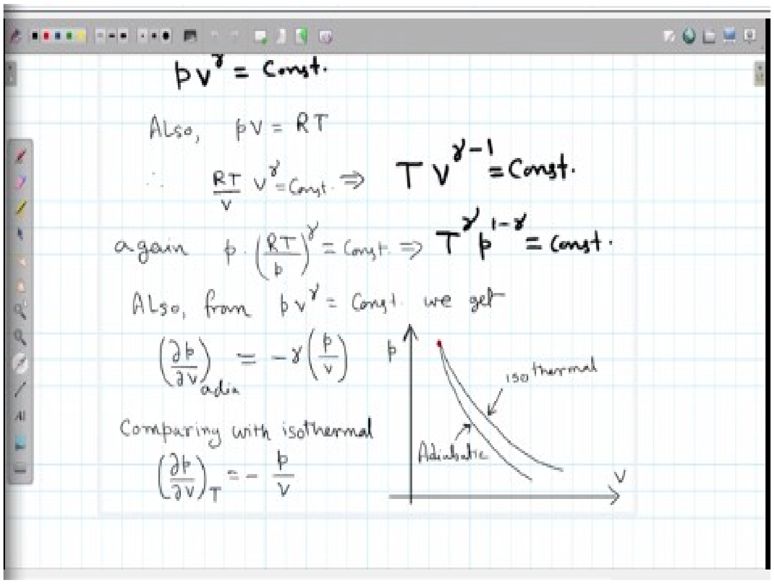

$pv^\gamma = \text{Const.}$

Also, $pV = RT$

$\therefore \frac{RT}{V} V^\gamma = \text{Const.} \Rightarrow TV^{\gamma-1} = \text{Const.}$

again $p \cdot \left(\frac{RT}{p}\right)^\gamma = \text{Const.} \Rightarrow T^\gamma p^{1-\gamma} = \text{Const.}$

Also, from $pv^\gamma = \text{Const.}$ we get

$$\left(\frac{\partial p}{\partial v}\right)_{adia} = -\gamma\left(\frac{p}{v}\right)$$

Comparing with isothermal

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{p}{v}$$

Now up to here this is only mathematical manipulation but let me tell you this relations will be extremely useful when will be doing several calculations on different cyclic processes. Now what is cyclic process? I will come back in a moment. Now also one thing we can do is we can start with this original equation which is pv to the power gamma is equal to constant and we can take a differential for an adiabatic process.

Now I have just put a suffix adia that that is represent that del p del v is taken for an adiabatic process. That means we are taking differentiation of this relation and this gives us minus gamma times p divided by v. So, typically we have to put the constant quantity here, now it so happens that in an adiabatic process the quantity which remains constant we have not defined that as of now.

We will come back to that may be towards the end of this week's lecture. Anyway, so what we care about is the slope dp dv is the slope in this pv indicator diagram for an adiabatic process. And compare this with an isothermal process which is minus p by v the slope of course there is a factor of gamma and typically gamma is great for any gas assembly the gamma is greater than 1. So, the slope of an adiabatic process is more than that of an isothermal process.

And I think we have discussed it already, we have made this statement already, now here we have proved it formally. So, if we just start from this initial point mark by this red dot here and if we go through an isothermal path or an adiabatic path in the same indicator diagram the adiabatic path will be steeper as compared to the isothermal path because there is a factor of gamma in the slope.

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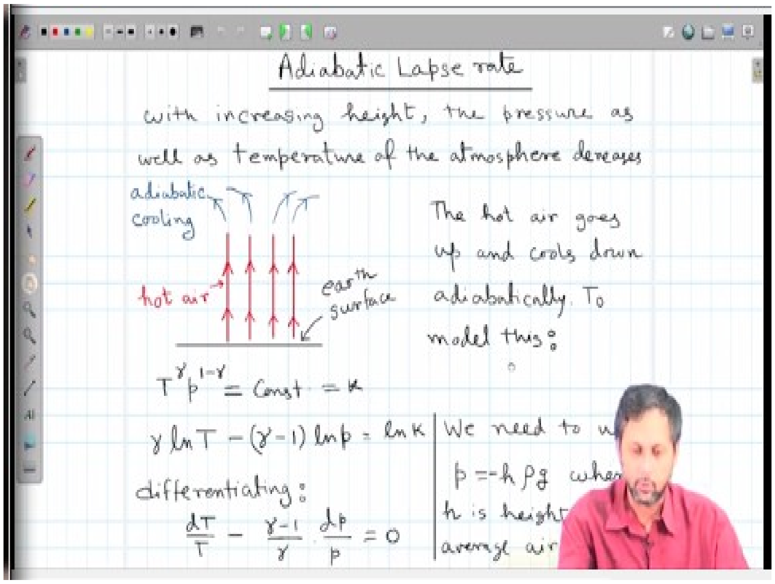

Adiabatic Lapse rate

with increasing height, the pressure as well as temperature of the atmosphere decreases

The hot air goes up and cools down adiabatically. To model this:

$$T^{\gamma} p^{1-\gamma} = Const = K$$

$$\gamma \ln T - (\gamma - 1) \ln p = \ln K$$

differentiating:

$$\frac{dT}{T} - \frac{\gamma-1}{\gamma} \cdot \frac{dp}{p} = 0$$

We need to u

$p = -h \rho g$ wher

h is height

average air

Now let us quickly look into another application of this adiabatic process or the equation of adiabatic process which is called the adiabatic lapse rate. Now what is that? Let us assume that this line here is the earth surface, now the air which is in contact with the earth surface it gets heated up. Because sunlight is falling on the earth surface the earth surface absorbs the sunlight and if the temperature goes up the surface of the earth becomes hot.

So, the air that is in immediate contact with the surface becomes hot and it goes up, as it goes up it expands and this expansion can be approximated by an adiabatic expansion which is in true sense we will see later on that the actual measured values are slightly deviated from this prediction when we predict a pure adiabatic expansion, which is obvious. But as a first principle calculation let us just take this process of cooling off by thermal expansion as an adiabatic process.

Now when that happens, the air becomes cool here and then once again it comes down and comes back towards earth's surface and the circulation goes on. I think we are all familiar with this process, all we have to do here we need to calculate an expression for temperature change as a function of height from the earth's surface. So, in order to do that we take this relation.

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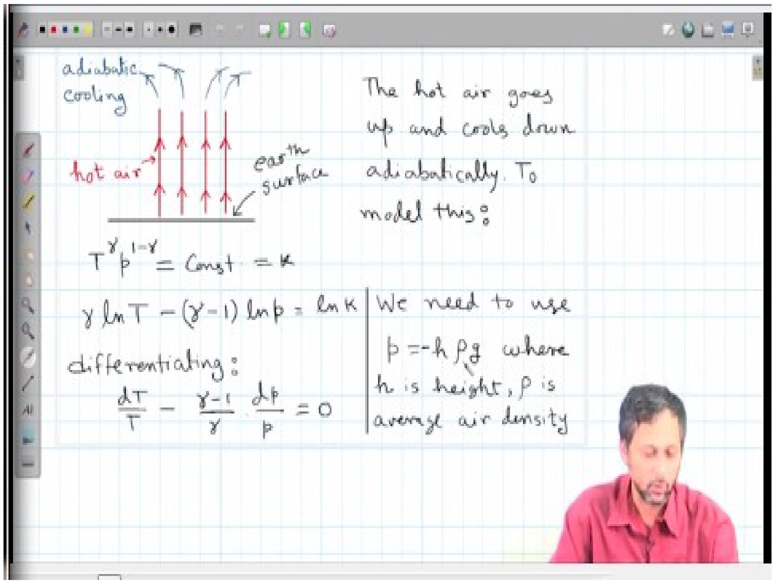

The hot air goes up and cools down adiabatically. To model this:

$$T^{\gamma} p^{1-\gamma} = Const. = K$$

$$\gamma \ln T - (\gamma - 1) \ln p = \ln K$$

differentiating:

$$\frac{dT}{T} - \frac{\gamma-1}{\gamma} \frac{dp}{p} = 0$$

We need to use $p = -h \rho g$ where h is height, ρ is average air density

So, we start from here T to the power gamma p to the power 1 minus gamma is equal to constant. And we take log of this relation which is gamma ln T minus gamma minus 1 ln p is ln k, k is a constant. Now if we differentiate this relation we get the right hand side becomes 0 immediately because this is a constant, so we get dT by T is equal to minus gamma is equal to 1 divided by gamma ln dp by p is equal to 0. Now all we have to do is instead of now we want it in terms of height, so instead of pressure we just use p is equal to minus h rho g. As h is the height from the earth's surface and as we go up the pressure decreases that is why there is a minus sign, rho is the average density of air and g is the acceleration due to gravity.

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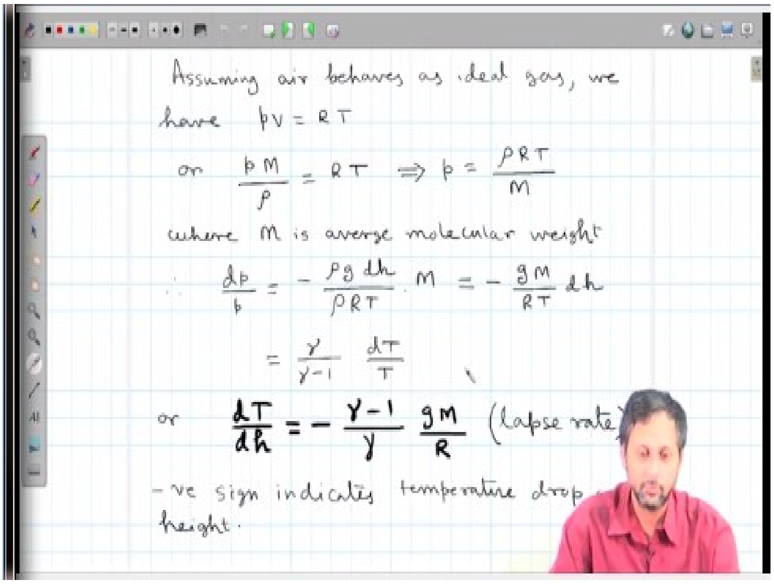

Assuming air behaves as ideal gas, we have $pv = RT$

or $\frac{pM}{\rho} = RT \Rightarrow p = \frac{\rho RT}{M}$

where M is average molecular weight

$$\therefore \frac{dp}{p} = -\frac{\rho g\, dh}{\rho RT} \cdot M = -\frac{gM}{RT}\, dh$$

$$= \frac{\gamma}{\gamma - 1} \frac{dT}{T}$$

or $\frac{dT}{dh} = -\frac{\gamma - 1}{\gamma} \frac{gM}{R}$ (lapse rate)

-ve sign indicates temperature drop height.

Now if we substitute that and also we have to keep in mind and we assume here that air is a ideal gas which is once again it is only an approximation. But for low density cases of course any gas behaves like an ideal gas in the low density limit. So, we have pv = RT. Now v we can simply write as M by rho where M is the molar mass or molecular weight and rho is the density. Now for air we can always define an average molecular weight we know the composition of air, it fluctuates a bit but it more or less remains same.

And please remember this calculations are only approximate, so we can define M and if we put p = RT rho divided by M as we have done here. So, this equation the right hand side, so dp will be is equal to minus rho g times dh and so dp by p will be minus rho g times dh divided by rho RT times M, M goes up here. So, rho, rho cancels out and we have g M divided by RT times dh. So, now this once again dp by p is equal to what? dp by p is equal to gamma divided by gamma minus 1 dT by T.

So, we have basically gamma times gamma divided by gamma minus 1 dT by T is equal to minus g M by RT dh. So, I am sorry this is not very conveniently written probably I should just take it down here, it will be more easy, I will do it, I will do this correction. And finally we get dT dh is equal to minus gamma minus 1 divided by gamma g M by R which is called the adiabatic lapse rate, this gives you the change in temperature as a function of h.

And negative sign indicates that as we go up in height temperature decreases just like pressure, as we go up the pressure decreases. Now here also it is kind of a linear relation and see the sorry this should be M here, my mistake. So, you see the beauty of this technique is rho cancels, so we do not have to worry about the density because the density changes as we go up as we know that air becomes lighter and lighter. So, the density changes with cancellation of rho in this part we have just got rid of that problem here, all we have to worry about is the average molecular weight.

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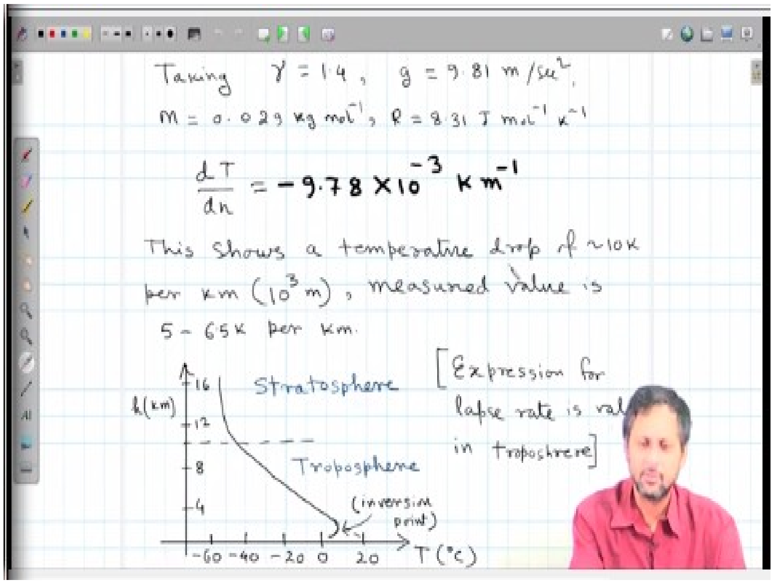

Taking $\gamma = 1.4$, $g = 9.81\ m/sec^2$,
$M = 0.029\ kg\ mol^{-1}$, $R = 8.31\ J\ mol^{-1}\ K^{-1}$

$$\frac{dT}{dh} = -9.78 \times 10^{-3}\ K\ m^{-1}$$

This shows a temperature drop of ~10K per km (10^3 m), measured value is 5 – 6.5K per km.

[Expression for lapse rate is val in troposhrere]

Now for air the average molecular gamma we can take as 1.4, g is well known, M is 0.029 kg per mole that is the average molecular weight and of course R is the universal gas constant. If we put this number in here in this expression we get dT dh = -9.78 into 10 to the power minus 3 Kelvin per meter inverse. So, that means if we go up by 1 kilometer which is 10 to the power 3 meters then you see the temperature change should be approximately 10 Kelvin, so 9.78 of approximately 10 Kelvin.

But in reality we generally get at average temperature change between 5 to 6.5 Kelvin which is understandable because we have made so many approximations. First of all with this process is not strictly adiabatic, secondly we have considered air as an ideal gas which might be a good approximation at heights where the air is really thin. But at least towards the close to the ground where the air is pretty dense, close the pressure is approximately 1 atmosphere this might not be a good approximation.

But anyway this is the real life experimental data and we see that in stratosphere above let us say 10 kilometers that there is a deviation. But in this part in the troposphere which is close to the earth's surface. The law is more or less followed except for the very close within first few meters of the or maybe first 50 meters of the ground level, there is a inversion point and this is a phenomena that is known as the inverted temperature inversion.

So, that is not very common, especially if we can experience this in a cold night when there is no wind flow we can sometimes realize that top floor of a high rise building is warmer as compared to the ground floor. So, that is a slightly different situation but typically except for this end part for most part all the way in the troposphere atmosphere. The adiabatic equation of adiabatic lapse rate is more or less followed although the value which is theoretically predicted is higher as compared to what we have measured experimentally, anyway. So, this is about adiabatic process the relations and the adiabatic lapse rate.

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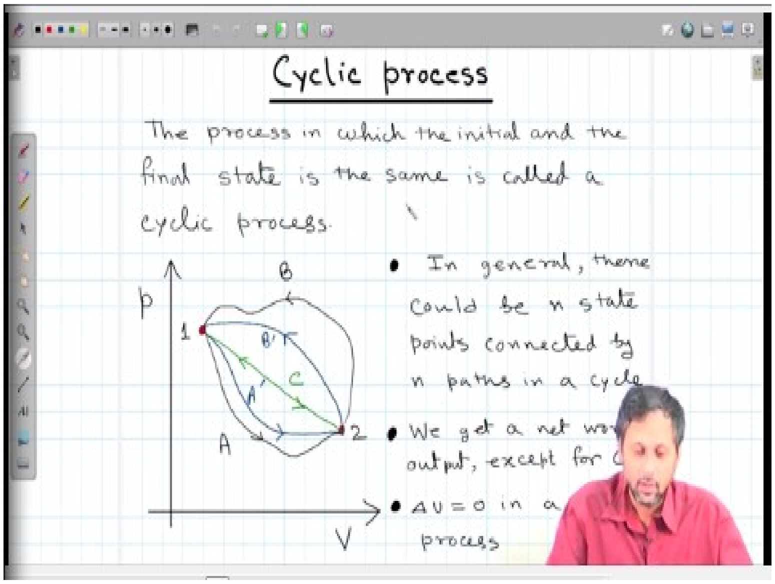

Cyclic process

The process in which the initial and the final state is the same is called a cyclic process.

- In general, there could be n state points connected by n paths in a cycle
- We get a net wo output, except for
- $\Delta U = 0$ in a process

Now let us focus on the cyclic process. Now cyclic process is going to be very important for this week's lecture. Why and what is the significance of cyclic process? We will come back in the next lecture. But today we will discuss about cyclic process it is what is the definition of a cyclic process and of course we look into some key aspect of a cyclic process.

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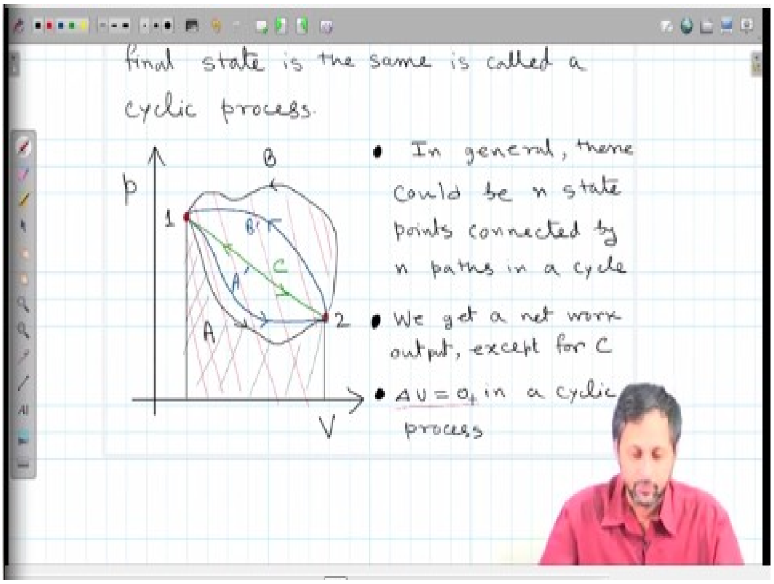

final state is the same is called a cyclic process.

- In general, there could be n state points connected by n paths in a cycle
- We get a net work output, except for C
- $\Delta U = 0$, in a cyclic process

Now cyclic process as the name suggest is a process in which if we draw this in that process if we draw the state points in an indicator diagram. So, we have one point at 1, the second point at 2, so what we can do is we can go from 1 to 2 in many different ways. We can either take path A and B or we can take path A prime and comeback by path B prime or we can take path C and come back by the same path.

I mean there are many ways there are infinite number of possible ways in which we can go from point 1 to point 2 and come back. So, but the idea is when we come back and please remember all these processes are considered quasi-static. So, once we complete the process or we start from 1 and come back to the point 1 by visiting point 2, this process is called a cyclic process when the initial and the final state is the same.

Now in general I have just given you 2 state points which are connected by possible combinations of paths or basically we take sets of 2 paths of different one in forward direction, one in the reverse direction let it be A, B, let it be A prime, B prime but they form the complete circle. But in general there could be any number of state points, we can have points like these only thing is and.

So, if we have n points we need to have at least n paths which are connecting this n points and the whole idea of a cyclic process is we start from point 1 go through all these n points fine, but we finally come back to point 1. So, if we do that that is the only criteria of a cyclic process that we start and end at the same state point. Now what happens is if we just go by path C and we follow path C while going from 1 to 2 and come back in the same path then what happens?

C, work done during this process, so please remember this is the pv indicator diagram and we are talking about a hydrostatic system which is some kind of a gaseous system enclosed and the process we are talking about is quasi-static. So, the work done the area under this curve if I just draw 1 curve here. If we just draw 1 curve here and 1 curve here, so let us say if we talk about path A, this area under this curve here will give you the work done.

Similarly while returning if we take path B, so this area marked by red will give you the work done. So, definitely if we take path A and come back by path B there will be a net work done either by the system or on the system, I hope you understand. So, that is true for any path except for case where we are following the same path while coming back and going back, I mean coming to point 2 and going back to point 1 that is the path C in this case.

Only in that case the work done while coming from 1 to 2 and while going back from 2 to 1 is exactly the same and the system does not perform any work at the end or no work is performed on the system. But all other cases when the path of incoming path and outgoing path is not the same, there have to be a net work done either on the system or by the system. So, for example when we are taking path A and then the work done is less as compared to when we are taking path B when we are going back, so the work has to be done by the system. So, we will come back to that in more details.

And of course one thing you have to keep in mind as we are going back to the same state point in the end of a cycle the net change in internal energy which is delta u is always 0 in a cyclic process. Because out of delta U delta Q and delta W, only delta U is a state function, the 3 parameters we deal with in the first law of thermodynamics out of that only delta U is a state function. So, irrespective of whichever path we take and even if we have not talked about dissipative forces here. But even if there are dissipative forces present in a cyclic process as long as the process is truly cyclic that means the initial and the final state points are the same we have delta U = 0.

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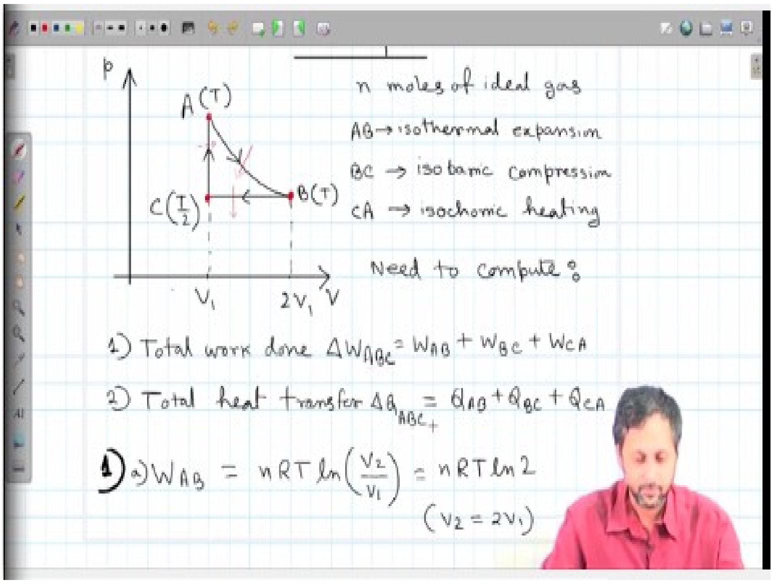

n moles of ideal gas
AB → isothermal expansion
BC → isobaric compression
CA → isochoric heating

Need to compute :

1) Total work done $\Delta W_{ABC} = W_{AB} + W_{BC} + W_{CA}$

2) Total heat transfer $\Delta Q_{ABC} = Q_{AB} + Q_{BC} + Q_{CA}$

1) a) $W_{AB} = nRT \ln\left(\frac{V_2}{V_1}\right) = nRT \ln 2$
$(V_2 = 2V_1)$

So, let us take an example of a cyclic process then we will understand it little better. So, let us assume that we have n moles of ideal gas enclosed in a piston cylinder arrangement. We start from point A and we have 3 state points point A, we go to point B, from there we go to point C and then finally we come back to point A. So, it is a valid cyclic process because we are going back in a circle.

I mean we could have come from A to C, C to B and B to A also it would have been a cyclic process. But what I am trying to tell you that finally ABC or ACB whichever we where we look at it is a cyclic process. Now what are these paths? AB is the isothermal expansion from a volume V 1 to a volume 2V 1. So, the volume becomes double, whatever the initial volume it becomes double.

Path B to C is a isobaric compression, so keeping the pressure same we go back to the original volume. And we will see later on that the temperature has to be reduced by a factor of 2 because the pressure is reduced by a factor of 2. And then finally point C to A is basically an isochoric heating. Here the volume is the same, only we have to increase the pressure and the best way of doing it keeping the same volume is by hitting the system.

So, the temperature goes up and the pressure also goes back to it is original value. Now in the cycle what we need to compute is the total work done which is given by delta W ABC and total heat transferred during this process which is given by delta Q ABC. So, delta W ABC we can break it down into 3 parts which is W AB, W BC and W CA. So, these are 3 segments of this path and we have to compute the work done.

Similarly for total heat transfer there are 3 segments and we have to compute the work done W Q AB, Q BC and Q CA. So, let us start with the work done 1 and we first compute W AB which is isothermal expansion from A to B and we have discussed it in the previous week's lecture which is W AB is equal to nRT ln V 2 by V 1 and in this case V 2 is equal to 2V 1, so it is simply nRT ln 2. Now for the isobaric compression, it is even simpler, it will be simply whatever the value of p we have not indicated that whatever the value of p of point B and point C the same actually.

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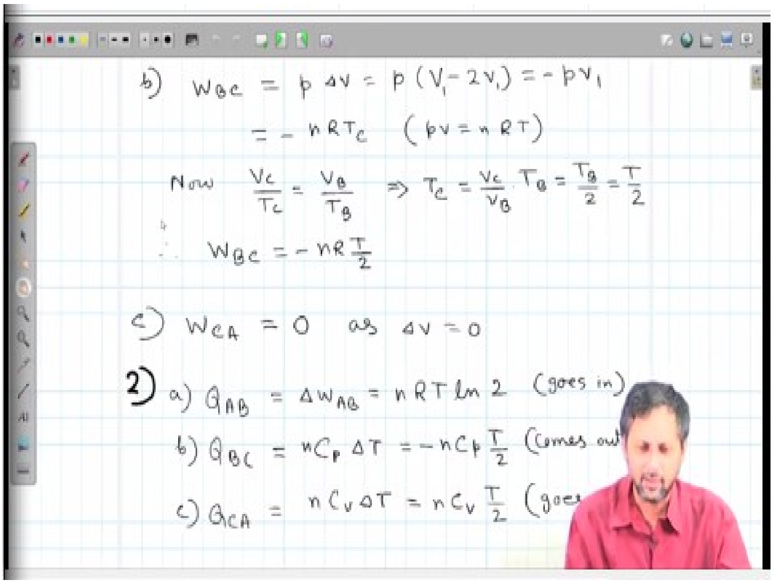

b) $W_{BC} = p\,\Delta V = p\,(V_1 - 2V_1) = -pV_1$

$= -nRT_C \quad (pV = nRT)$

Now $\frac{V_C}{T_C} = \frac{V_B}{T_B} \Rightarrow T_C = \frac{V_C}{V_B} T_B = \frac{T_B}{2} = \frac{T}{2}$

$\therefore W_{BC} = -nR\frac{T}{2}$

c) $W_{CA} = 0$ as $\Delta V = 0$

2) a) $Q_{AB} = \Delta W_{AB} = nRT \ln 2$ (goes in)

b) $Q_{BC} = nC_p\,\Delta T = -nC_p\frac{T}{2}$ (comes ou

c) $Q_{CA} = nC_v\,\Delta T = nC_v\frac{T}{2}$ (goes

That times delta V which is V 1 minus 2V 1 because the volume is compressed, so it is minus pV 1 and because it is a n moles of ideal gas we have minus nRT C. T c being the temperature of point C, I am just indicated it by this one because the volume V 1 corresponds to this point C here. So, minus pV 1 will be equal to minus nRT c. Now V c by T c is equal to V B by T B which gives T c is equal to T B by 2 T B being the temperature of point B and please remember it is the initial process AB is an isotherm.

So, let the temperature of A becomes T, so the temperature of B is also T, so temperature of C has to be T by 2. So, W BC is equal to minus n R T by 2 and W CA is equal to 0 because it is a isochoric process in which there is no change in volume. So, as delta V = 0, we have the W AC = 0. Now for the heat transfer, now for a isothermal process as the temperature does not change, there is no change in the internal energy. And we have W AB or Q AB = W AB, sorry I should not put delta here I have not used delta for this segments, so Q AB is equal to W AB is equal to n R T ln 2.

And please remember this is a positive definite quantity n R T this is all positive, ln 2 is also positive. So, the heat as per the convention we have already discussed while discussing first law the heat goes in, this is very important, the heat is actually going in to this system here. Now Q BC, Q BC is the temperature or that is the heat that is needed for going from point B to point C. And as we see here the volume decreases keeping the pressure constant, so heat must be negative. So, heat must be taken out from the system and exactly what we get here, so it is a constant pressure process, so it is n C p delta T which is minus n C p T by 2.

Because change in temperature delta T is equal to minus T by 2 and here the heat comes out. So, if I just indicate this by an arrow like this, this arrow actually tells you that the heat is coming out of the system. And finally the last process CA which is once again heat going in because here the temperature increases keeping the volume constant and the pressure increases. So, it is a constant volume process so we have n C v delta T which is n C v T by 2 and the heat here goes in. So, what we see and now if I compute the total of delta W ABC and delta Q ABC what do we see?

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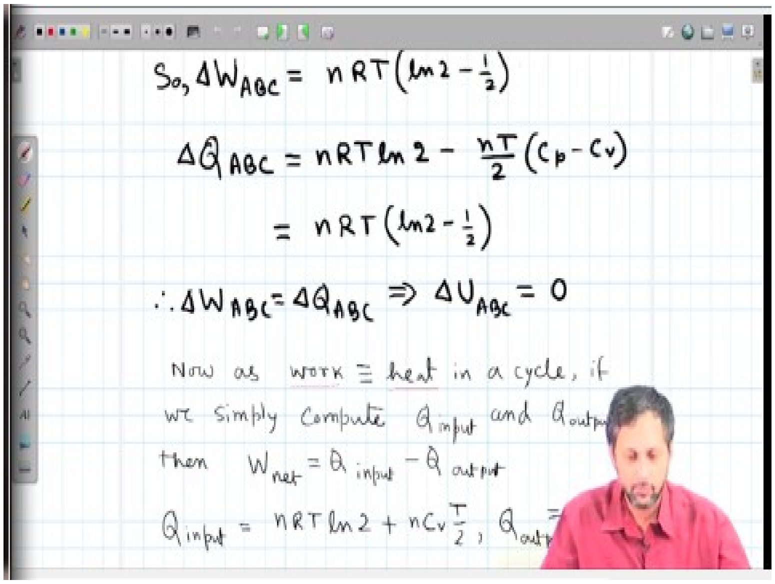

So, $\Delta W_{ABC} = nRT\left(\ln 2 - \frac{1}{2}\right)$

$$\Delta Q_{ABC} = nRT\ln 2 - \frac{nT}{2}(C_p - C_v)$$

$$= nRT\left(\ln 2 - \frac{1}{2}\right)$$

$$\therefore \Delta W_{ABC} = \Delta Q_{ABC} \Rightarrow \Delta U_{ABC} = 0$$

Now as work ≡ heat in a cycle, if we simply compute Q_{input} and Q_{outp}

then $W_{net} = Q_{input} - Q_{output}$

$Q_{input} = nRT\ln 2 + nC_v\frac{T}{2}$, Q_{out} =

We see delta W ABC is equal to n R T ln 2 minus half and delta Q ABC is equal to n R T ln 2 minus n R T by 2 C p minus C v which is once again R. So, n R T ln 2 minus half, so what we see is delta W ABC that means total work done in this cyclic process is equal to total heat that is exchanged during this cyclic process and this is absolutely obvious. Because finally delta U ABC is equal to 0, it has to be as we have already discussed it is a state function, so delta U ABC has to be equal to 0.

Now as the work performed during the cycle is equivalent to heat that is exchanged during a cycle. If we simply compute the input heat and the output heat, so then we can by the difference of that we can compute the net work. Because heat, so what we see is total heat is equal to total work done. So, whatever heat that goes in and whatever heat that comes out the difference if that is positive that means the system has done some positive work that means work is done by the system.

And if that is negative then the work is done on the system which will be negative. So, the net work output from that system during a cycle will be defined as Q input minus Q output. Now in this case Q input is equal to n R T ln 2 plus n C v T by 2. So, the addition of this one and this one and Q output is equal to minus n C p T by 2. So, we will take it up in the next lecture from here.

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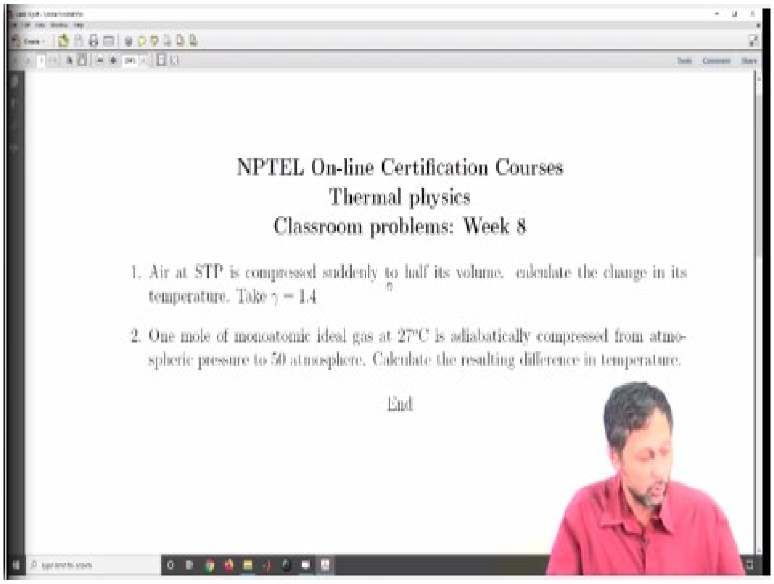

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Thermal physics
Classroom problems: Week 8

1. Air at STP is compressed suddenly to half its volume. calculate the change in its temperature. Take $\gamma = 1.4$
2. One mole of monoatomic ideal gas at 27°C is adiabatically compressed from atmospheric pressure to 50 atmosphere. Calculate the resulting difference in temperature.

End

Now for the remaining few minutes of today's lecture let me show you some problems on these adiabatic processes or at the relations we have derived during at the beginning of the class. So, at present I have only 2 problems, in this problem set as the lecture progresses we will have more and more problems, so let us solve this first 2 problems. Air at STP is compressed suddenly to half it is volume, calculate the change in it is temperature by taking gamma is equal to 1.4.

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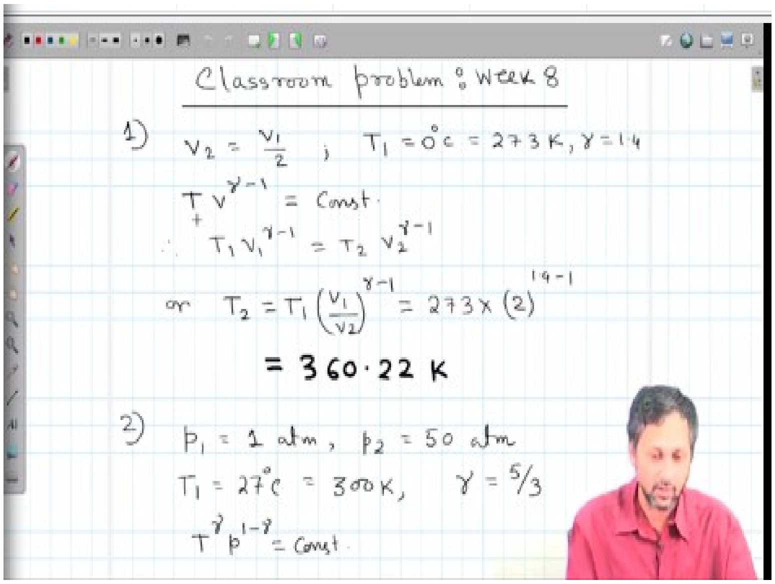

Classroom problem : Week 8

1) $V_2 = \frac{V_1}{2}$; $T_1 = 0^\circ C = 273\,K, \gamma = 1.4$

$TV^{\gamma-1} = Const.$

$\therefore T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

or $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 273 \times (2)^{1.4-1}$

$= 360.22\,K$

2) $p_1 = 1\,atm, \; p_2 = 50\,atm$

$T_1 = 27^\circ C = 300\,K, \quad \gamma = 5/3$

$T^{\gamma} p^{1-\gamma} = Const.$

So, V 2 is equal to V 1 by 2 air is compressed to half it is volume, T 1 at STP which is 0 degree centigrade which is 273 Kelvin, gamma is equal to 1.4. So, we have volume and temperature, so we choose the first relation which is or rather I should not call it the first relation, this relation where it is the relation of temperature and volume, so we get TV to the power gamma minus 1 is equal to constant.

So, we have T 1 V 1 gamma minus 1 is equal to T 2 V 2 gamma minus 1, so T 2 is equal to T 1 V 1 divided by V 2 gamma minus 1, so basically 273 into 2 to the power 1.4 minus 1 that is 0.4, we have 260.22 Kelvin. The second and the last problem for today's lecture 1 mole of monoatomic ideal gas at 27 degree Centigrades is adiabatically compressed from atmospheric pressure to 50 atmosphere pressure. Calculate the resulting difference in temperature.

So, initially it is at 27 degree centigrade, so we have T 1 is equal to 300 Kelvin, gamma is equal to 5 by 3 for a monoatomic ideal gas p 1 is equal to 1 atmosphere, p 2 is equal to 50 atmosphere. Here it is a relation between pressure and temperature that we should choose and we have T to the power gamma, p to the power 1 minus gamma is equal to constant.

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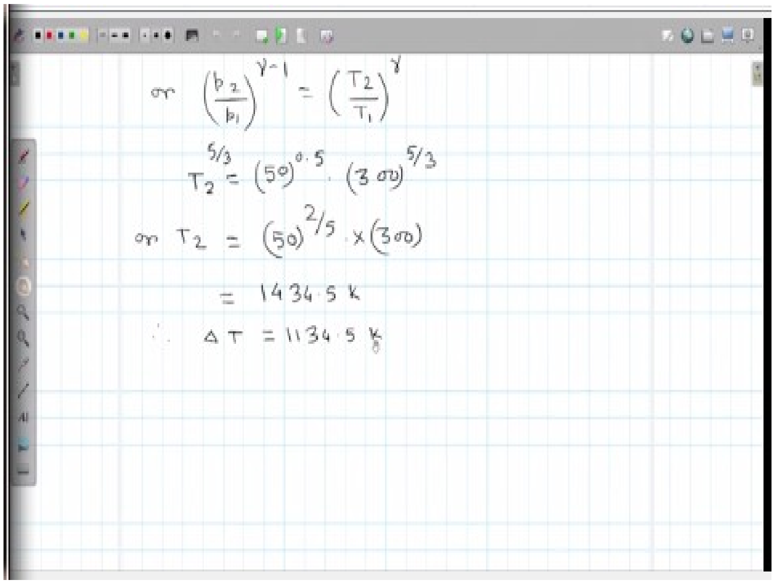

or $\left(\frac{p_2}{p_1}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)^{\gamma}$

$T_2^{5/3} = (50)^{0.5} \cdot (300)^{5/3}$

or $T_2 = (50)^{2/5} \times (300)$

$= 1434.5 \text{ K}$

$\therefore \Delta T = 1134.5 \text{ K}$

So, p 2 by p 1 whole to the power gamma minus 1 is equal to T 2 by T 1 whole to the power gamma and we have T 2 to the power 5 by 3 is equal to 50 whole to the power 0.5 times 300 whole to the power 5 by 3. So, if we rearrange we get T 2 is equal to 50 whole to the power 2 by 5 into 300 which is 1434.5 Kelvin, so the difference will be 1134.5 Kelvin. So, this concludes this week's lecture and sorry not this week today's lecture. And in the next lecture we will continue from with the cyclic process and we will define something called the efficiency of a cyclic process, till then thank you.