Thermal Physics
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Lecture - 34
General Description of Work Done and Specific Heat

Hello and welcome back to another lecture of this NPTEL lecture series on thermal physics. Now in the last lecture we have introduced first law of thermodynamics which is the energy conservation principle applicable for thermodynamic systems. And we talked about the functions like internal energy the total work done and total heat absorbed in a thermodynamic system. So, the first law is actually an interconnection between these three types of energy.

And we have also discussed we have also you know solved some of the problems which helped us for a better understanding of the first law of thermodynamics. Now we will continue from there but for today's lecture we will start with the work done in a closed gaseous system.

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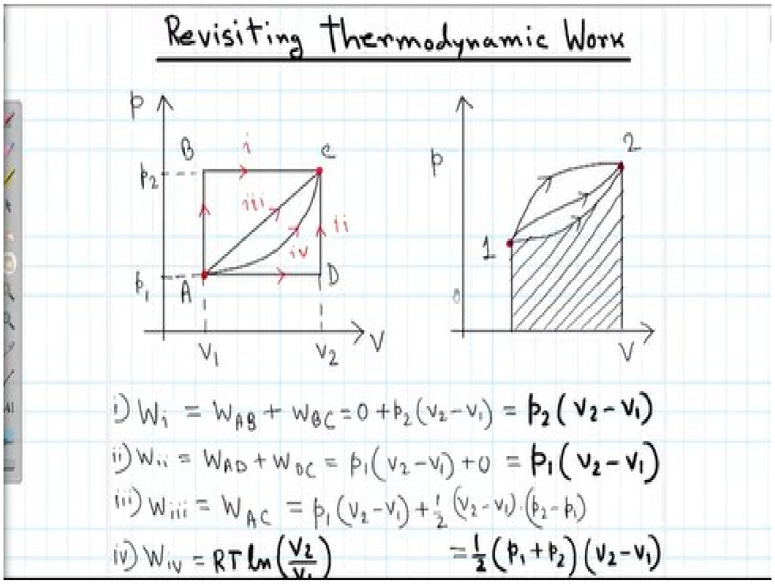

Revisiting thermodynamic Work

i) $W_i = W_{AB} + W_{BC} = 0 + p_2(V_2 - V_1) = p_2(V_2 - V_1)$

ii) $W_{ii} = W_{AD} + W_{DC} = p_1(V_2 - V_1) + 0 = p_1(V_2 - V_1)$

iii) $W_{iii} = W_{AC} = p_1(V_2 - V_1) + \frac{1}{2}(V_2 - V_1)\cdot(p_2 - p_1)$

$= \frac{1}{2}(p_1 + p_2)(V_2 - V_1)$

iv) $W_{iv} = RT\ln\left(\frac{V_2}{V_1}\right)$

So, let us start without any further delay. So, let us assume the system in question goes from the initial state marked with A to the final state marked with C point C in four different paths. Path one is along the rectangular along the path ABC so A and C are basically two points on the two corner points of a rectangle in this PV indicator diagram. So, one path is along one side ABC the other path is along the other side ADC marked as 1 and 2.

And two other path there are two other paths path number 3 is the diagonal along AC and path number 4 is marked here which is basically this one isotherm which is connecting point A and point C. So, all what we need to do is? We need to compute the work done in these four different cases. Now for case 1 it is pretty straightforward. So, you see this is W 1 is actually W AB + W BC and W AB in this vertical path the volume does not change volume is constant at V 1.

So, the initial volume is V 1 initial pressure is P 1 final volume V 2 final pressure P 2. So, we get everything in terms of P 1 V 1 and P 2 V 2. So, in this case the volume is 0 as a volume change is 0 so the work done is also 0. So, only work done is work done along the path BC. Now the pressure here will be constant at P 2 and the work done will be P 2 times V 2 minus V 1. So, the total work done it was is once again P 2 times V 2 - V 1.

Now along path 2 which is ABC we have W AD + W DC once again W DC will be 0 because there the volume does not change and W AD is nothing but P 1 times V 2 minus V 1. So, the total work done will be P 1 times V 2 minus V 1. For W 3 which is along the diagonal which is actually a combination of you know this work done is actually a combination of this area plus this triangular area. So, we have a rectangle with area P 1 times V 2 minus V 1.

And we have a triangle with base V 2 minus V 1 and height P 2 minus P 1. So, the total work done will be P 1 into P 1 times V 2 minus V 1 which is this rectangle here and half V 2 minus V 1 times P 2 minus P 1 simplifying we get half P 1 plus P 2 times V 2 minus V 1 and finally the isothermal work done along this path 4 is RT lon V 2 minus V 1 which we have proved in the last class. So, what we have learnt from this first of all we have learned once again that work done especially for a I mean for a thermodynamic system.

This is a path function so you see the although the initial and the final points are exactly the same in all these four cases this point A and point C, we have different values of work done for four different cases. And also, that work done is in a PV indicator diagram it is the area under this curve. So, for example in this right hand side diagram I have indicated that this is my initial state 1 final state 2 from 1 to 2 we can go into three different paths.

So, just to give you an illustration for the lowest path I have shaded the area which makes which actually is a measure of the work done while going from A to B sorry 1 to 2 along this path. Now similarly along if the work done, we have to compute the work done along this path we have to compute this area here and this is exactly what we have done in these all four cases. So, think about it you know this one is not so obvious because here we do not get a geometrical you know equation for this one.

Of course, we get a geometrical equation for this one but it is not very intuitive we cannot just do it by I estimate but for other cases you see P 2 times V 2 minus V 1 is nothing but this full area. Whereas P 1 times V 2 minus V 1 is this area and the third one is a combination of this area plus half of this rectangular area. So, these two things I wanted to ensure I mean I wanted to make you understand.

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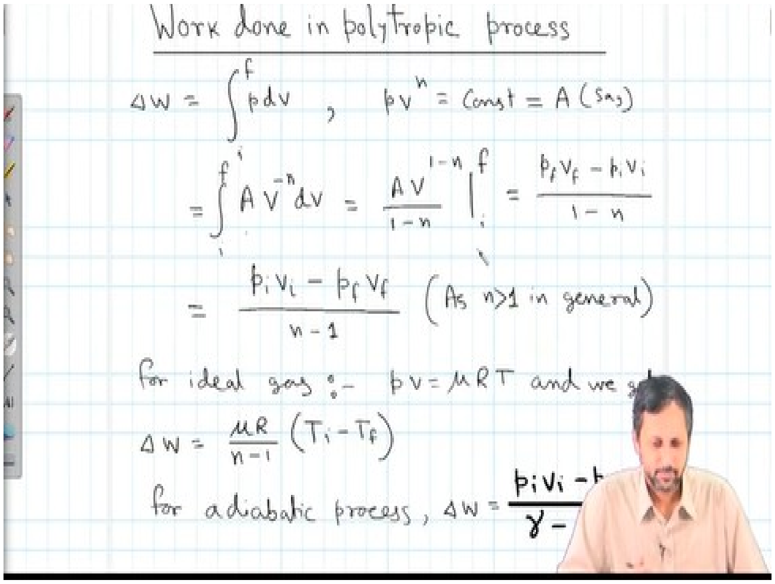

Work done in polytropic process

$$\Delta W = \int_i^f p\,dV \quad , \quad pV^n = \text{Const} = A \text{ (Say)}$$

$$= \int_i^f A V^{-n} dV = \left. \frac{A V^{1-n}}{1-n} \right|_i^f = \frac{p_f V_f - p_i V_i}{1-n}$$

$$= \frac{p_i V_i - p_f V_f}{n-1} \quad (\text{As } n>1 \text{ in general})$$

for ideal gas :- $pV = \mu R T$ and we

$$\Delta W = \frac{\mu R}{n-1} (T_i - T_f)$$

for adiabatic process, $\Delta W = \frac{p_i V_i -}{\gamma -}$

So, now let us revisit work done in a polytropic process. We have used this expression which we have derived here in the last class there was a mistake actually there was a sign error which I have also pointed out. So, just correct it yourself I mean I hope you understand I will try to correct it in the notes but I after going through this I think you can do it yourself also. So, we have i to F the initial state i final state F pdv, pdv being the work done.

So, and the equation of a polytropic process is pv to the power n is equal to constant let us say this constant is equal to A so, we can take any constant. So, i to f A v so P is actually A divided by v to the power n which is A v to the power minus n so i to f pdv is actually A v to the power minus n dv sorry A A v to the power minus n dv which is A v to the power 1 minus n divided by 1 minus n i to f which we can write please remember p v to the power n is equal to p i.

So, sorry p v to the power sorry A v to the power n v to the power minus n is equal to p so A v to the power one minus n is nothing but p v. So, we can write this as p f v f minus p i v i divided by 1 minus n. Now please remember n is typically greater than 1 so it is customary to write this as p i v i minus p f v f divided by n minus 1. So, just this is the correct expression and there was a sign error as I have said in the one of the problems, we have discussed in actually problem 2 part c there was a problem.

So, please correct that accordingly. Of course, we got the final answer right but intermediate stage there is a sign error. Now for ideal gas pv is equal to mu RT and we get delta w is equal to mu R divided by n minus 1 T i minus T f because we can substitute pv is equal to mu RT. So, this quantity in the which I have written as p i v i minus p f v f is actually T i minus T f. That is the temperature change and for general adiabatic process we have to substitute n I mean if the process is an adiabatic process where the what is an adiabatic process will come back very soon.

So, n will be replaced by gamma and we have delta w is equal to p i v i minus p f d f divided by gamma minus 1 and typically we know gamma is cp by C v which is greater than 1.

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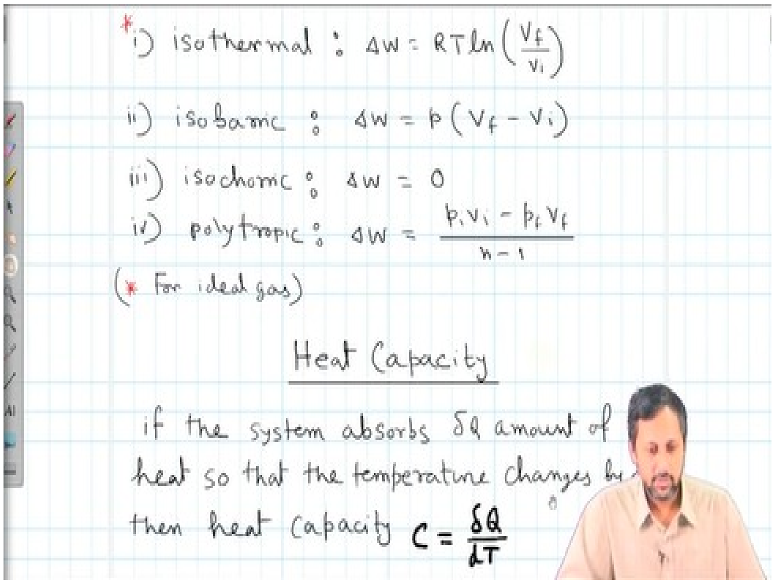

*i) isothermal : $\Delta W = RT \ln\left(\frac{V_f}{V_i}\right)$

ii) isobaric : $\Delta W = p\,(V_f - V_i)$

iii) isochoric : $\Delta W = 0$

iv) polytropic : $\Delta W = \frac{p_i V_i - p_f V_f}{n-1}$

(* For ideal gas)

Heat Capacity

if the system absorbs δQ amount of heat so that the temperature changes b

then heat capacity $C = \frac{\delta Q}{dT}$

With this let us quickly revisit the work done for some of the standard quasistatic processes. For isothermal delta w is equal to n r sorry not n RT ln v f minus v i but please remember this is strictly valid for an ideal gas. If the gas is non-ideal the equation of state is p is not equal to pv miuns n RT this is not valid. But the other three expression for isobaric process delta w is equal to p times v f minus v i isochoric process delta w is equal to 0 because delta v dv = 0.

And for polytropic process we just computed delta w is equal to p i v i minus p f v f divided by n minus 1. These are very general and this is valid for any gas assembly. So, let it be ideal gas let it be non-ideal gas these relations are valid. Given that the equation of a polytroph for even for a non-ideal gas is given by this relation p v to the power n is equal to constant. So, we will prove that very soon towards the end of this lecture.

And let me tell you although we will prove it for a I mean ideal gas assembly. This is in general valid for even for non-ideal gas assembly as well. Now let us move into one very important aspect for a gas assembly or a fluid system hydroscopic system which is the heat capacity. Now without going into much of the details we have already discussed about the heat capacity when we were discussing equipartition of energy.

And we have written one expression which is if E is the total energy of the gas assembly as per the equipotential theorem, then CV is equal to d dT. So, this is the expression we have used and frankly speaking we have done it only for ideal gas assembly. Now we did that without deriving this particular relation. Now it is time that we have a systematic derivation of that particular relation so that we have a better understanding.

Now in the definition of heat capacity with which I am going to give you right now is a very general definition and that can fit any system let it be a gas assembly let it be a stretched wire let it be I mean any system you can think of. If the system absorbs a heat delta Q and the temperature changes by delta dT then the heat capacity C is written as dQ by delta Q by dT. So, this is a very general definition there is nothing special about it. And we will apply this definition as and when required for the system of our interest.

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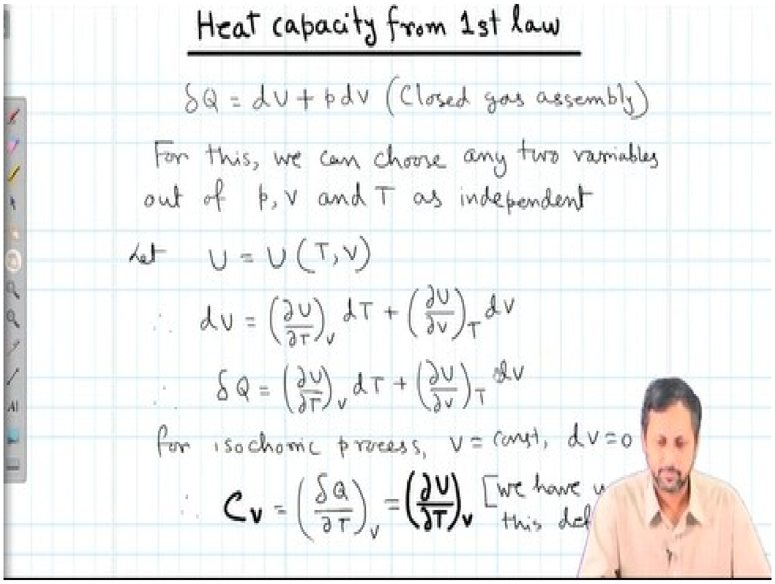

Heat capacity from 1st law

$\delta Q = dU + p\,dV$ (Closed gas assembly)

For this, we can choose any two variables out of p, V and T as independent

let $U = U(T, V)$

$\therefore\ dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

$\therefore\ \delta Q = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV$

For isochoric process, V = const, $dV = 0$

$\therefore\ C_V = \left(\frac{\delta Q}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V$ [we have this def

Now let us see what we can learn about heat capacity or what expressions we can derive for heat capacity starting from the first law. Now for a closed gas assembly the first law of thermodynamics takes the form dQ = dU + pdv that we have discussed in the last class dQ being the heat absorbed dQ being the change in internal energy and pdv is the work done by a gas assembly. Now for if we take and we have discussed what are the possible forms of delta w that is this particular term.

When we are considering other thermodynamic systems but for now, we are concentrating primarily on gas assembly or fluid assembly as a whole where the parameters of interest are pressure volume and temperature. Now out of these three parameters pressure volume and temperature we are free to choose any two as independent as they are related by the equation of state. Equation of state relates pressure volume temperature for any gas assembly.

So, let us for now assume that U which is the internal energy is a function of pressure sorry temperature and volume. This is the case then we can write U is equal to U of T, V so dU is equal to dU dT del U del T v dT plus del U del vT dv. So, this is the chain rule. I think you we are all familiar with this chain rule of differential calculus. So, dQ in this case will be del U del T v dT because so what we have to do is we have to go back to this particular relation.

We have to substitute for dU and then also we have pdv here. So, what do we do? We substitute for dU and we write delta Q is equal to del U del T v dT plus del U del v T dv sorry there should be another term o minus tech del U del T sorry I just ignored this term here plus p dv. So, this is the full expression I will correct that. So, for isochoric process when we have v is equal to constant then dv is equal to 0. Now when dv is equal to 0 immediately the second term in the inside this bracket all goes to 0.

Because dv is equal to 0 so anything multiplied by 0 is equal to 0 and we have heat capacity at constant volume C v which is del Q dT or rather del Q del T at constant volume is nothing but del U del T at constant volume. And this is the definition or this is the equation relation we have used when we were discussing equi partition theorem. U being in this particular case even U is the internal energy and that particular case we just wrote E capital E in place of U.

But it means absolutely the same thing. This is the internal energy that means pure for an ideal gas it is pure kinetic energy and as we have already seen for real gases there will be a potential energy part involved. But finally, C v is equal to del U del T v so we were what we used was proven at this point.

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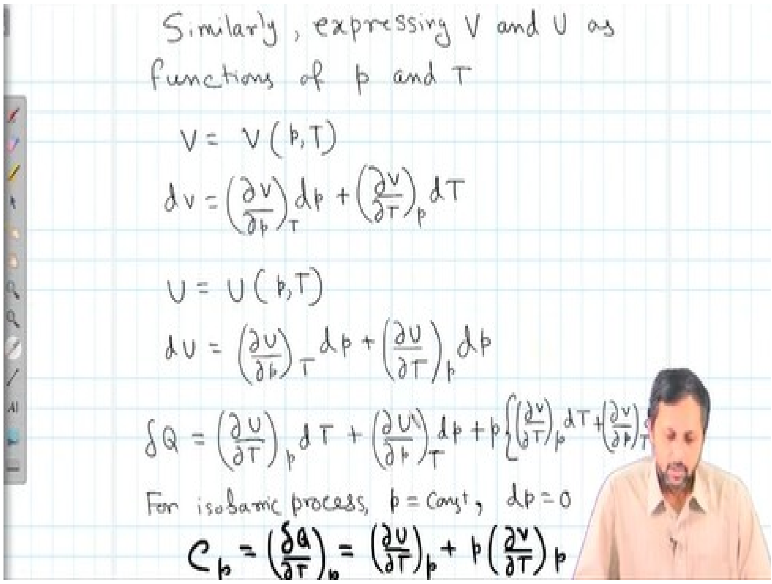

Similarly, expressing V and U as functions of p and T

$$V = V(p,T)$$

$$dv = \left(\frac{\partial v}{\partial p}\right)_T dp + \left(\frac{\partial v}{\partial T}\right)_p dT$$

$$U = U(p,T)$$

$$dU = \left(\frac{\partial U}{\partial p}\right)_T dp + \left(\frac{\partial U}{\partial T}\right)_p dp$$

$$\delta Q = \left(\frac{\partial U}{\partial T}\right)_p dT + \left(\frac{\partial U}{\partial p}\right)_T dp + p\left\{\left(\frac{\partial v}{\partial T}\right)_p dT + \left(\frac{\partial v}{\partial p}\right)_T\right.$$

For isobaric process, $p = \text{Const}$, $dp = 0$

$$C_p = \left(\frac{\delta Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial v}{\partial T}\right)_p$$

Now also what we can do is we can express both V and U as functions of pressure and temperature. So, we can write V is equal to V of T p so dv is equal to del V del p T dp plus del v delta p dT. Similarly, U is equal to U of T and p so dU is equal to del U del T del U del p T dp plus del U del T p dp. So, when we substitute all that into this in this particular form of first law that is d delta Q is equal to delta U or the dU plus p dv.

So, we have delta Q is equal to del U del T p dT plus del U del p T dp plus p del V del T p dT plus del V del p T dp. So, this is how we rearrange this term. Now for an isobaric process for which p is equal to constant we have dp is equal to 0 and so this term goes to 0 this term goes to 0 and we see both these terms as dT in common that dT goes down and that leaves us with C p which is del Q del T p that is equal to del U del T p plus p del V del T p.

So, in this case we have derived an expression for C p which is the heat capacity or when we are talking about one mole of gas assembly one mole of gas. This is the molar heat capacity and this will be the molar heat capacity per unit sorry for constant volume and this will be the molar heat capacity of at constant pressure. But frankly speaking at this point we have not discussed about the system size.

We do not know whether it is 1 moles or 100 moles or 10 moles we do not know anything about it so this relation is in general valid. When we are talking about molar heat capacity all we have to remember that this v will be the molar volume this U will be the molar internal energy pressure does not matter because it is an intrinsic variable anyway.

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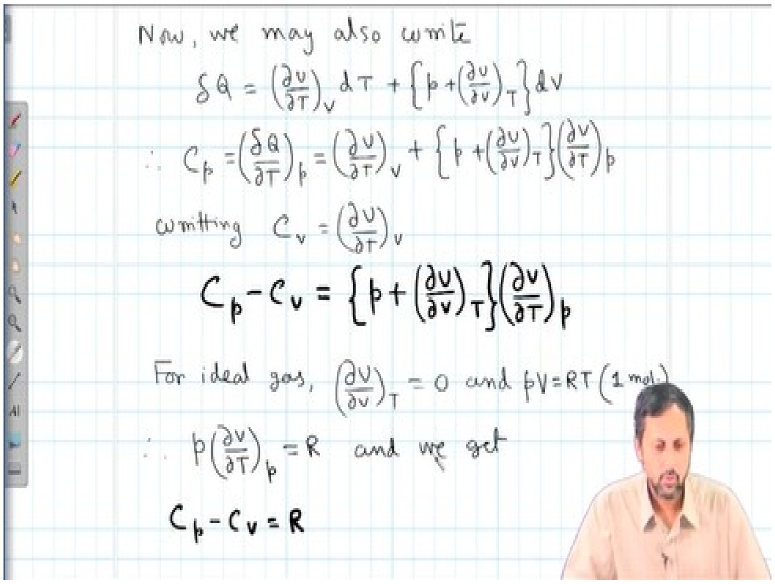

Now, we may also write

$$\delta Q = \left(\frac{\partial U}{\partial T}\right)_V dT + \left\{p + \left(\frac{\partial U}{\partial V}\right)_T\right\} dV$$

$$\therefore C_p = \left(\frac{\delta Q}{\partial T}\right)_p = \left(\frac{\partial U}{\partial T}\right)_V + \left\{p + \left(\frac{\partial U}{\partial V}\right)_T\right\}\left(\frac{\partial V}{\partial T}\right)_p$$

writing $C_V = \left(\frac{\partial U}{\partial T}\right)_V$

$$C_p - C_V = \left\{p + \left(\frac{\partial U}{\partial V}\right)_T\right\}\left(\frac{\partial V}{\partial T}\right)_p$$

For ideal gas, $\left(\frac{\partial U}{\partial V}\right)_T = 0$ and $pV = RT$ (1 mol)

$\therefore p\left(\frac{\partial V}{\partial T}\right)_p = R$ and we get

$$C_p - C_V = R$$

Now from this particular relation so we just we can go back to this particular relation and so basically delta Q = del U del T v dT + p + del U del v T dv. So, C p if we start from this particular relation then what will be the expression for C p. So, we have got one expression for C p here now if we go back here, what will be the expression for C p from this if we start from here. So, C p will be once again del U del T dot sorry del Q del T p which is see here this term is independent of pressure.

So, we have del U del T v dT goes down dT goes down here so we have C p is equal to the first term will be del T, del T at constant v + p + del U del p + del U del v T and there will be a T coming down d T coming down here at constant p. So, this equation I mean this term divided by d T at constant p will be del v del T p. Now this term is what del U del T v which is nothing but C v so, this is C p = C v plus an additional term here.

And if we just rearrange it and we can write then we can write C p - C v = p + del U del v T times del v del T p. This is one of the most important relations that we get from the first law of thermodynamics that is the difference in heat capacity. Now for ideal gas assembly what happens is del U sorry this my handwriting here is not that good so this is del U, I hope you understand this is U so do not mix it up with U and v.

So, we have del U del v T which is equal to 0 and once again we have pv = RT. Now this is for one mole because we have taken the what you call the equation of state for one mole of ideal gas. So, we have so what we have to do is now so for ideal gas the internal energy is not a function of volume. For ideal gas internal energy is a function of temperature only. So, that is why we have del U del v at constant temperature is equal to 0.

So, that essentially means if we change the volume of an ideal gas keeping that same temperature the internal energy of the gas assembly does not change. Then in this expression for ideal gas this second term becomes 0, this term goes to 0 as according to this relation. Now then we are left with p times del v del T p and starting from pv = RT if we just compute del v del T at constant pressure, we have p del v del T p = R because p is a constant here and we get C p - C v = R.

This is the famous relation that was first derived by Mayer and also for dilute gas system experimentally it was verified that the specific heat capacity. The difference of two specific heat capacities of an ideal gas is equal to the universal gas constant. Now this is only for ideal gas and also please remember that we generally we talk about two specific heats only two types of specific heat that is C p that specific heat at constant time pressure and C v which is specific heat measured at constant volume.

But please remember even if we make everything free like if we make the system to, I mean if we allow the system to change pressure and volume simultaneously and then we change the temperature even then there will be a specific heat associated with that process. We will come back to that in a moment.

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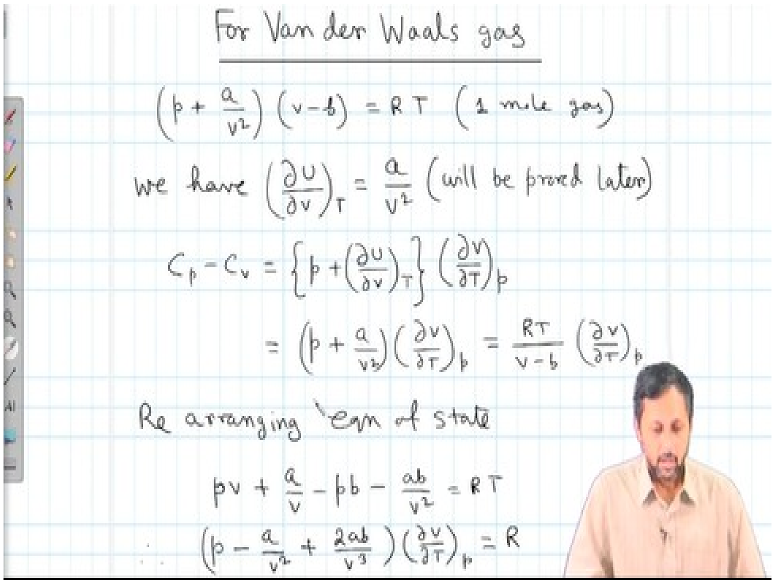

For Van der Waals gas

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT \quad (1 \text{ mole gas})$$

we have $\left(\frac{\partial U}{\partial v}\right)_T = \frac{a}{v^2}$ (will be proved later)

$$C_p - C_v = \left\{p + \left(\frac{\partial U}{\partial v}\right)_T\right\}\left(\frac{\partial v}{\partial T}\right)_p$$

$$= \left(p + \frac{a}{v^2}\right)\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{v-b}\left(\frac{\partial v}{\partial T}\right)_p$$

Re arranging eqn of state

$$pv + \frac{a}{v} - pb - \frac{ab}{v^2} = RT$$

$$\therefore \left(p - \frac{a}{v^2} + \frac{2ab}{v^3}\right)\left(\frac{\partial v}{\partial T}\right)_p = R$$

After we discuss this will be the last topic of the day will be the polytropic process where both pressure and volume can change simultaneously. Now we have discussed it for ideal gas. Now let us do it for Van der Waals gas. The Van der Waals gas one mole of Van der Waals gas the equation of state is p + a by v square times v - b = RT that we have discussed already. And also, for Van der Waals gas we have del U del v T = a by v square.

Now I cannot prove it to you but I promise that it will be proved later on when the time comes, we will talk about internal energy change we will talk about dual function expansion and then it will be proved. Now C p - C v = p + del U del v T del v del T p so this term we already know this is a by v square or this is given to us that it is a by v square so we have C p minus C v is equal to p plus a by v square del v delta T p and as p plus a by v square we can write as RT by v - b we simply write RT by v minus b del v del T p.

So, rearranging equation of state if we just you know open the bracket and do this multiplication, we have for the for equation of state we have pv + a by v - pb - ab by v square = RT. Now we have to compute del v del T p now for del v delta T p that means p remains constant if we take the differentiation with respect to temperature then the first term will be simply p minus second term will be minus a by v square plus two and the third term is a constant p times p is a constant.

And the fourth term is 2 ab by v cubed whole multiplied by del v del T p is equal to R right hand side we have only R. Now what we can do is? We can write this as p plus a by v square minus 2 a by v square so basically in we have 1 minus a by v square we write 1 plus a by v square minus 2 times a by v square. So, if we do that then also, we can rearrange the second term here or the all these four terms here.

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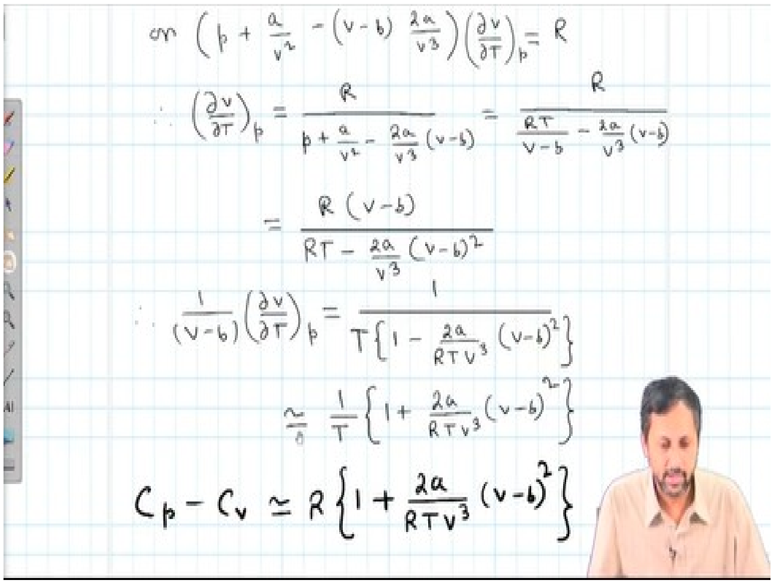

$$\text{or}\ \left(p + \frac{a}{v^2} - (v-b)\frac{2a}{v^3}\right)\left(\frac{\partial v}{\partial T}\right)_p = R$$

$$\therefore \left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p + \frac{a}{v^2} - \frac{2a}{v^3}(v-b)} = \frac{R}{\frac{RT}{v-b} - \frac{2a}{v^3}(v-b)}$$

$$= \frac{R(v-b)}{RT - \frac{2a}{v^3}(v-b)^2}$$

$$\therefore \frac{1}{(v-b)}\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{T\left\{1 - \frac{2a}{RTv^3}(v-b)^2\right\}}$$

$$\underset{0}{\simeq} \frac{1}{T}\left\{1 + \frac{2a}{RTv^3}(v-b)^2\right\}$$

$$C_p - C_v \simeq R\left\{1 + \frac{2a}{RTv^3}(v-b)^2\right\}$$

And we can write p plus a by v square minus v minus b 2a by v cubed whole multiplied by del v del T p is equal to R. So, rearranging we have del v delta T p is equal to R divided by p plus a by v square minus 2a by v cube v minus b once again for p plus a by v square we substitute RT by v minus b and then after rearranging we get R times v minus b divided by RT minus 2a by v cubed divided by multiplied by v minus b whole square. Now let us go back here we have C p - C v = RT by v minus b del v delta p.

So, if we just write RT by v minus b del v delta p or rather 1 by v miuns b del v delta p this will be 1 divided by c we can take out T common from this for these two terms and we can write this as sorry RT common from these two terms R will be cancelling out with this this R over here. So, we have T here 1 here and we have to divide it by the second term with RT and we can write 2a divided by RT v cubed v minus b whole square.

And this quantity we see a is already a small constant; we have 1 over R 1 over v cubed. So, this term as a whole is a small term smaller than 1 definitely. So, what we can do is? We can write this as 1 minus 2a by RT v cubed v minus b I mean in take it in the numerator and we can write this whole term to the power -1 and we using binomial expansion keeping only the first term we can approximately write this term is equal to 1 by T 1 plus 2a by RT v cubed v minus b square.

So, then we have to multiply this with RT so we have we multiply here with RT so T, T cancels leaving only R behind and the final expression we get is C p - C v = R multiplied by 1 plus 2a by RT by v cubed v minus b whole square. Now if you look at this term you see this is very interesting here at v is equal to b that means that very low volume when the volume is compressed so that the volume is equal to core volume.

And if you remember for volume is actually fourth time for a Van der Waals gas core volume, is I think three times I do not remember often but it is somewhat very close to the exact molecular volume that time the second term becomes 0 and we have C p - C v = R. Alternatively when we have very high value of v when the v becomes very high then the second term all together goes to 0 that mean or we can basically neglect the second term and we can have C p - C v = R.

So, when it comes to heat capacity difference in heat capacities at very high volume a Van der Waals gas behaves like a real gas which is understandable. But even at very low volume in a very compressed state the relation between or the difference in heat capacity becomes equal to that of a ideal gas.

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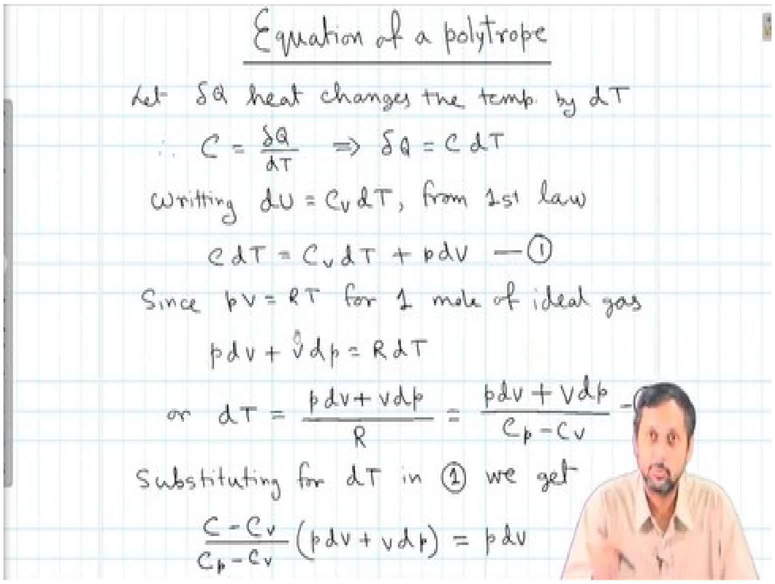

Equation of a polytrope

Let δQ heat changes the temp. by dT

$$\therefore C = \frac{\delta Q}{dT} \Rightarrow \delta Q = C\,dT$$

Writing $dU = C_v dT$, from 1st law

$$C\,dT = C_v\,dT + p\,dV \quad \text{— ①}$$

Since $pV = RT$ for 1 mole of ideal gas

$$p\,dv + v\,dp = R\,dT$$

$$\text{or} \quad dT = \frac{p\,dv + v\,dp}{R} = \frac{p\,dv + v\,dp}{C_p - C_v}$$

Substituting for dT in ① we get

$$\frac{C - C_v}{C_p - C_v}(p\,dv + v\,dp) = p\,dv$$

So, with this we come to the last topic of the day which is equation of a polytrophic process. We have already written that equation of a polytrophic process is pv to the power n = constant here we are going to prove it problem. Now as I have already mentioned that in a polytropic process we do not put any bar on the pressure volume and temperature. So, we let all three parameters change freely.

So, that means the heat capacity C which is dQ by dT is nothing but I mean it is not equal to C v not equal to C p. It is a totally independent or rather it is a new type of heat capacity. So, we can write dQ = C dT so writing dU = C v dT please remember that we can do because dU = C v = del u del T at constant volume. So, we it is okay if we write dU = C v dT we can write so dQ = C dT from first law we can write C dT = C v dT + p dv.

Since p v = RT please remember we are deriving the polytrophic equation of a polytroph for an ideal gas. So, this is our equation of state. We can differentiate this equation and write pdv + vdp = RdT. So, we can rearrange and write dT = pdv + vdp divided by R once again R = C p - C v for an ideal gas so it is pdv + vdp = C p - C v. Now substituting this dT this is let us call it equation two. Now substituting this into equation one which is C dT = C v dT + pdv we have C - C v C p – C v times p + pdv + vdp = p dv.

So, this is if we substitute this one into this one basically, we substitute for dT and we get after rearranging I have not shown the rearrangement but you can check it yourself.

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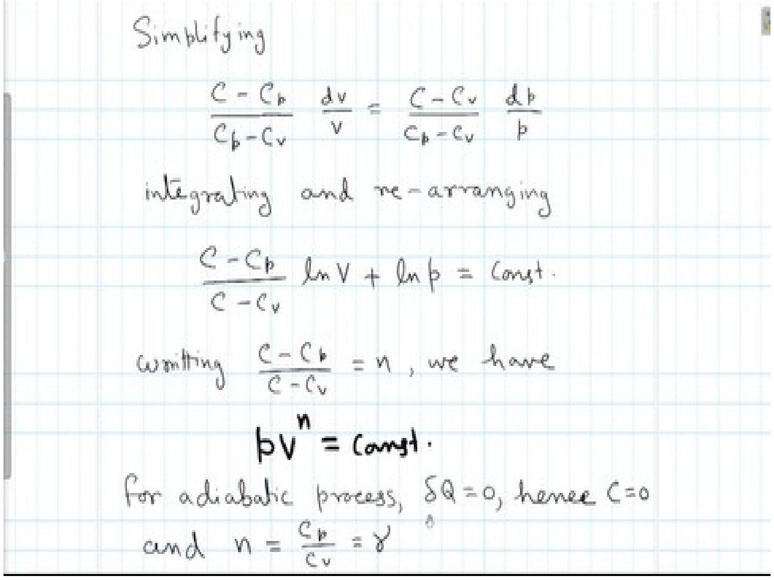

Simplifying

$$\frac{C-C_p}{C_p-C_v}\frac{dv}{v} = \frac{C-C_v}{C_p-C_v}\frac{dp}{p}$$

integrating and re-arranging

$$\frac{C-C_p}{C-C_v}\ln V + \ln p = \text{Const.}$$

writting $\frac{C-C_p}{C-C_v} = n$, we have

$$pV^n = \text{Const.}$$

For adiabatic process, $\delta Q = 0$, hence $C=0$

and $n = \frac{C_p}{C_v} = \gamma$

We can write this in this particular form C minus C p divided by C p minus C v times dv by v C - minus C v divided by C p minus C v d p by p and after integration and after some more rearrangement we get C miuns C p divided by C minus C v ln v plus ln p is equal to constant. Now writing this C minus C p my divided by C minus C v is equal to n we get p v to the power n = constant which is the general equation of a polytroph for an ideal gas. Now also we have to remember that what happens in an adiabatic process.

In an adiabatic process dQ is equal to 0 so that dC is equal to 0 because dQ is equal to C dT so if dQ is equal to 0 C has to be equal to 0 so we have n is equal to so if we have C is equal to 0 we have n = C p - C v is equal to gamma. So, this is where we have p v to the power gamma is equal to constant for a adiabatic process. So, that is where we stop in today's lecture and in the remaining lecture of this week we will be talking about the elastic properties.

And how they correlate with some of these some properties of the or some parameters of the ideal gas assembly or rather gas assembly. That coming that is coming in the next lecture and also will be doing some problems to understand this the whatever we have discussed so far in a better manner. Till then thank you.