Thermal Physics
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Lecture –3
Average Speeds in an Ideal Gas Assembly

Hello and welcome back. So, in today's lecture we are first starting with the concept of speed like there are different mean or average speed inside a gas assembly and we will be first discussing those okay.

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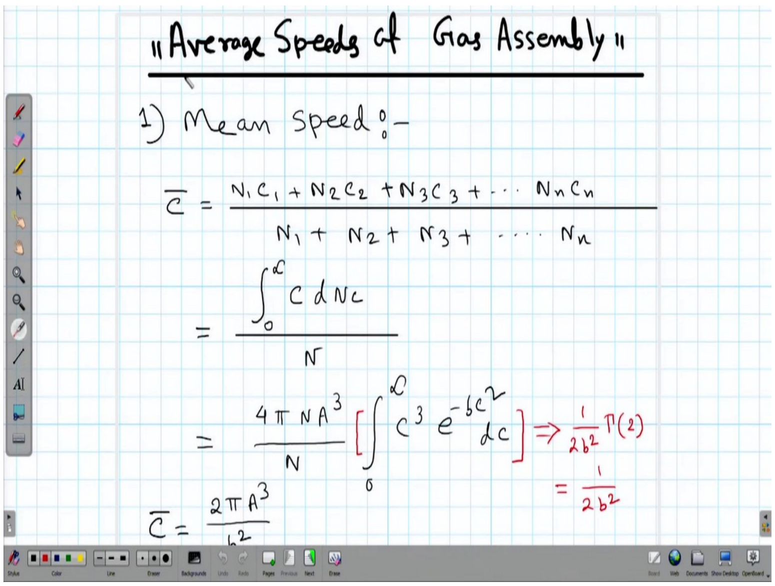

So, let us focus on this there are as I have said there are several different cons I mean say we can diff define at least three different speeds in a average speed in a gas assembly. The first one is the mean speed what do you mean by mean speed? Mean speed is simply if there are N 1 number of particles moving with C 1 speed average speed of C 1 or speed of C 1 absolute speed of C 1 N 2 number of particles moving with absolute speed of C 2 so on and so forth and n small n number of particles moving with speed C n then the mean speed of the gas assembly is defined as N 1 C 1 plus N 2 C 2.

So, basically this sum now this sum can be brought down in form of an integration which is zero to infinity C dN C where dN C is the distribution from or rather the expression of dN C is I am not going to write at this here as of now but as you know this is from Maxwell or rather I would write from Maxwell's distribution, right. So, once we try to compute this integral it will be 4 pi NA cubed at present I am just writing in terms of A and b we are not writing the full expression for A and b and then this integration becomes 4 pi N cubed in the denominator numerator and capital N in the denominator with whole multiplied by this integral 0 to infinity C cubed e to the power minus b C square dc.

Now this integration can once again can be brought into the form of a standard integral in terms of the gamma function and this is nothing but 1 by 2 b square gamma 2 which is gamma 2 is equal to 1. So, we have 1 by 2 b square. So, once we substitute for this one we have C bar which is the mean speed is equal to 2 pi a cubed by b square.

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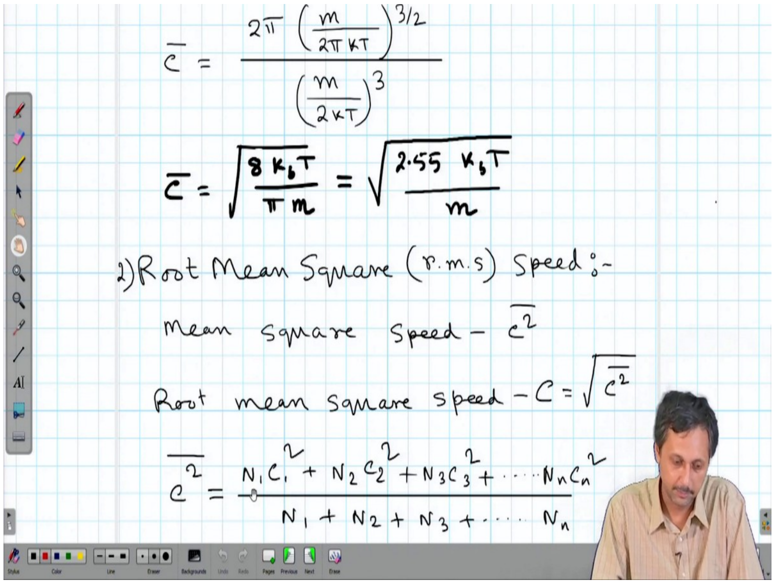

Now this is the time when we can actually put the values of A and b and once we do that we put the put the value of A as m by 2 pi k T whole cubed whole to the power three by 2 and b as m by 2 k T whole cubed sorry it is b square and A cubed. So, it will be right b will be m by 2 pi k T whole square not whole cubed sorry there is a small mistake here yeah root by yeah it will be simply m by 2 pi k T. And finally we have C bar is equal to root over 8 k B T by pi m which once again if we evaluate the factor 8 by pi is nothing but 2.55 k B T by m right.

Next we define the root mean square or rms speed which is once again is a mean speed and so first we define the means mean square speed which is C square average and then we define the root mean square speed as capital C which is equal to root over of mean square speed which is C square average. Now all we have to do is we have to find out the expression for C square average which is N 1 C 1 square plus N 2 C 2 square all the way to capital N subscript small n C n square divided by N 1 plus N 2 plus N 3 all the way to N n.

Now this once again the denominator is nothing but the total number of particles that is capital N and the numerator is integration of C square dN C right. So, the integration as numerator is C square dN C integration 0 to infinity and the denominator is this one.

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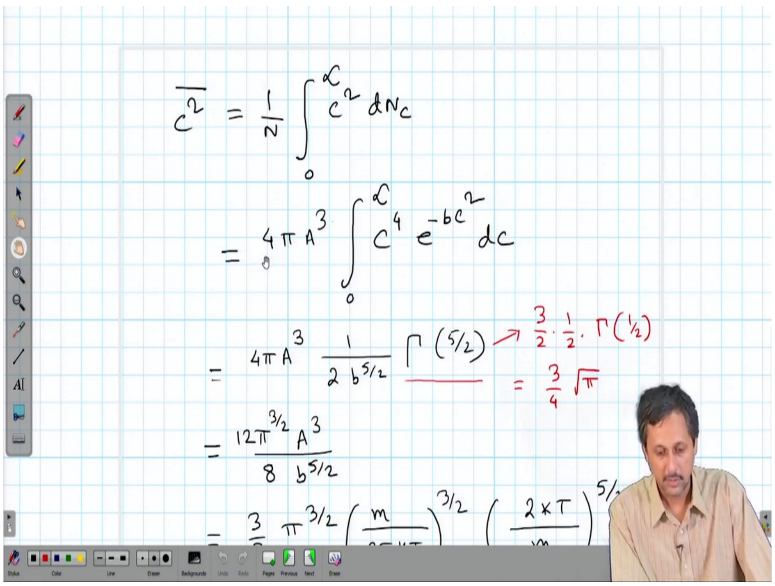

So, once again we compute this integral in terms of A and b at present. So, we write 4 pi A cubed and this integration becomes a to the power 4 e to the power minus b C square dC and right. So, here you see the power of C has increased. So, the gamma function it will be 1 by 2 b to the power 5 by 2 gamma 5 by 2. Now gamma 5 by 2 is nothing but 3 halves times half times gamma half and gamma half is nothing but root pi right.

So, once we put this in we have 12 pi whole to the power 3 half a cubed by 8 p whole to the power 5 half simplified in terms of A and b it gives you the expression of C square which is equal to a C square average which is equal to three k B T by m and then the root mean square value is capital C is equal to root 3 k B T by m also it is convenient to write this as small C with a subscript of rms okay.

So, we can either write capital C or we can write C rms and the expression is root over three k B T by m.

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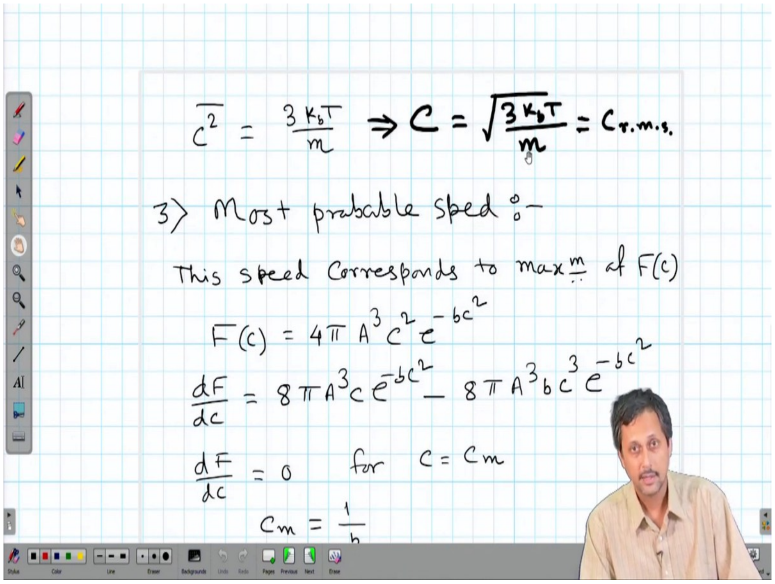

Now there is another possible mean speed that we can define that is the most probable speed what do you mean by most probable speed? Most probable speed means if we draw the Maxwell Boltzmann velocity distribution function you see it looks like it has a I mean it goes up then reaches a maxima and then comes down. Now the speed that corresponds to this maximum point is called the most probable speed.

Now in order to get that what we have to do is we have to take the function F C capital F C that is the Maxwell velocity speed distribution function and we have to take the derivative of that function with respect to C and then we have to set this equal to 0. Now once again we just to avoid complication we are not writing A and b explicitly we are just writing I mean with the expression for A and b at present we are simply differentiating the expression 4 pi A cubed C square e to the power minus b C square.

And this derivative you see once we set this equal to 0 many terms cancels from the left side from the left hand side and the right hand side because it will be this one equal to this one when we put the F dc is equal to 0 you see 8 by A cubed and 8 by A cube cancels out yeah and once C cancels out with this is C square leaving behind a C cube leaving behind C square and finally after some simplification you have to of course put the values of value of b here because you get C m is equal to 1 by b and this is nothing but root over 2 k B T by m right.

So, oh ok now I think we just have to make a small correction b square will be m by 2 k T yes perfect m. So, I it was right. So, b is this. So, b squared will be m by 2 k B T perfect. So, we get all three expressions one is for the most probable speed which is root over 2 k B T by m then we have the rms speed which is root over 3 k B T by m and finally we and at first we have deci we have derived the average speed which is or the mean speed which is root eight by pi k B T by m or 2.55 k b T by m whole root.

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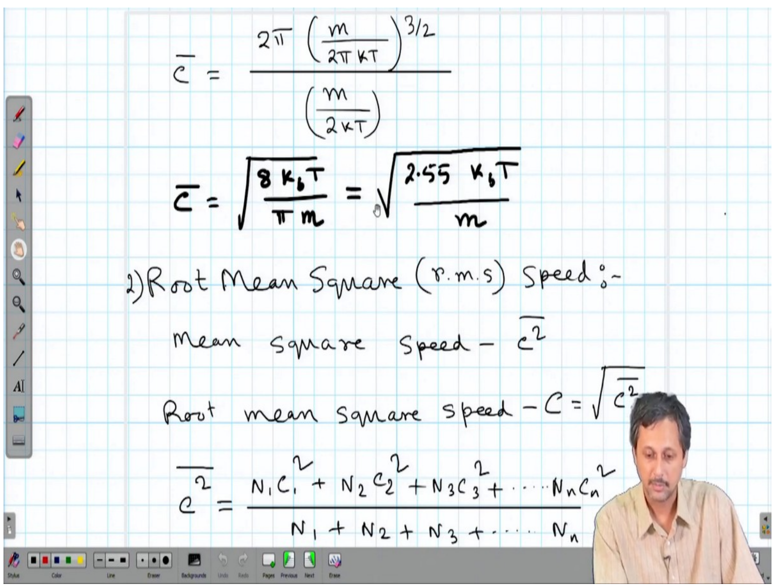

Now in terms of the speed distribution function where does this three speed I mean where does it fall in the in the distribution curve. Now of course we see very easily that this one is the lowest it comes with a prefactor of 2 with k B T by m whole root this one is the highest because this comes with a prefactor of 3 factor of three and this is something in between these 2 speeds which is which comes with a factor of 2.55.

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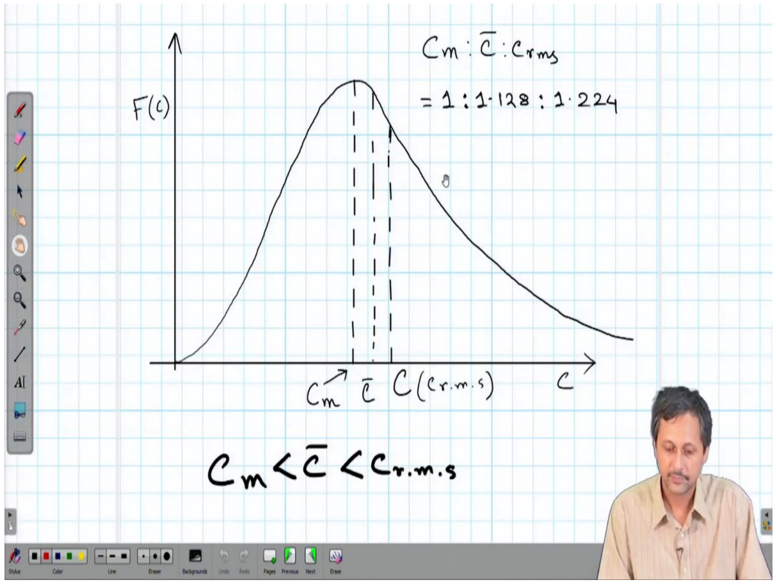

So, once we plot that I mean it is just a a quantitative plot it is not exactly I mean I am not using exact numbers here but if I if I draw the distribution function like this the first the lowest one will be the most probable speed which is which corresponds to this point the top most point of the distribution function and this we denote with C m. Then we have the mean speed which is C bar and then we have the rms speed which is either capital C or C rms.

And we see C rms is greater than C bar which is again greater than cm right and of course you can compute the ratio of these three speeds and you see that of course cm is the smallest and we have this relation that cm is to C bar is to crms is 1 is to 1.128 is to 1.1224 okay. So, it is obvious that you know they are very close to each other of course there is a small difference we will look into the numbers now because we just but we will do some problems.

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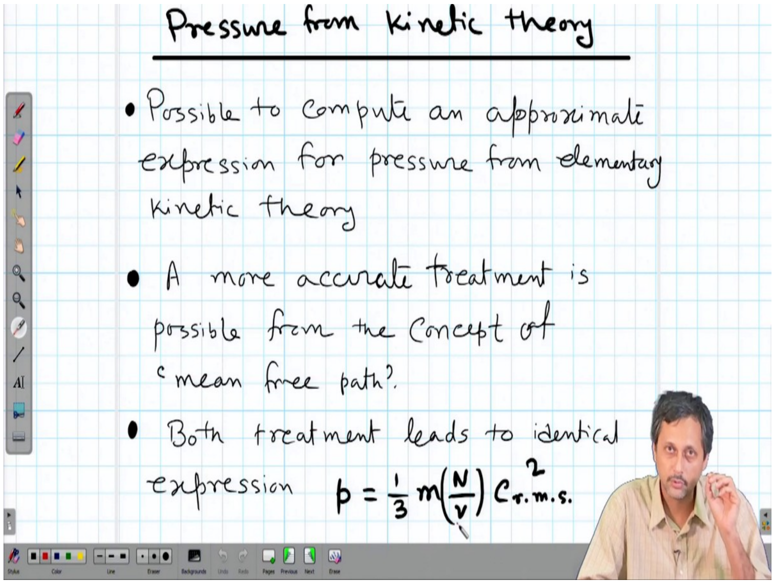

But before that let me quickly tell you that there is also there is a possibility of derive a pressure expression from kinetic theory. Now this pressure expression is kind of approximate that is why I am not doing it now what I will do is I will just wait till we go into the section of mean free path and collision because that picture gives you something more realistic view of the situation because we all know that gas fire I mean that is one of the fundamental assumption of kinetic theory.

That the gas particles are at random collide randomly of course the changes direction those things will not be was not considered in the elementary treatment of pressure while deriving from kinetic theory. But little more advancement from the elementary kinetic theory is when we introduce collision. But surprisingly for both the treatments from the elementary kinetic theory and from the concept of mean free path we get the exact same expression for the pressure which is p is equal to one sorry p p is equal to one third m N by v C rms square right.

This n is the total number of gas particle which is present v is the total volume of the container. So, N by v sometimes written with a small n which is the density or per unit volume I have just avoided this because that small n is very confusing sometime we get it I mean we mix it mix it up with the number of moles that is present. So, once we will we have to write this density explicitly probably we will try to use some slightly different symbol for it but for now you I just I have just written N by v ok.

So, with this what we are going to do is we are going to do some problems I have problem set prepared for you I mean it is not complete yet I have at present I simply have added four problems right just a minute it is a bit slow anyway.

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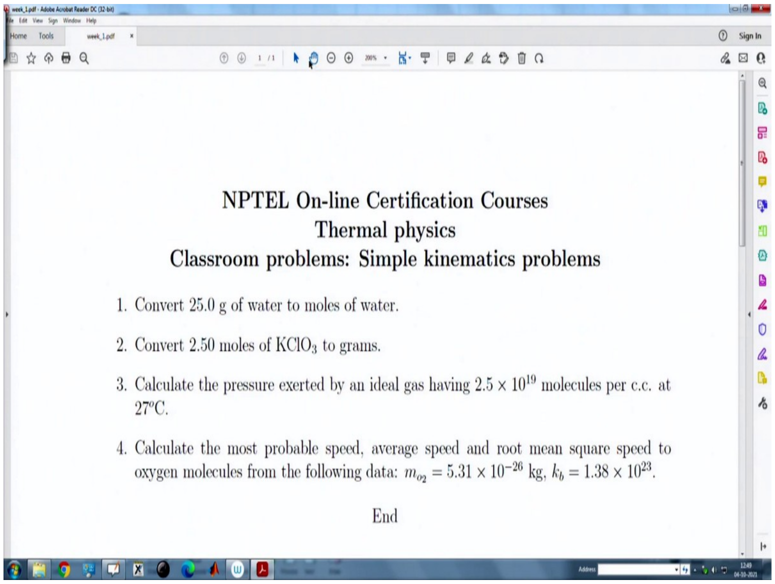

So, let it we can go into the problems the first 2 are really elementary in nature we have to convert a specific amount of 25 grams of water to moles of water and we have to convert 2.5 moles of KCLO3 to grams, grams are actually kgs because kg is something that we will be using explicitly I have just taken this example but gram to kg conversion we all know. And then there are 2 very basic problems from the kinetic theory of gases whatever we have discussed so far ok. So, let us go back here right.

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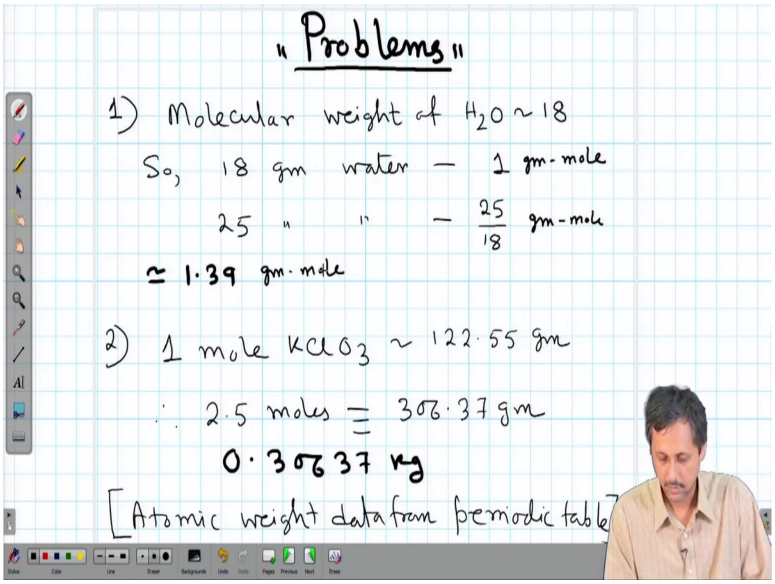

So, for problem 1 its super simple we have the molecular weight we know that the molecular weight of water is 18 approximately I mean the exact expression for molecular weight if we have to. So, basically in for both these problems we have to compute the molecular weight. So, what we have to do is we have to add up this atomic weight data from the periodic table. So, for exact numbers or exact value of this mole or you know weight we have to go to third fourth or sometimes fifth decimal place accuracy.

But here only the approximate calculation and for most part of our discussion we can use the approximate number. For example 18 for water is something very I mean that gives results within the error of I mean limits of our calculation and computational error. So, we will just take 18 as the molecular weight of water that means 18 grams of water makes one gram mole ok. So, there is a you know I probably should have written one gram mole instead of one mole because ideally one mole is expressed in kgs.

So, 18 kgs of water will be one mole probably I should make this small correction here yeah. So, this is one gram mold sorry wrong color yeah one gram mole right. So, this is once again gram mole and this will also be gram mole only right. So, there is also a problem here this should be ideally one mole of KCLO 3 should be expressed in kgs. So, this is actually just convert this to one gram mole once again do not worry I will make the necessary changes in the problem set also.

But I think the idea is clear that we have one gram mole of KCLO 3 which corresponds to 122.55 grams for problem 2 and then 2.5 sorry it should be not moles but gram moles once again yeah this will be 306.37 grams which is 0.30637 kg okay. So, of course if we want to compute the number of moles we have to express everything into kgs but the basic calculation does not change ok.

So, now let us look at problem 3 which is calculate the pressure exerted by an ideal gas having 2.5 into 10 to the power 19 molecules per cc at 27 degree centigrade right. So, what is given is the number of molecules per cc once again please note that we are dealing with number of molecules per cc and the temperature is given in centigrade's.

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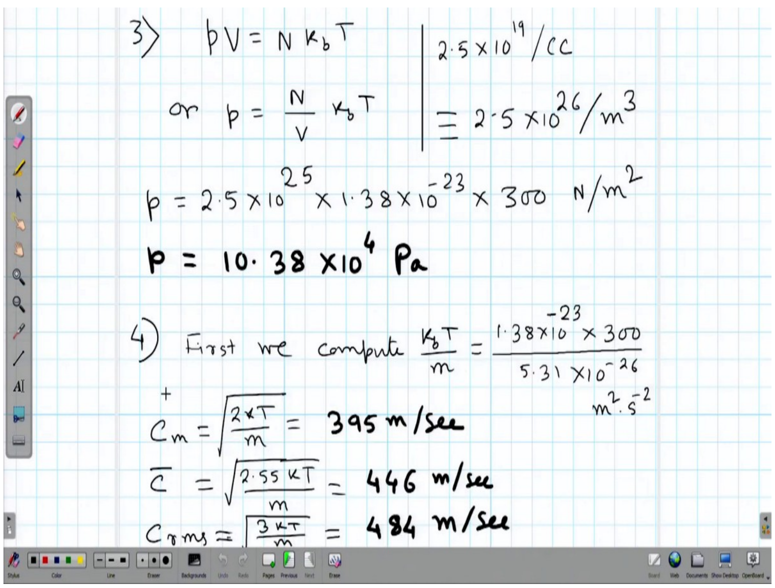

Now how to compute this compute the pressure please remember the ideal gas expression is pv is equal to N k B T right p V is equal to N k B T. Now 2.5 into 10 to the power 19 molecules per cc is actually sorry it should be once again there is a small mistake it should be 25 because the factor between cc and meter cube. So, it will be 2.5 into 10 to the power 25 per meter cube the factor between cc to meter cube is 6. So, I tend to be 10 to the power 6. So, we have to just multiply this number with 10 to the power 6 in order to get the number of molecules per meter cubed right.

Now then from here we can just rewrite this as p is equal to N by v k B T and N is nothing but the number of molecules per unit given volume right. Now here again we have to consider SI units. So, we are converting the temperature to Kelvin 27 degrees is equal to this is nothing but 27 degrees right and the Boltzmann constant k B is also given as 1.38 into 10 to the power minus 323 what is the unit of this? This is Joules per Kelvin right.

So, this is also than in an SI unit we have to always keep this in mind unit is very important which was not ok in this problem there was a problem with the mole and gram mole but I will correct it in the final version final version of the classroom problem set. Here there is no such confusion and once we compute we get and the unit of this pressure after we substitute p the values of N by V as 2.5 into 10 to the power 25 Boltzmann constant and the temperature is 10.38 into 10 to the power 4 Pascal's.

Of course the unit is Newton per meter cube which is nothing but Pascal's right ok. So, for the fourth problem it is once again a very easy problem we just have to calculate the most probable speed average speed and root mean square speed of oxygen molecules from the following data m the mass of one oxygen atom is given once again you see this mass oxygen molecule is given in kgs here and then we have the Boltzmann constant which is 1.38 into 10 to the power 23.

I forgot to include the units but the units will be joules per Kelvin okay. Now what we have to do in order to calculate all these three velocity or three speeds is we have to compute the quantity k B T by m right this quantity and once we put the numbers in and we get you know we get a number for this k B T by m. Now what is C m this is simply root 2 k b. So, I should write k B actually here k b T by m this is k B T by m and k dt by m. So, C m is root 2k k T by m C bar is equal to 2.55 kt by m and C rms is equal to 3kT by m and the numbers are this.

So, you see they are close like but not very close they are significantly different for oxygen molecules even at room temperature 27 degrees is essentially the room temperature. So, you see the spread is from 395 meters per second. So, almost 400 meters per second to almost 480, 85 meters per second. So, this is a considerably widespread of course it depends on what molecule you are dealing with what temperature you are dealing with but this spread is significant for most of the practical purpose. So, we will stop here today and in the next class we will be talking about degrees of freedom of a gas molecule.