Thermal Physics
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Lecture – 29
Measurement of Temperature: Celcius Scale, Ideal Gas Scale, Absolute Zero

Hello and welcome back to another lecture of this thermal physics course in NPTEL platform. Now, in today's lecture, as promised we will be talking about temperature scales, the measurements of temperature actually. Now, before we do that, let me briefly tell you there is of course, we all know about this three laws of thermodynamics and the history of those laws are pretty old, it dates back to 17th century, but there is a more recent law which originated sometime in 1930s and it is called the Zeroeth law of thermo dynamics.

Now, Zeroeth law of thermodynamics actually formulates the basis of temperature the principles of temperature measurement, although temperature measurement is something that was standardized even before this law came into shape, but this law somehow I mean, if we start from this law, somehow it is easy to realize the concept of temperature measurement and temperature scale. So, without further delay, let us start with this.

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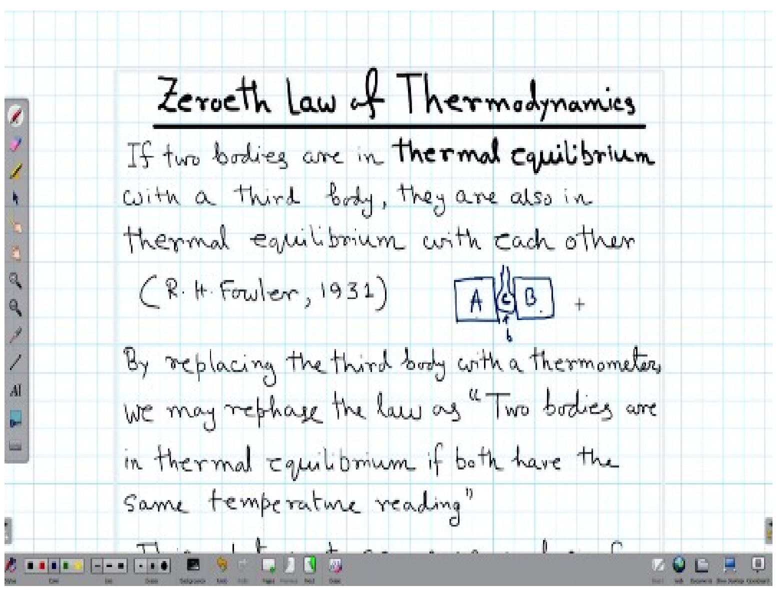

Zeroeth law of thermodynamics, now, what does this law state? This law states that if two bodies are in thermal equilibrium with a third body they are also in thermal equilibrium with each other.

So, that means, if we have for example, three bodies we have A here, we have B here and both of them are in contact with a third body C, let us say this one which is in thermal equilibrium now, in thermal equilibrium means, in thermodynamic term that there is no exchange of heat between this bodies which are in thermal equilibrium.

And this also means that there they have the same temperature. So, this is something that you know, we will exploit in the process of measurement of temperature. So, if we just replace this third body, let us say instead of this third body, we bring this two objects close enough. And we place a constant volume gas thermometer, what is that we will come back in a moment. So, let us say this is the bulb of this, bulb B sorry, let us call it some think let is call it b of the gas thermometer. So, these acts as the third object C.

So, eventually the system A B C, once they are all in thermal equilibrium, the temperature of this object C will be equal to the temperature of object A and object B. So, if we can somehow measure the temperature of this even if this C as we have said that this C is a thermometer, so, that essentially means that by looking at the reading of this thermometer scale C, we can measure the temperature of this two objects A and B which are equal to each other. So, this is the concept of thermometric measurement. So, now let us a look at the concept of temperature measurement in a more systematic way.

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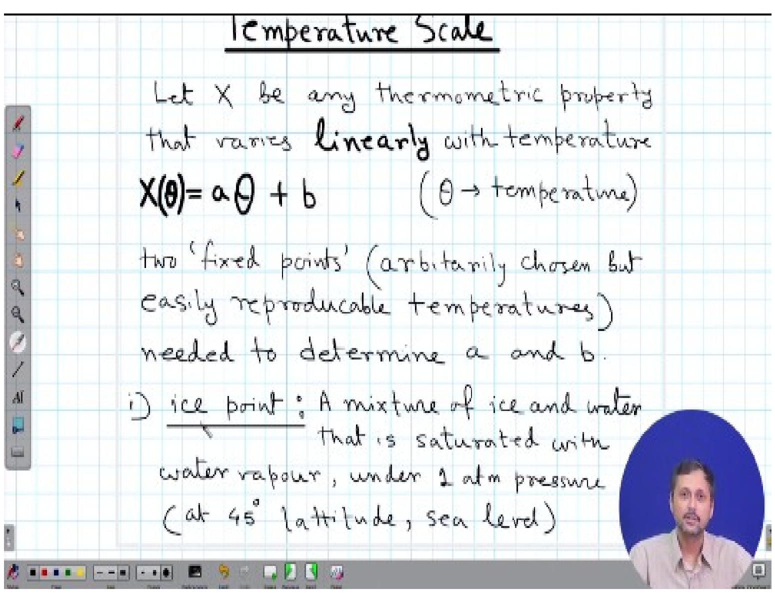

So, what happens in temperature measurement, let us say we have any thermometric property X, the only criteria that are rather the I should not call it this is the criteria, but we want the variation I mean we want to choose X such that the variation of X with temperature is linear. What do you mean by linear? I mean, that X as a function of theta we will have linear variation which is a theta plus b theta being the temperature I should actually write this as X of theta.

So, this is how the quantity theta or quantity X should vary with the temperature theta. Now, why we are writing this temperature theta? Because, so, far we have written temperature with C sorry with T, but we will see very soon that T is the absolute temperature scale and any other temperature scale has to be represented by another symbol. So, we can use theta we can use t, but because t sometimes people mix it up with you know time and in thermodynamics we also have some time in the discussion of thermodynamics we have you know we have to consider time explicitly.

So, that is why it is better to put theta so, that there is no confusion. Now, we see that it is a linear equation, but what do we mean by a linear equation a linear equation has two unknown constants a and b. So, that means, we need two fixed points in order to so, that means this two should be known temperature. Now, these 2 fixed points there is no rule how to choose these two fixed points. So, these are arbitrarily chosen, but easily reproduced temperatures.

And obviously, you know as a kind of an obvious choice is the melting point of water or ice and the boiling point of water. So, these are called the these are the two standard fixed points that has been chosen for the centigrade scale, we will come back to the definition of centigrade scale once again we are all familiar with it anyway, but let me recap. So, the ice point which is the melting point of ice, now, this is something tricky because when we say melting point of ice or you know, boiling point of water.

We will see later on that these temperatures, they vary with pressure. So, it depends strongly on pressure. So, these are not exactly fixed numbers. So, we will do we will be studying phase transition and equation called you know, Clausius equation of first kind, second kind. So, then we will discuss all this changes of melting point and boiling point in detail, but for now, it is important to note that they are not exactly fixed numbers, but changes with temperature changes with pressure.

So, it is important that we have standard pressure standard before we can define this ice point or steam point. So, the pressure standard is under one atmosphere pressure which is achieved at a latitude of 45 degrees on Earth's surface at sea level pressure also varies as a function of height. So, if we go if the elevation of the place changes the pressure changes. So, sea level is something that is universal standard on I mean this is kind of a standard in the earth surface sea the water level more or less is the same.

So, that is why sea level and latitude of 45 degrees is where one atmosphere is defined, but in general one atmosphere can we maintain artificially inside a laboratory. So, we do not have to go to 45 degree latitude and at sea level to you know measure the ice point. So, ice point is you know this is the mixture once again this is the pressure we have defined and this is not exactly the temperature of ice itself, but it is the temperature of equilibrium temperature of ice water mixture with that is saturated with water vapour. So, this is called the ice point.

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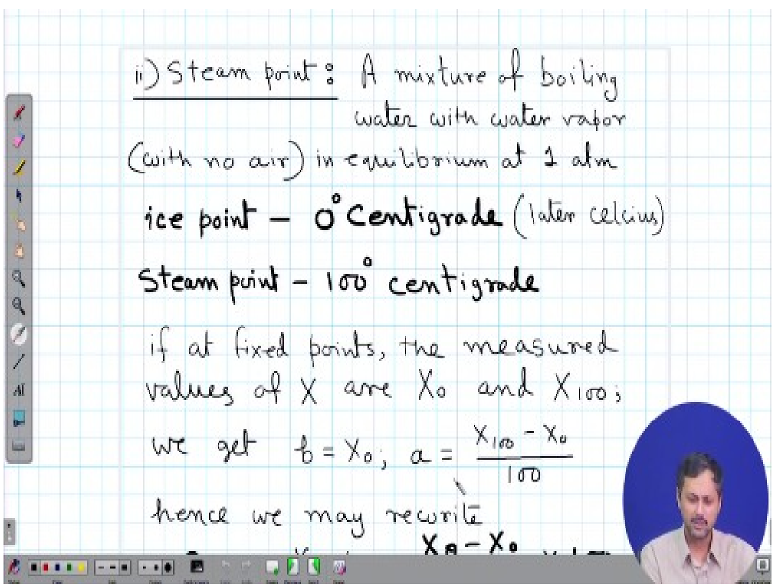

Similarly, the steam point is a mixture of boiling water with water vapour in a container where there is no air present it is only boiling water and water vapour under in equilibrium under one atmosphere pressure. So, this is how the steam point is defined. Now, it so happens that in the centigrade scale we are talking about this is where the ice point is defined as 0 degrees and the steam point is defined as 100 degrees of centigrade and then you know the name centigrade has some other meaning in French.

So, that is why later on degree Celsius the word degree Celsius or as written simply Celsius here but it is actually called degrees Celsius. Instead of using Centigrade worldwide people start using the term degrees Celsius, so they are essentially the same thing. Now, if we know the values of X add these 2 fixed points and we call them X 0 and X 100. Then, going back to this equation that, you know X is equal to a theta plus b. From here, we see, when we put b is equal to 0 or sorry, theta is equal to 0, we immediately see that X 0 is equal to b. So, this is exactly what is there, b is equal to X 0. And similarly, a will be X 100 minus X 0 divided by 100.

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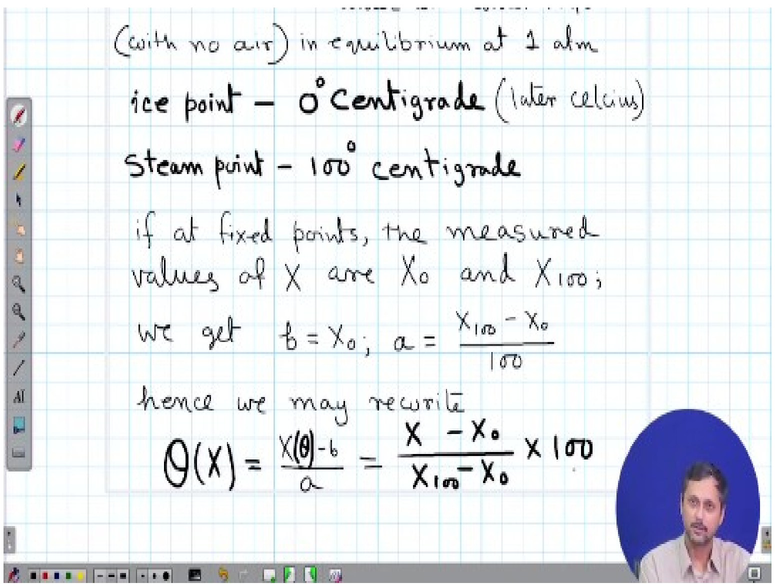

So, we know the 2 constants in terms of the values of this quantity X, right now we have not defined X, X could be anything, we will come back to that. So, this quantity, we can define the constants a and b in terms of this quantity, and eventually we can write theta X is equal to it is it just a minute once again, it should be X theta minus b, which is we will just write it X you know that will be easier. So, it is X minus X 0 divided by X 100 minus X 0 multiplied by 100. So, this is the temperature in the Celsius scale or centigrade scale as we might call it.

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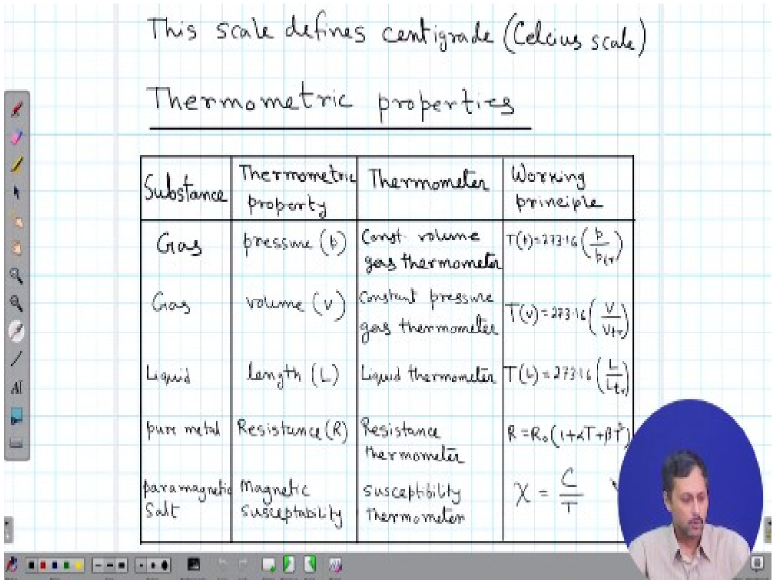

Now, what are this thermometric standards or thermometric materials that we use, so, the oldest thermometer was using only gas so, we had the we can use the pressure of the gas in a constant volume gas thermometer or we can use the volume of the gas in a constant pressure gas thermometer both ways it gives a reliable result for now, just ignore this right hand column, we will come back to that in a moment I mean after we finish the discussion, I mean certain few more topics will a few more pages of notes and then we will come back to this one again.

Similarly, we can use a liquid where the thermometric property we exploit is the length L and this is something that is absolutely familiar to all of us we have all used this you know medicinal thermometer, which we you know we at some point of our life we must have seen those. What is the changeable property? Changeable property is the length the typically those you know for measuring body temperature Fahrenheit is the scale that we use.

But you know centigrade scale is also there, if you look carefully in that thermometer that we have the mercury thermometer where the column length of the mercury column length changes, so, let us say this is the thermometer at you know at before body contact that mercury level is supposedly here and then after body contact it increases and there is an indication where you can see both Fahrenheit and Centigrade temperature of your body.

So, we have liquid similarly, we can have pure metal where the resistance of the metal is measurable thermometric property that we exploit and we call it the resistance thermometer. And also we can use paramagnetic salt fed the magnetic susceptibility conserve as susceptibility I mean as a thermometer basically. So, this is the working formula for the centigrade scale of Celsius scale that we have adapted.

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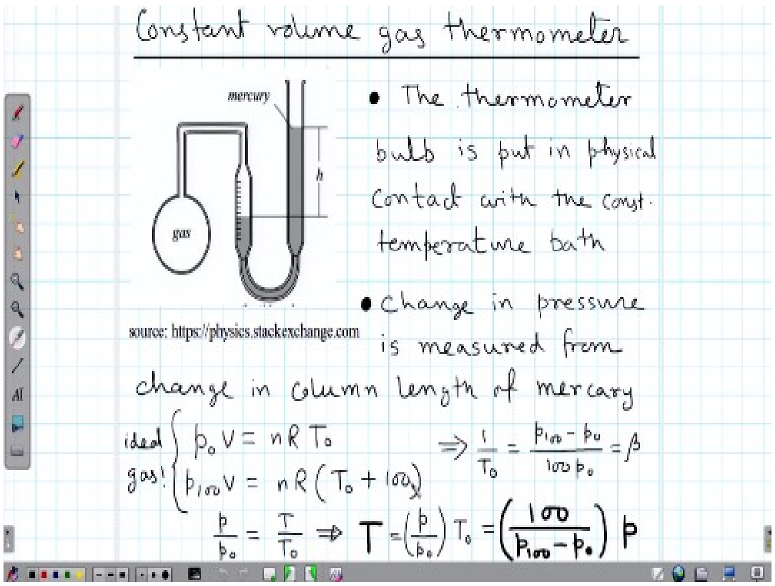

Now, in a constant volume gas thermometer, what is the typical construction I mean, I have given you a very simplified view of the constant volume gas thermometer over here, I mean, it could be more complicated, maybe there could be many more components that is present, but let us focus on this. So, we have the primary construction is this we have a chamber or bulb where the gas resides and this is placed inside the heat bath for which the temperature we need to measure.

For example, if we want to measure the temperature of a slab, we have to place this bulb on the slab and then what happens the principle is very simple as the volume is kind of constant and so, we this is exactly when not exactly the constant volume construction, but this is just for representative purpose. So, what happens is, as the volume you know, volume is constant when the pressure increases and so let us assume that lines actually capillary.

So, you know, the volume change is not considerable as I mean the volume of this capillary tube is not considerable as compared to the bulb. So, what happens is the excess pressure will be on this mercury column here and the mercury will shoot up. So, the difference it is not the absolute height, but the difference the change as we you know take the temperature from one point to the other, the change in this height gives you the pressure change.

And correspondingly we can let us assume that we have ideal gas which is a bad assumption anyway but we will nonetheless use this for now, let us assume that we have an ideal gas inside this thermal meter. So, we can write p 0 V is equal to nRT 0 and similarly, we can write p 100 V is equal to nR T 0 plus 100. Now, here why we are writing T 0 that is one thing, because we will very soon we will see that this temperature of this gas thermometer what we measure is actually you know this is an ideal gas.

This ideal gas temperature is equivalent to the thermodynamic temperature. So, we are writing T 0. So, this is the equation. So, from the second equation, we see just a minute, no, actually from this here from the first and the second equation, we see p 100 minus p 0 by 100 p 0, and we 1 by T 0 is equal to p 100 minus p 0 divided by 100 p 0, and we call this beta. So, now, for any unknown temperature T, if we read a pressure p, then we can write p by p 0 is equal to T by T 0 because you see p and T they are proportional to each other.

So, p by p 0 is equal to T by T 0, which keeps T is equal to p by p 0 times T 0 and then I replace it for 1 by T 0 and I finally get 100 divided by p 100 minus p 0 multiplied by p. So, this is the working formula of constant volume gas thermometer and this works because we have V as a constant. So, V does not intervene into this equation. Now, there are a few problems first of all the there are certain corrections that has to be taken into account when we are talking about a constant volume gas thermometer.

So, I am not going into the details of this because this is something that you can actually read from any textbook and it is understandable because there are certain things that has to be taken care of for example, the bulb itself this bulb over here, it does not have a fixed volume. So, the volume might change. So, this is one problem, then certain you know, although we are calling it constant volume, but there is certain amount of volume expansion is happening because of the displacement of this liquid column.

So, all these things has to be taken into account. So, the actual working formula although this is the fundamental working formula, there are the actual working formula will be more complicated, but what is important that the pressure or sorry the temperature can be accurately measured using a constant volume modern gas constant volume gas thermometer. Actually, in modern gas thermometers, the mercury column is not even used. So, the displacement whatever small displacement or small pressure change takes place that will actually displace the plate of a capacitor and the capacitance change will be measured that will be more accurate anyway.

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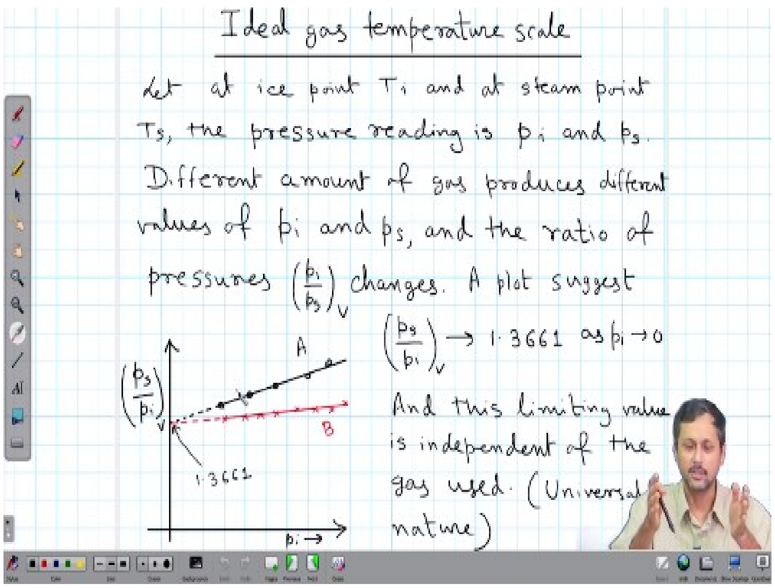

Now, we will try to define an ideal gas temperature scale. Now, please let us go back quickly here and we see this equation is p 0 V is equal to nRT 0 assuming this gas is ideal in nature which is never the case we please realize that we have already discussed it in detail that there is nothing called ideal gas. So, but still it is possible using this ideal gas thermometer to established an ideal gas temperature scale. So, let us assume that at the ice point, let us call it T i and the steam point T s, the pressure reading is p i and p s respectively.

Now, inside this bulb, if we take different amount of gas, we will have different values of the pressure for at the same given temperature, because you see finally, the if we change the amount of gas what I mean by this is although the volume is constant, but if we change the amount of gas not the volume, the amount of gas if we you know, pressing more gas, the pressure change will be more if we have less gas. So, basically we are not changing the volume here, but the number of moles n.

So, if we have more n than the change in p will be more with temperature. So, by this construct, by this method, we can take a series of measurement of p i and p s. So, it can range over I mean it can go over a wide range of values initial pressure, the pressure at ice point and pressure at steam point. So, now, if I try to plot this if we compute the ratio p s by p i and try to plot this as a function of p i the exactly the way it has been done here, see what happens we have let us say for one gas A we have this points.

And that low pressure side this gas, this points followed on a straight line which extrapolates to a value of essentially 0 p i, 0 p i is something that is not achievable experiment absolutely not possible. Because of course, if the pressure is 0, we cannot measure it there is no deflection in the mercury scale or whatever pressure gauge we are using then there will be no deflection. So, that is why it has to be achieved using this extrapolation technique.

So, we start from this finite values we measured the finite values and we just assuming that I mean as shown here it behaves almost like a straight line, we can extrapolate it to 0 pressure and if it is repeated with another gas B let us say we have for example hydrogen and argon so, A let us assume this is for hydrogen and B is for argon. So, in this case, we see that although the values the absolute values of p s by p i at a finite pressure is different.

They extrapolated values they point to once again to the 0 position or sorry 0 pressure extrapolation points to exactly the same value. And this value is one point with the best accuracy which was available, it is 1.3661. So, this is the limiting value and this limiting value is independent of the nature of the gas. Now, this is a very important observation because, and we can safely state that at 0 I mean in the limit of 0 pressure all the gases behave like ideal gas that is what we have seen the remember this virial expansion.

At very low pressure, the second or third or fourth or any higher virial coefficient will have 0 contribution. So, it will be p V is equal to nRT simple ideal gas equation. So, this one this ratio what we get which is universal in nature, the 0 extrapolation of 0 pressure can be taken as the ideal gas ratio for p s by p i. So, once again, remember that p is proportional to T.

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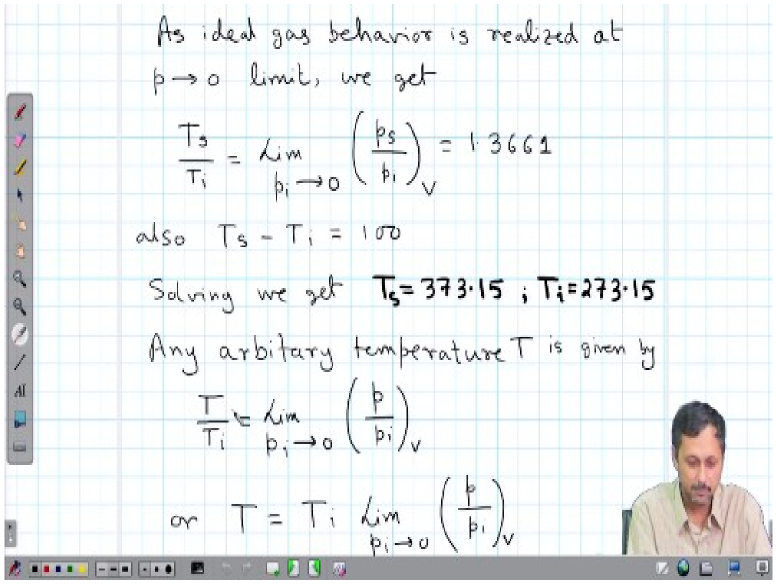

So, we can safely write T s by T i is equal to limit p i tends to 0 p s by p i, which is 1.3661 and we already know that T s minus T i is equal to 100 that is by the definition of the centigrade or degrees Celsius scale. So, we have 2 equations in T s and T i and by solving we get T s is equal to 373.15 and T i is equal to 273.15. So, these are the temperature of the steam point and ice point respectively in the ideal gas scale, why it is called ideal gas scale?

Because in this limit, the limiting value is actually a value which corresponds to the ideal gas behavior of any I mean rather low pressure ideal gas behavior of the working gas and it is independent of the working gas we take 2 gases we take multiple gases, it will eventually extrapolate to this 0 pressure value. So, this T s and T i are the values of steam and ice point and ideal gas, for any arbitrary temperature what do we have? We have please remember like we have T s by T i is equal to limit p i tends to 0 p s by p i.

Similarly, we have T by T i is equal to limit p i tends to 0 p by p i for constant volume. So, we can write T is equal to T i times limit p tends to 0 p by p i. So, this is the equation the working formula for a constant volume gas thermometer.

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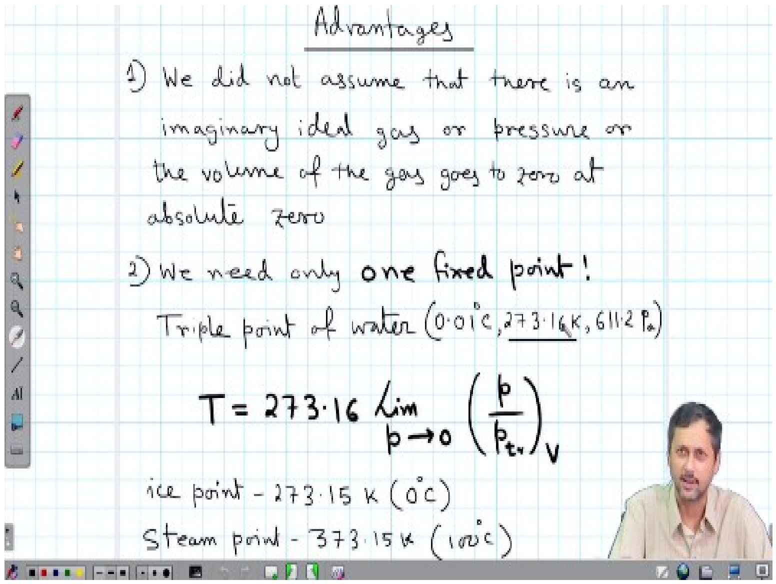

Now, the advantage of this construction or advantage of this ideal gas scale here we did not assume that there in reality there is an ideal gas we did not assume that an ideal gas is an imaginary concept. So, and we never have assumed that the volume and pressure goes to 0 at the absolute 0. So, what we do is if we you know if we go to a temperature which is so, the corresponding temperature where the pressure will go to 0 if we try to compute then we will see that it will be minus 273.15 degrees centigrade.

Now, this is purely I mean an extrapolation or deduction from a measurement at finite temperature on a real gas. So, the absolute temperature scale the absolute 0 which is minus 273.160 centigrade 273.15 degrees centigrade is a experimentally measured number, but experiments are conducted on a real system at a finite temperature. Also, we see this particular formula over here we see that we actually do not need to work in self I mean 2 fixed points we do not need T s and T i instead we can use a single fixed point T i.

And can get the value of temperature just by measuring the pressure at the fixed point and the arbitrary temperature point. So, this is one advantage is that instead of two fixed points in the original this formula over here for example, yeah instead of two fixed point X 0 and X 100 we can work with one fixed point in terms of T i and later on it was found that as I have already discussed the T i is a ice point or T s is the steam points. So, these points are difficult to maintain because difficult to reproduce.

I mean I should not say difficult to reproduce, but the accuracy of them are questions because they are so, sensitive to pressure change and purity of water all these things because, you know getting pure water itself is a big problem some time. So, that is why later on there is only this one fixed point is replaced by the triple point of water, which is very close to this ice point, but not the ice point exactly. So, this is a very common misconception that triple point and ice point these are actually both 0 degrees centigrade.

It turns out that ice point is 0 degree centigrade under one atmosphere pressure and triple point is 0.01 degrees centigrade which is equal to 273.16 Kelvin not 15. And it is at a pressure of 611.2 Pascal, which is like 4 orders of magnitude lower as compared to the atmospheric pressure. Now, what is triple point, triple point is the point where all 3 phases of water can coexist, that means ice water and water vapour they can coexist. So, triple point is something that has to be discussed separately when we will be discussing phase transition.

And we will just leave this for now, except for the fact we do not have to remember anything about this at this point, except for the fact that the triple point temperature is 0.01 degrees centigrade or 273.16 is the universal standard for fixed point in the ideal gas temperature scale and in this temperature scale ice point once again is 273.15 degree centigrade which is 15 Kelvin which is 0 degrees centigrade. Steam point is 273.15 Kelvin which is 100 degree centigrade.

So, please remember ice point and triple point. They are separated by 0.01 degree centigrade, but this is something can be very significant at over time. So, with this we will move to the problem set here I have added just a minute here.

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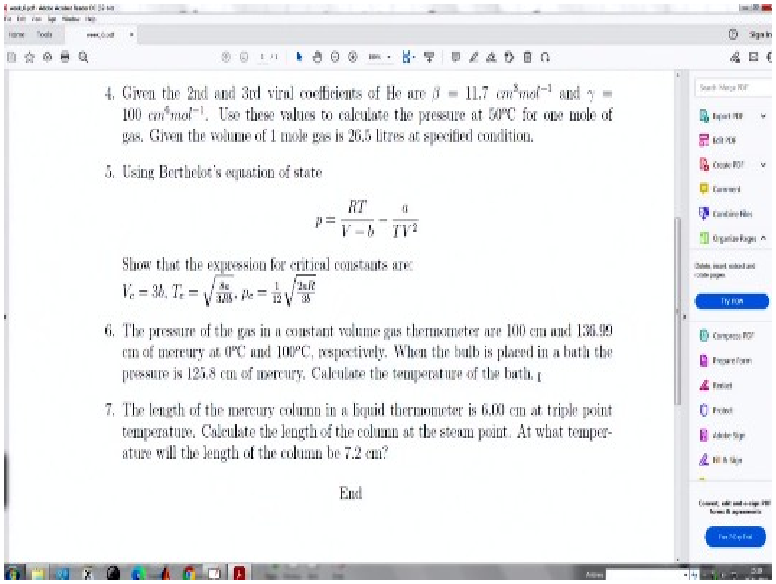

4. Given the 2nd and 3rd virial coefficients of He are $\beta = 11.7\ cm^3 mol^{-1}$ and $\gamma = 100\ cm^6 mol^{-1}$. Use these values to calculate the pressure at 50°C for one mole of gas. Given the volume of 1 mole gas is 26.5 litres at specified condition.

5. Using Berthelot's equation of state

$$p = \frac{RT}{V-b} - \frac{a}{TV^2}$$

Show that the expression for critical constants are:
$V_c = 3b$, $T_c = \sqrt{\frac{8a}{27Rb}}$, $p_c = \frac{1}{12}\sqrt{\frac{2aR}{3b}}$

6. The pressure of the gas in a constant volume gas thermometer are 100 cm and 136.99 cm of mercury at 0°C and 100°C, respectively. When the bulb is placed in a bath the pressure is 125.8 cm of mercury. Calculate the temperature of the bath.

7. The length of the mercury column in a liquid thermometer is 6.00 cm at triple point temperature. Calculate the length of the column at the steam point. At what temperature will the length of the column be 7.2 cm?

End

I have added two problems for today's class Problem number 6 and 7. The first problem is the pressure of the gas in a constant volume thermometer are 100 centimeters and 136.99 centimeters of mercury at 0 degree and 100 degree Centigrades respectively, when the bulb is placed in a bath, the pressure is 125.8 centimeters of mercury calculate the temperature of the bath. Now, in here, it is all given in centimeters of mercury.

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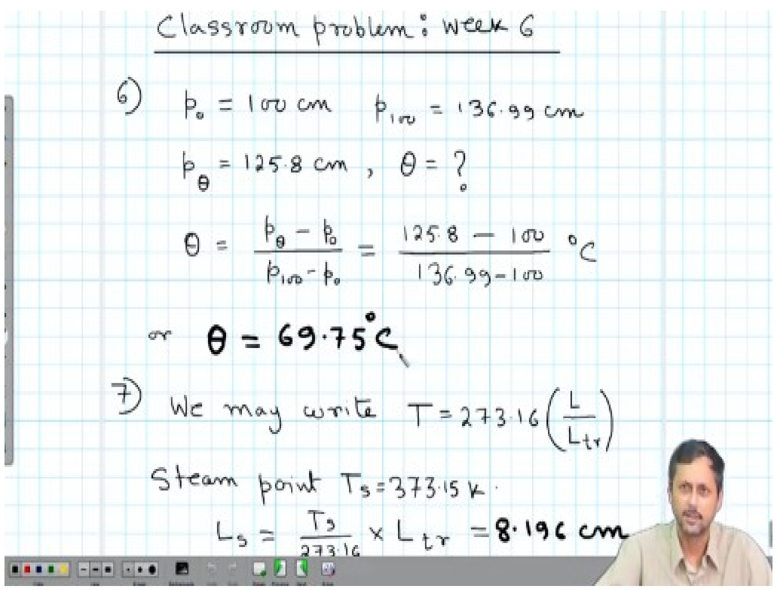

And we the ice point and steam point is given. So, what we will do is we will simply use the formula theta is equal to p theta minus p 0 divided by p 100 minus p 0, p 0 is equal to 100 centimeters p 100 is equal to 136.99 centimeters, you might say why you are not converting it to SI units, I would say useless because whatever conversion I am going to do I can do that I mean I can just take 76 centimeters of mercury as one atmosphere pressure converted to Pascal's we can do all this conversion, but finally, this is a ratio here and it will nicely cancel out.

So, it is a ratio of pressure we are calculating. So, p theta minus p 0 will be 125.8 minus 100 divided by 136.99 minus 100. And if we compute this ratio it will be theta is equal to 69.75 degree Centigrade.

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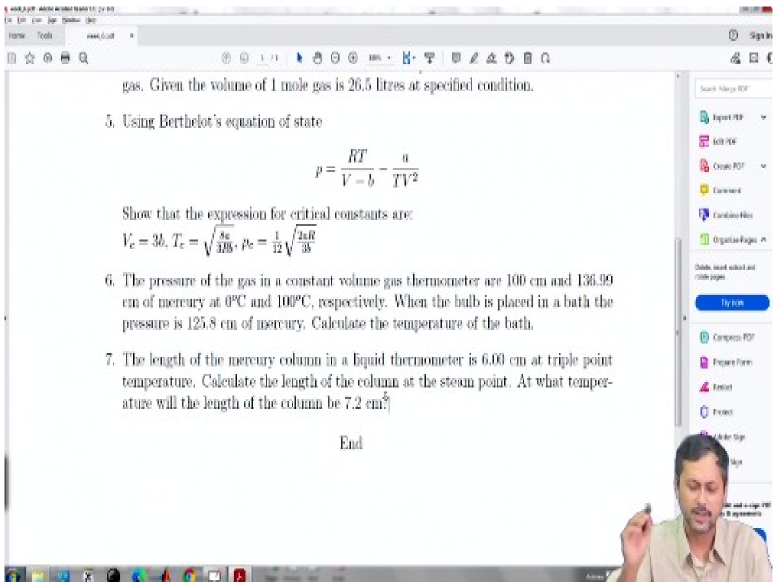

So, the we move to the next problem, which is the length of mercury column in a liquid thermometer is 6 centimeter at triple point temperature that means 273.16, calculate the length of the column at the steam point and there is a second part to it at what temperature the length of the column will be 7.2 centimeters. So, we have to first calculate I mean we get the working formula.

Let us go back a few slides you see the right hand column now, I think we understand the right hand column. Let me quickly go through it once again. So, for constant pressure for constant volume gas thermometer, we have T phi is equal to 273.16 p by p triple. Similarly, we can have a constant sorry constant volume gas thermometer and for constant pressure gas thermometer where the volume changes at constant pressure once again the working formula is p V = nRT.

So, instead of p here we are changing V. So, the relation will be very similar and we have TV is equal to 273.16 V by V tr, tr stands for triple, triple point. And for liquid we have a similar formula that T of L is 273.16 L by L tr triple. So, we have given in this problem the triple point length is given and we have to compute the length at a given another given temperature that is the steam point. And for this the other two we will come back to that in the next class.

Here so, we may write T = 273.16 L by L triple. So, the steam point which is T s = 273.15 Kelvin for that L s will be equal to T s times L tr divided by 273.16. So, that is exactly it is done here. T s is 373.15 divided by 273.16 into L, L tr is given as 6 centimeters and we have 8.196 centimeters. I have just kept up to 3 decimal but, you know probably the measurement error will be more than this accuracy so we can finally have maybe 8.12 centimeters also. This is one thing you have to keep in mind.

You might keep in the numerical calculation, you might keep as many terms as you want, but finally, you are limited by the accuracy of your measurement. So, in this experiment we have not discussed about what is the scale we are using and what is the accuracy of that scale, but most likely it will be in the second decimal place and keeping up to 0.196 centimeters is a waste, so, probably 8.12 centimeter is a better answer.

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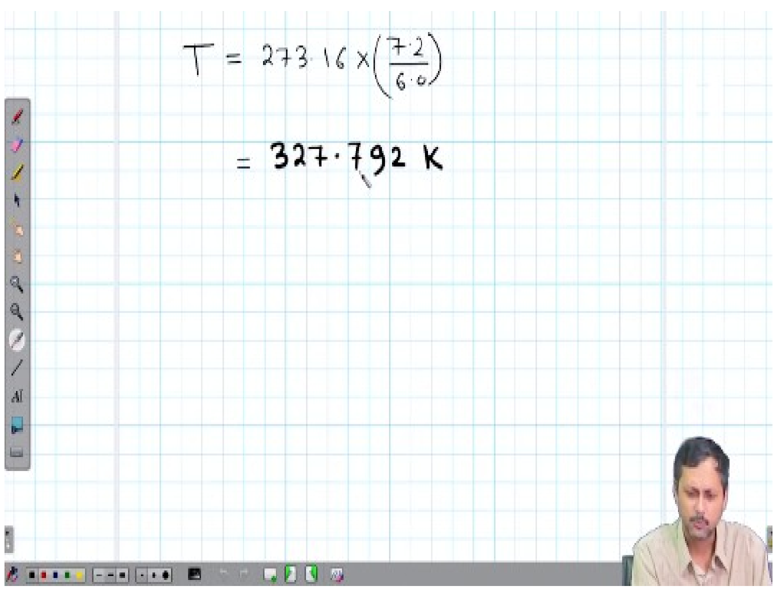

And for the last part, the length is given us 7.2 centimeters corresponding temperature is straightforward we all I have to do is we have to put the value of L here. So, it will be 7.2 divided by 6 multiplied by 273.16. This is 327.792 Kelvin.

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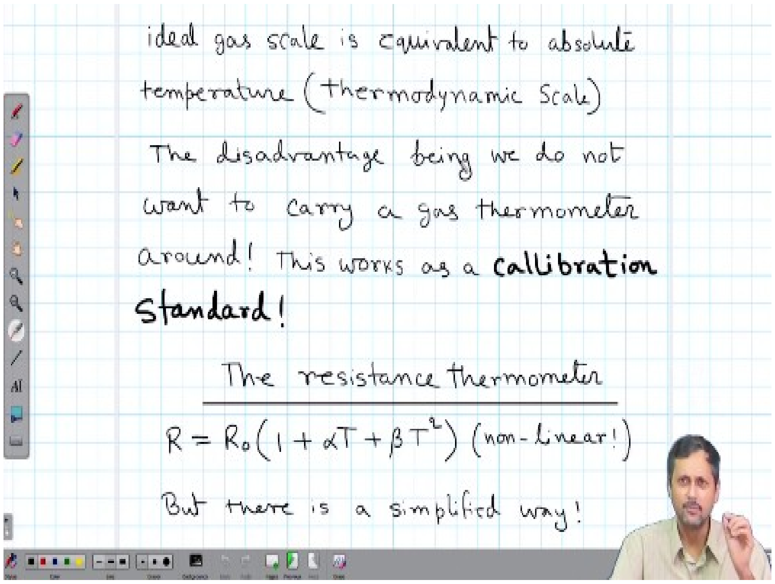

So, just one more thing and we will stop here that is the ideal gas scale, what we have calculated or what we have calculated using a gas thermometer is actually equivalent to the thermodynamic scale or absolute scale of temperature that was later demonstrated. So, we will probably if we have time, we will discuss thermodynamic scale when we will be discussing second law of thermodynamics, not now, the disadvantage of this being a constant volume gas thermometer is a hefty bulky unit.

And it is slow to react and there are many disadvantages it has to be maintained at a constant temperature for a long time to for thermal equilibrium measurement is difficult. So, it is not something that we carry around. So, what this is actually used as a calibration standard and we have handy thermometers, we have seen that we have liquid thermometers sometime hanging from our school or colleges you know classroom that shows the room temperature, we use the doctors thermometer medicinal thermometer at home.

Similarly, in laboratory we use something called the registered thermometer, we typically use the platinum registered thermometer with the working formula which is R is equal to R 0 1 plus alpha T plus beta T square. Once again, there is a problem this is nonlinear, but there are ways of rectifying this. So, you know, bringing it to a linear form and all so, in the next lecture, which will be the last lecture of this week, we will be discussing about the resistance thermometer is necessary connections and the use of resistance thermometer. So, that is why we stop today. See you all again in the next lecture bye.