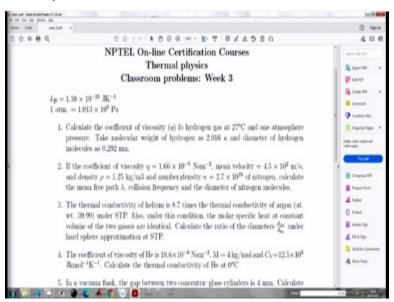
Thermal Physics Prof. Debamalya Banerjee Department of Physics Indian Institute of Technology-Kharagpur

Lecture-13 Topic-Thermal Conductivity: Transport of Thermal Energy

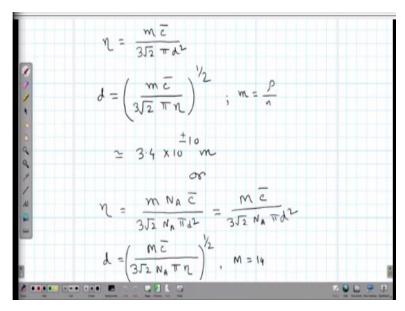
Hello and welcome back to another lecture on this NPTEL course of thermal physics. In today's lecture, we will be talking about diffusion but before that let me quickly go to the last problem of the last lecture that is lecture number 12. We ended the lecture with a problem from the classroom problem set, so let us have a look.

(Refer Slide Time: 00:53)



So, that was problem number 2 and there was a confusion, actually I got confused whether we really need this number n to calculate the diameter of the nitrogen molecule. So, it so happens that actually I figured out that there is a way of not using the value of n.

(Refer Slide Time: 01:08)



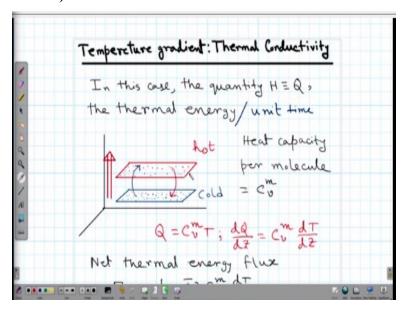
So, one way of doing it is of course you start from this particular relation over here. So, you start from this particular relation where d can be written in this term and we need to know m small m. Now rho is given, m is given, so as m is equal to rho in the last class I stated that, yes, m can be calculated by rho by n and we can put this value m back here and we can calculate the diameter from this relation. Now it is so happens that there is another way although we do not need to know m in another way in which we do not need to use this value of n directly, right.

So, let us look at it the exact same relation, all we want to do is we want to put N A in the numerator and N A in the denominator. Now once we do that, once we put N A in the numerator m times N A gives you M, M being the molecular mass or molecular weight of the molecule of interest. And in the denominator we have N and N A is also another constant that we all know we are familiar with. So, this relation becomes instead of this particular form we have capital M c bar divided by 3 root 2 N A pi eta whole to the power half.

And we know that for nitrogen molecule M = 14, not exactly 14 but very close to 14, 13.99 or something I really do not remember this number. But once again please remember that these are only approximate calculation, we cannot we should not maintain an accuracy for more than 2 or maybe 3 decimal places in this calculation, please keep that in mind. Because all these transport properties first of all we will discuss it in towards the end of this discussion on transport properties that these relations are only approximate, their first principal calculation.

They are accurate to the order of magnitude, they are accurate to the fundamental dependence but the 3 factors are mostly not right, we will come back to that. And secondly, please remember that we are not maintaining the accuracy of higher decimal place all throughout the calculation. So, it is not recommended that you maintain maybe more than 2 or maybe 3 decimal places of accuracy throughout these calculations. So, with this let us switch to the today's lecture, lecture number 13 which will be on the next transport property that is thermal conductivity.

(Refer Slide Time: 03:59)



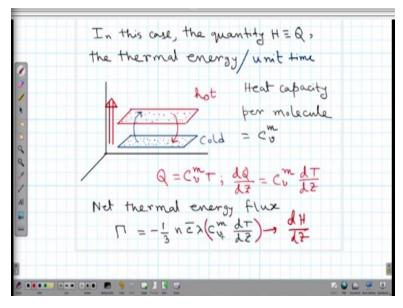
Now what is thermal conductivity? Thermal conductivity is the case when the quantity H, H is if you remember the generic transport quantity or generic we just took this H has a general property that is being transported by molecular flow between these 2 layers. Now in this case, H will be equal to Q, Q being the thermal energy. So, it is not only the thermal energy, we have to remember that this is the thermal energy per unit.

So, this thing we have to keep in mind, I forget to write that but I hope it is clear. So, what is happening once again? Here the hot layer is actually hot and the cold layer is actually cold because the temperatures are drastically different sorry I should not say drastically, the temperatures are different, the hot layer has a higher temperature, cold layer has a lower temperature.

So, the temperature gradient once again is pointing upwards. So, the temperature gradient if we draw it like this, so the temperature gradient is this, so this is our temperature gradient. And once again the molecule exchange is going on between these 2 layers and that mutual distance is anything less than two third lambda. So, that means from the theory that we have learned we can assume that there is a free transport of molecules without any interaction between these 2 sets of layer.

So, hot layer, the molecules from hot layer brings in more temperature to the cold layer, molecules from cold layer takes it actually these are colder molecules, they are going into the hot layer and bringing down the temperature there. So, eventually once again nature wants to drive us towards equilibrium and the molecules that are being transferred between the hot and the cold layer, they are transferring the thermal energy between these 2 layers. Now the heat capacity per molecule is actually let us define the heat capacity per molecule as $C \ v \ m$, this quantity. Now $Q \ will be equal to <math>C \ v \ m$ times $T \ and \ dQ \ dz = C \ v \ m$ times $dT \ dz$.

(Refer Slide Time: 06:55)

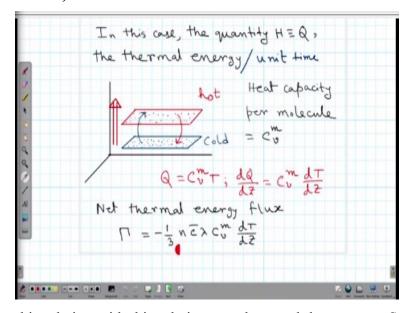


So, we go back to the original expression for that we have derived for the flux and we have to replace the dH dt which was originally here. Basically this was your sorry dH dz with this C v m m times dT dz. Now and then what do we do? So, let us remove this, so what do we do now?

(Refer Slide Time: 07:23)

We compared this with the Fourier's law of heat conduction and we find out that Q is equal to which is minus k dA dT dz. And please remember that here the definition is we are already computing in terms of molecular flux. So, it is not only per unit time but also per unit area, so in this relation we have to put dA is equal to 1 unit once again.

(Refer Slide Time: 07:55)



Then we compare this relation with this relation over here and then we get. So, then we get k is equal to one third m n c bar C v m. So, this is the relation of thermal conductivity with microscopic properties like the mean free path, I mean we could not call mean velocity as a microscopic property. Similarly n is not exactly a microscopic property, it is a macroscopically

measurable property. But there are 2 microscopic properties in terms of the lambda which is

mean free path and C v m which is the heat capacity per molecule.

Now we can slightly rearrange this and we can write k is equal to one third lambda c bar and we

write n times C v as equal to C v v, so this is nothing but the heat capacity per unit volume which

is kind of an unconventional unit of heat capacity. But what we can do is we can substitute n =

rho by m and once again we can write n is equal to rho by m and we can multiply the numerator

and the denominator with m N A which is the Avogadro number.

Then the whole thing reduces to one third lambda c bar rho divided by M times C v and what is

the C v? This C v is nothing but N A C v m which is molar specific heat at constant volume

which is a much more convenient quantity to work with. Because this is something that we can

measure experimentally, this is something which is more familiar to us, we have already

described we have discussed C p - C v = R for the relation C p - C v = R for ideal gases when

discussing the Equipartition theorem. So, this is something that we would like to work with.

So, we have 2 relations, one is microscopic in nature which is 1, okay, both are microscopic

because both because both has lambda. One has the microscopic heat capacity and the other one

the last line here we have the macroscopic heat capacity but it still has lambda in it. So, we have

2 relations for kappa or k whatever you might call it, I have intentionally put it in a little more

curly manner because in some books it is written as kappa, in some books it is written as k, I

thought I will just go in between and write a curly k, I hope that this will you will not get

confused.

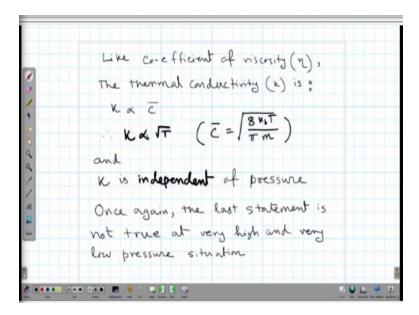
And also it is important to write in a slightly different notation because we already have 2 k's, the

small k is actually k B if is the Boltzmann factor, I sometimes write k B I sometimes write just k

and then capital K is the Kelvin temperature scale. So, it has to be better if we can distinguish

this thermal conductivity from this 2 notation.

(Refer Slide Time: 11:18)

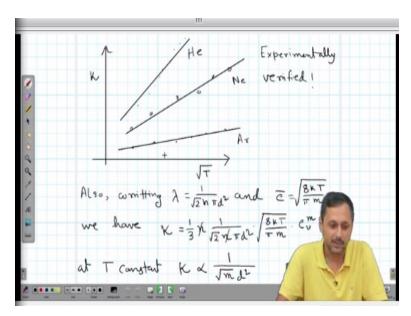


Next, once again if we compute the temperature dependence similar to the coefficient of viscosity, what do we see? We see that k is actually proportional to c bar which is the mean velocity. And mean velocity is nothing but, so c bar is nothing but 8 kT see once again there is a k. So, I should actually write small k B T by pi n. So, that is what, this k and this k we should not confuse.

Anyway, so this is a constant and then we see that k is this k is proportional to root over T. And once again this k is independent of pressure as we see from this expression here. But that independence holds once again only when at not very high or not really low pressure. Once again at very high pressure what happens? The mean free path is comparable to the intramolecular separation and the description or the entire formulation that what we have worked out is does not exist, is not valid anymore.

And at very low pressure what happens is that the mean free path is comparable to the container length, then if we reduce the pressure further the number density reduces, keep reducing. So, then k starts decreasing with decreasing pressure and it turns out that there is a very interesting application of, this can be actually use to measure low pressure, we will come back to that in a moment. So, these are the important parameters or important relations we have.

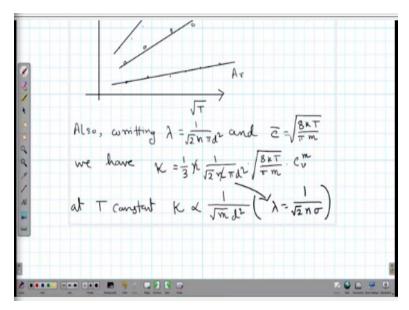
(Refer Slide Time: 13:18)



Once again experimentally it is also been verified that root T is equal to when we plot k versus root T for inert gases, typically inert gases are mono atomic gases and they are very good candidate for ideal gas. So, if we really want to examine ideal gas, we have to examine the properties of inert gases measured experimentally. So, you see argon, neon and helium all this they kind of follow the linear behaviour when plotted against.

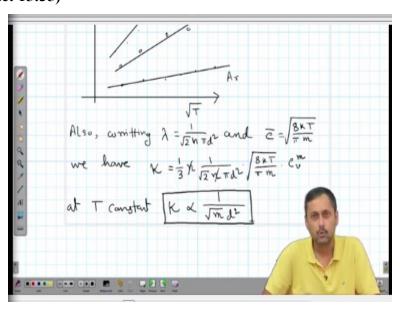
The thermal conductivity when plotted against root of temperature the kind of follow the linear behaviour, right. Now also there is a interesting relation here, if we write lambda is equal to 1 over root 2 n pi d square and c bar is equal to 8k T by pi m. Then we see that at constant temperature, if the temperature remains constant, you see this n and this n cancels out nicely. See, this is where as I was said at very low pressure this n dependence or 1 by.

(Refer Slide Time: 14:36)



So, this comes because we are writing lambda is equal to 1 by root 2 n c. So, this relation breaks down when we write when we go to very low pressure. So, this n and this n does not cancel anymore, lambda becomes a constant at very low pressure. So, as n decreases k decreases, I hope we understand this, fine. So, let us go back to this. So, we have this relation here and we said when the temperature is constant this whole thing one third 1 by root over 2 pi and inside that we have 8k root pi root m and C v m which is once again a molar or molecular level heat conductivity or heat capacity, these are all constants. So, what we get?

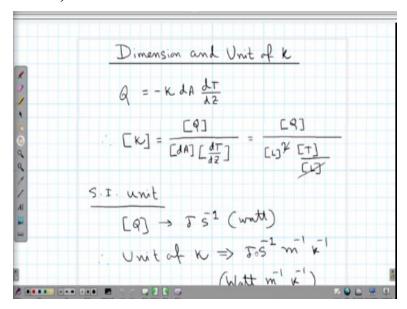
(Refer Slide Time: 15:35)



When the temperature is constant, we have a dependence of k with root m times d square, inversely proportional to root m times d square. So, this can also be verified experimentally. So,

basically if we plot k versus root m d square for different gases, of course for 1 gas it will be the same and if we maintain the temperature constant and if we plot this for different gases, we kind of get a straight line. So, those data are available, you can look into the book of Langley thermal physics by Langley and then this representative data set is already there.

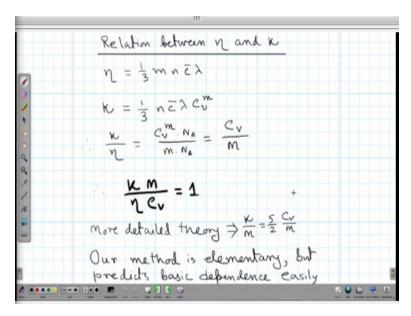
(Refer Slide Time: 16:17)



Next up is the dimension and units of k. So, we look at the original Fourier equation, we see that it can be rearranged as the Q divided by dA dT dz. Now what is the dimension of Q? Dimension of Q is actually heat capacity per second and then we see in the denominator there is a length square here, this one is actually length square, this one is temperature y by length, so 1 length cancels out.

So, we have Q divided by L T types of dimension. So, in SI units we simply have the units of Q as Watt which is joules per second. So, unit of k will be joules per second per meter per Kelvin. This k has to be written in a slightly better way. So, basically Watt per meter per Kelvin or joules per second per meter per Kelvin, these are the standard SI units for this thermal conductivity.

(Refer Slide Time: 17:27)

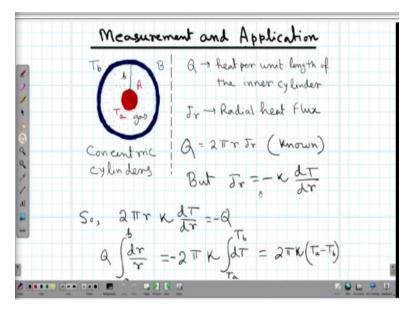


And the 2 transport properties that has been discussed so far one is and one is k, is there any relation between them? It so happens there is, so we just write out the fundamental relations of eta and k that is eta is equal to one third m n c bar lambda and k is equal to one third n c bar lambda C v m which is once again the molecular level heat capacity. So, k by eta is equal to C v by m divided by m once again we multiply both the numerator and denominator with N A.

And we get C v by M which is once again a constant for a given substance. So, we can write k M by eta C v is equal to 1. Now which again in a more detailed way if it is looked into in a more detailed theoretical manner? Then we see that the equality factor is not 1 but 5 by 2 but once again as I have mentioned many times that our method is elementary but it predicts the basic dependence very easy.

So, we get the basic dependence of parameters like eta and k B if that these are linearly dependent on mean free path, linearly dependent on mean velocity, linearly dependent on density. These dependencies it can be predicted very accurately but may not be the free factor one third may not be the right free factor for that. But so this is what we have in hand and this is what we have to study. So, towards the end of this discussion as I said I will give you a little more detailed account of this.

(Refer Slide Time: 19:28)



But for now let us move on to the measurement and application of the heat conductivity. Now this is very important application the same method which is used to measure the heat capacity also leads us to a very interesting application of this thermal conductivity. So, let us first focus on this geometry here. So, we have 2 concentric cylinders, one is a solid cylinder that is going into the center, so there concentric. The inner one is actually a solid cylinder with diameter small a and the outer one is a hollow cylinder with a diameter small b.

Now the temperature of the outer cylinder is kept constant at T b which is in contact with the heat path. Now let us say this is in contact with the environment directly. Let us assume that this is environment, so whatever the temperature of the room and please remember that our environment is the best heat path probably that we can environment and the air and sea or river, these are the best heat bath we can think of. So, the inner cylinder is actually it is we are passing constant current and please remember that these are long cylinders, we are just drawing a cross section in it, we are taking the top view and just drawing a cross section.

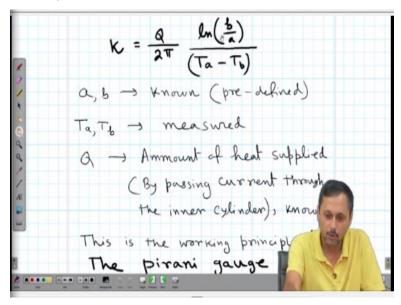
So, we are passing a constant current, these are conducting metallic cylinders. We are passing a constant current through the inner cylinder with essentially is a ware. So, that ware is heated up and it is maintained at a temperature T a. Now let us assume that the Q is the heat per unit length of the inner cylinder that will be generated due to this current. And if J r is the radial heat flux then we have this simple relation that Q = 2 pi r J r which is a known quantity. Because once we

know Q r then we know J r actually, this is the very relation. But J r according to the Fourier law is once again proportional to J r = -k times dT dr.

So, this is dT dr being the temperature gradient, so they are maintained at temperature T a and T b. So, that means there is obviously a temperature gradient and there has to be if there is a very high vacuum between these 2 it might this experiment will not work. So, there has to be some gas molecule which will be transferring the temperature from this end to the other end. So, this is simply given by J r = -k dT dr. So, from these two relations we can equate 2 pi r k dT dr = -Q.

So, this is the relation over here, so basically we take this relation and this relation together and we write this, we rearrange and we integrate. So, what are the integration limits? So, we have one integration that is Q sorry this is dr by r integrated between a to b and dT is integrated between T a and T b. So, the right hand side simply gives 2 pi k T a - T b, please remember this minus sign is already taken care of, it should be T b - T a we just changed the sign and write T a - T b.

(Refer Slide Time: 23:27)



So, and the left hand side is nothing but lon of b by a. So, rearranging we can write k = Q by 2 pi lon b by a divided by T a - T b. Now in this expression here, what is known a, b is known which is predefined. Once we finalize the setup we have defined a and b that is done. T a and T b they are measured. So, they are measured during the experiment itself. So, idea is there has to be the

outer cylinder T b is already at a constant temperature because it is in contact with let us say the environment which is the heat part in our case.

And T a, the temperature has to be constant; it has to be a steady state condition when the temperature of the inner cylinder or the inner valleys changing this experiment is normal. So, in the steady state T a and T b they are measured and they are constant. Q is the amount of heat that is being supplied and this heat is supplied by passing unknown amount of current through this inner cylinder a. So, if we know the current, if we know the resistance we can very easily measure the amount of heat that is given to this inner cylinder.

So, all these quantities in the, please remember Q is heat given per unit length, anyway, so this is anywhere known. So, this entire right hand side is known and we can measure the value of k. Now it so happens the same geometry is used and similar working principle is applied in a Pirani gauge. Now what is the Pirani gauge? Pirani gauge is a gauge that is capable of measuring low vacuum. In one of the lectures previously we have discussed about different pumps that can give you different ranges of vacuum but I never mentioned how to measure that.

It so happens that there is something called a Pirani gauge that will work for moderate vacuum and there is something called a penning gauge or ionization gauge that will work for very low vacuum. So, let us look into the diagram again and try to understand what is the possible working principle of a Pirani gauge. Now in a Pirani gauge exact same geometry is applied and the concept is very similar.

We are passing a known current through the central ware and once the temperature of the inner ware has to be maintained at a constant value. So, the amount of current that is being supplied is a measure of what is the thermal flux that is flowing between the inner cylinder and outer cylinder. Now let us assume that we have started to reduce the pressure, now we will reach a situation at which the mean free path.

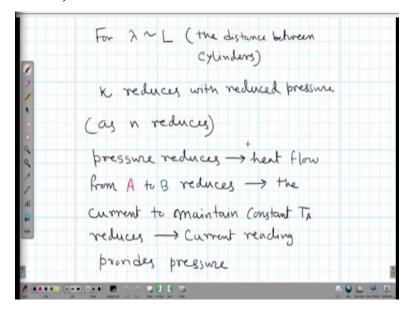
So, typically the distance between here to here, it will be of the order of, so this distance let us say, it will be of the order of let us say 5 nanometer 5 millimeter which is something or maybe 1

centimeter I should not write 5 millimeter, I should probably write 1 centimeter, 10 millimeter. It could be little less, it could be little more that it is of the order 1 centimeter. Now, if we start reducing the pressure it will very soon, you know, not very soon, after a while the mean free path. So, once again as we decrease the pressure the mean free path will keep increasing that we have discussed in the previous lectures.

Now at some point the mean free path will be comparable to 1 centimeter and we have seen that for very low pressure situation the mean free path are few meters. So, 1 centimeter is 100th of a meter, so in a moderate vacuum itself it will reach it will be comparable to 1 centimeter. Now as I have already mentioned once the mean free path is comparable with the dimension of the container it cannot increase further because that is the limit that is the upper limit. Now what happens k will be the thermal conductivity will be a function of pressure.

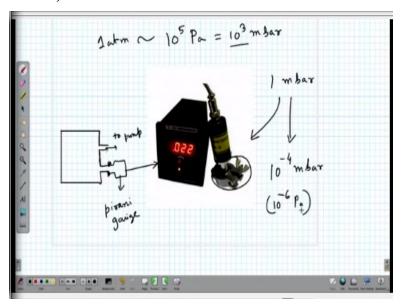
Now as the pressure decreases the number of molecules decreases. So, the heat flux which is J r radial heat flux definitely decreases because if there are less molecule less likely the heat will be transferred from the inner wear to the outer wear. So, it will take less current to maintain the inner wear at a constant temperature. And that current can be calibrated and that value of the current can be accurately calibrated, so that the pressure can be measured.

(Refer Slide Time: 28:47)



So, what I have here? I have written whatever I have said in just now is written here. So, once again I will repeat that, pressure reduces heat flow from A to B reduces. So, pressure reduces means n reduces, please understand, when n reduces the heat flow from A to B reduces the current to maintain constant T A reduces and current reading provides pressure and this all is happening when lambda is comparable to L. So, lambda cannot increase any further, so finally the current reading provides pressure. Now the time is almost over. So, what do we do?

(Refer Slide Time: 29:31)



We just show you some pictures of the pirani gauge. Let me try to bring a picture here, see this looks better. I will do one thing, I will just bring this to the. So, this is a picture of a pirani gauge the meter and the gauge, you see this I hope you can see this here. You see this cylindrical thing down here this is actual gauge, now what happens is? We have let us say there is a vacuum chamber; I will just draw this try to draw it quickly. So, let us say this is my vacuum chamber and we have.

So, let us say on this side we have a opening for the pump and there is a second opening that is for the gauge. Now this opening there are particular vacuum clamps, there are mains for it, typically, so this one is 2 pumps and then this is a proper vacuum clamp. You see there is a arrangement here I think I hope you can see, there is a clamp arrangement, this is bit hard to draw and show you.

But basically once we see it will it is very easy, so we have one connector from this side and the other connector comes in from the other side. And this is on top of this in between we place a oring and on top of this we put this c-clamp we call it c-clamp and then we just simply tightening. So, that is where the pirani gauge will be connected. So, what happens here, we just connect the gauge here and with gauge it will be connected to this meter?

So, whatever pressure is inside this chamber, so this is my pirani gauge. So, typical pressure atmospheric pressure is 1 atmosphere is roughly 10 to the power 5 Pascal. Which if I am not very wrong let me quickly check first Pascal to millibar, I think it is discussed in the last class Pascal to millibar. I think the conversion is 0.01, so it will be, so if I put 1E5 10 to the power 5 Pascal, it should be 1000 millibar.

So, it will be equal to 1000 or rather 10 to the power 3 millibar. That is where the pirani gauge not working, we start vacuum when the pressure reaches of the order of 10 millibar or rather 1 millibar the gauge starts to work. Before that it will not show any reading here. Now 1 millibar all the way down to 10 to the power -4 millibar which is what 10 to the power minus 4 millibar will be 10 to the power -6 Pascal, this gauge will work.

Beyond that this gauge is not very sensitive, beyond this 10 to the power minus 4 millibar it is out of the range for a pirani gauge and we have to use something called a penning gauge or a ionization gauge for that. We will discuss that in some more details in future and also there are problems on thermal conductivity which we will take up in the next lecture. You also have diffusion after thermal conductivity we have to talk about diffusion. So, and diffusion, there are 2 more topics, this will be covered in the upcoming lectures of this period, thank you.