Thermal Physics
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Lecture-11
Topic-Transport in Fluids: Introduction

Hello and welcome back to the week 3 lecture of this NPTEL course on thermal physics. So, in today's lecture we are going to talk about transport processes. Now so far whatever we have discussed about the gas assembly, this has been an equilibrium phenomenon. Now what do you mean by equilibrium? Of course equilibrium will be more formally represented very soon but let me tell you equilibrium means when inside the system there is no gradient present. What do we mean by gradient? Gradient essentially means differences.

Now before we proceed to the content of today's lecture let me tell you that there is a very common misconception between or mix up between an equilibrium and a steady state. Now what is an equilibrium and what is a steady state? Let me try to explain to you very briefly. Equilibrium is once again what is equilibrium I will tell you in a moment but let me give you an example of a steady state which is not equilibrium.

So, let us assume that there is a drum under a tap and water is falling inside the drum and after certain time the drum will be filled to it is top and then the water will start pouring out from the sides. Or you can think of there is a hole at the bottom of the drum, let us assume that there is a hole at the bottom of the drum, so water is coming in and water is going out. Now somehow if we can make these 2 rates equal, the rate at which water is coming in from the tap and rate at which the water is flowing out of that hole.

If we can make them equal then what happens? The water level inside the drum does not change with time and that essentially means not only the amount of water changes. But also after a certain time all the system will have the equal in the same temperature all the water that is coming in and that is going out will have equal temperature, so that system is in thermal equilibrium.

Of course system does not have any pressure gradient, so it will be in a kind of a mechanical equilibrium which is once again I will define in a moment in a more formal way. But I think you understand the situation that water is pouring into the drum, water is going out of the drum. But let me tell you this is not a chemical equilibrium system, so let us look into it formally.

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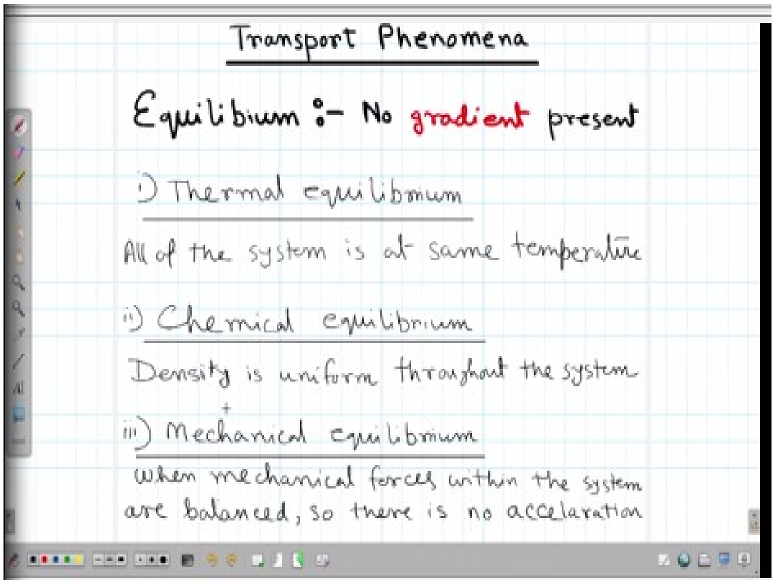

Transport Phenomena

Equilibium :- No gradient present

i) Thermal equilibrium

All of the system is at same temperature

ii) Chemical equilibrium

Density is uniform throughout the system

iii) Mechanical equilibrium

When mechanical forces within the system are balanced, so there is no accelaration

So, what is equilibrium? Equilibrium as I said already that there is no gradient present. Now there are 3 main types of equilibrium, one is the thermal equilibrium in which there is no all the system is at the office sorry but I do not know what I did oops! just a minute. So, the thermal equilibrium is a state in which all the system is at the same temperature. Now if we look back to the example of this drum which will**.** Let me just try to draw this for you then you will have a better feel yeah.

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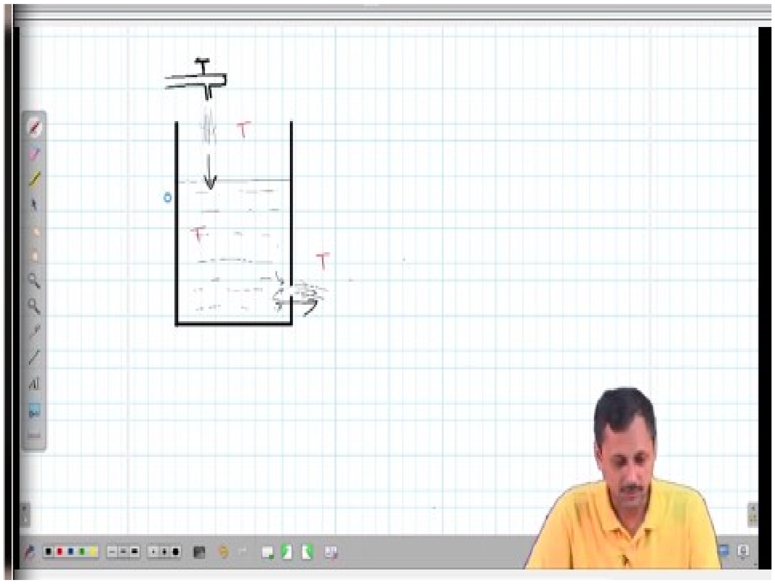

So, we have a drum, very bad drawing, just a minute let me try to work on it, yeah this looks better. So, this is my drum of water and then we have a tap here that is pouring water inside some sort of a tap arrangement. So, I hope you understand and at the same time there is a small hole in the system. So, what is happening? Water is coming in, pouring in and water is going out and in between of course there is a fixed water level.

And as I have said that after a certain time we will be achieving a steady state where the amount of water that is pouring out. Basically we have to adjust the flow rate, so that the amount of water that is pouring in will be the amount of water that is pouring out per unit time. And then what happens is this entire system will be after some time at equal temperature, the temperature of the water that is coming in, the temperature of the water that is sorry I should use a different colour probably, yeah.

Temperature here and the temperature the water that is going out, so these are all in perfect thermal equilibrium. Now but that is not all, equilibrium means we also have to have chemical equilibrium and mechanical equilibrium. Now what is chemical equilibrium? Chemical equilibrium is I do not know why this is gone, then density I do not know why it is gone but I think we can manage, yeah.

So, density is uniform throughout the system. Now, this is one way of saying this also there is other way of looking at it that says chemical equilibrium is when there is no, so basically if there is no particle movement in the system but which is not the case for the example we are taking. What is happening? Just a minute I do not know, density this is fine. So, mechanical equilibrium is when the mechanical forces within the systems are balanced, so there is no acceleration.

Now in a thermodynamic system the mechanical equilibrium is realized when the velocity is uniform, let us assume a gas assembly. When we have a gas assembly we assume that there is Maxwell's Boltzmann distribution for example that there is no velocity gradient. So, basically if we take randomly if we select randomly 1 region in the container and we measure the average velocity we will look at the velocity distribution, we get exactly the same distribution if we measure it in a another portion of the container.

So, that means the velocity distribution in 2 parts of the container are equilibrium are equal that is what we call the mechanical equilibrium. Now let us examine the system in question once again how? The software crashed, give me a second, we will open it again. So, the system in question, we see of course it is in thermal equilibrium, the temperature is uniform throughout the system. But then we see that there is a molecular flow, the molecule is coming in and molecule is going out.

So, that means there is a density gradient present, that is why the molecule is I mean it is not exactly in a density gradient in it is traditional sense. But of course there is a flow of molecule which of course is not the case when the system is in chemical equilibrium. And then we talk about mechanical equilibrium, now if we just focus on this portion, so the moving particle or let us say this portion where the water is going out.

This outgoing water definitely I mean close to this exit, the velocity will be more as compared to the rest of the system. So, the system although it is in thermal equilibrium and it is in a steady state, so that the entire description of the system is not changing, the height of the water level is not changing by any means it is not in chemical or mechanical equilibrium, it is fine. So, now we understand that what is the meaning of equilibrium, so basically we are talking about these 3 different types of equilibrium.

Also there is something called a phase equilibrium which will be discussing towards the end of this lecture series when we will be discussing phase rule and all. But in general if there is mechanical thermal and chemical equilibrium then we call the system thermodynamic system to be in equilibrium which is not the case here. So, this is a system which is in it is at steady state for sure but not in equilibrium.

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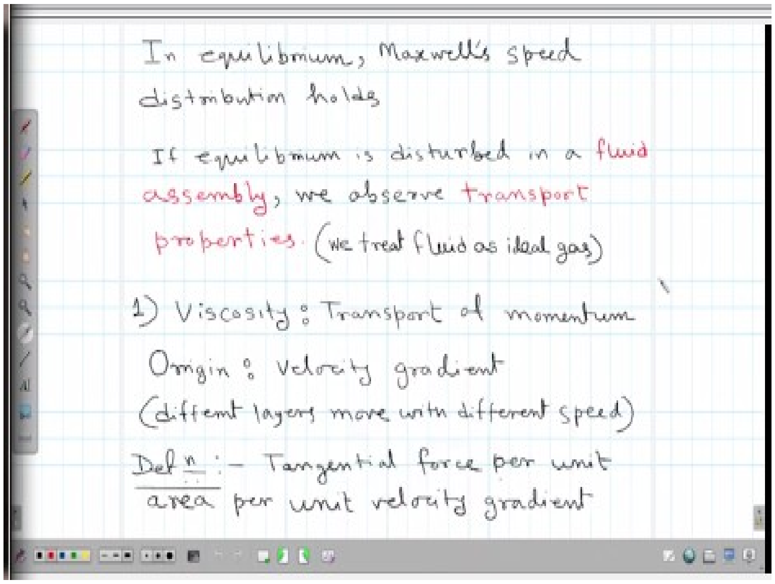

In equilibrium, Maxwell's speed distribution holds

If equilibrium is disturbed in a fluid assembly, we observe transport properties. (we treat fluid as ideal gas)

1) Viscosity : Transport of momentum

Origin : velocity gradient

(diffemt layers move with different speed)

Def n :- Tangential force per unit area per unit velocity gradient

Now let us move on, whatever we have discussed so far in terms of Maxwell, well speed distribution law the mean free path. Well, mean free path is not exactly a very equilibrium phenomena but of course we have assumed that the overall the equilibrium the velocity distribution is more or less uniform. So, these are all equilibrium distribution or equilibrium phenomena.

Now if somehow the equilibrium is disturbed in a fluid assembly, here we talk about fluid in general which we know could be liquid, could be gas but in general here we treat fluid and ideal gas as equivalent terms to each other. Now, so let me state it again when the equilibrium in a fluid system is disturbed then we have something called the transport phenomena. Now what are the 3 major transport phenomena we are going to investigate in this lecture? First of all we will be talking about viscosity which is basically transport of momentum from one part of the liquid or one part of the fluid in question to the other part.

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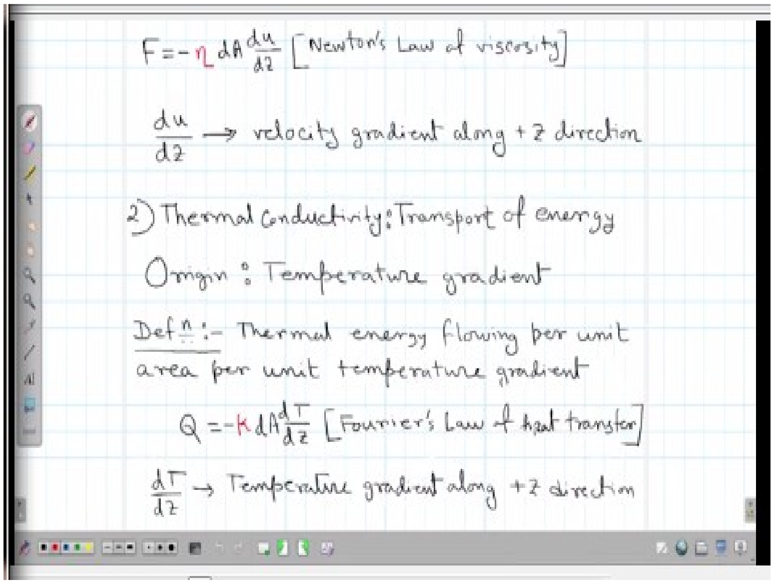

$F = -\eta \, dA \frac{du}{dz}$ [Newton's Law of viscosity]

$\frac{du}{dz} \rightarrow$ velocity gradient along +z direction

2) Thermal conductivity: Transport of energy

Origin: Temperature gradient

Defn:- Thermal energy flowing per unit area per unit temperature gradient

$Q = -K \, dA \frac{dT}{dz}$ [Fourier's Law of heat transfer]

$\frac{dT}{dz} \rightarrow$ Temperature gradient along +z direction

Then we will have 2 other things, one is the thermal conductivity which deals with the transport of thermal energy from one part of the system to the other part.

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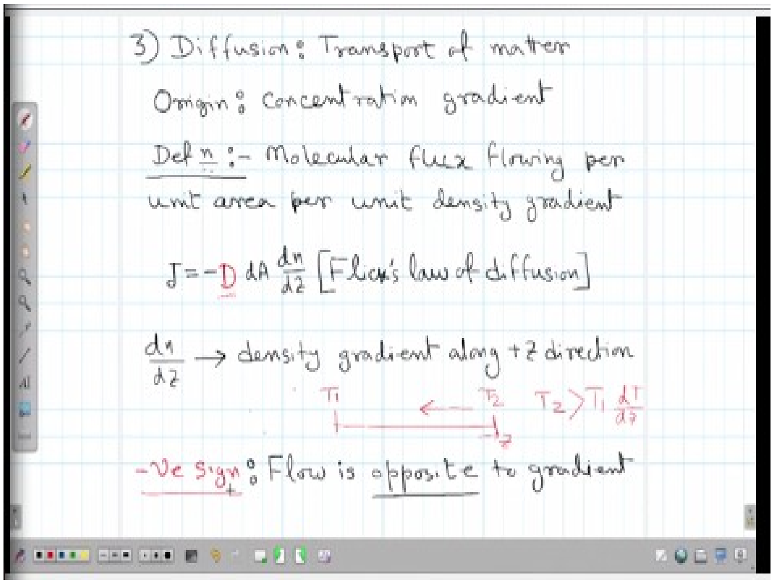

3) Diffusion: Transport of matter

Origin: Concentration gradient

Defn:- Molecular flux flowing per unit area per unit density gradient

$J = -D \, dA \frac{dn}{dz}$ [Flick's law of diffusion]

$\frac{dn}{dz} \rightarrow$ density gradient along +z direction

$T_2 > T_1$ $\frac{dT}{dz}$

-ve Sign: Flow is opposite to gradient

And finally we will be talking about diffusion which is the transport of mass from one part of the fluid to the other part. So, let us look at this 3 systematically one by one. So, for viscosity we have which once again is a transport of momentum. The origin of this viscosity and as I have said already that you go back to the definition of equilibrium, when equilibrium exist I mean it is not exactly the definition of equilibrium but it is a condition that when equilibrium exist there is no gradient present.

So, whenever there is a gradient there is equilibrium being disturbed and we have one of these 3 transport properties. For example whenever there is a velocity gradient that means the system is not in mechanical equilibrium; so that means it has different parts of the system exhibits different average velocity that means there is a velocity gradient. So, that is when the basically different we can assume as if different layers of the layer of the fluid is moving with different speed that is when we have viscosity.

And the definition of viscosity is the tangential force per unit area per unit velocity gradient. So, if the viscous force is F we can write according to Newton's law of viscosity we can write F is equal to minus eta dA du dz where dA is the elemental area we are talking about, force on elemental area dA, eta being the coefficient of viscosity and du dz is the velocity gradient along z direction. Of course we are talking about only one dimensional cases and these all the 3 phenomena we are talking about, it will be derived and discussed in one dimensional case only.

But it can be very well generalized to 3 dimensions as well. Now next is the thermal conductivity and the origin of thermal conductivity is **e** when there is a temperature gradient into the system. So, that means when we have 2 parts of the system at 2 different temperatures, then from the hotter part the heat is being transferred to the colder part. And that is obvious we have seen that even in a solid if we have 1 metal rod, 1 end is in a oven the other end I mean the there is a uniform.

Let us say this is a metal rod, metal body, so if this end is put on a oven the other end slowly heats up and of course because of the heat dissipation the temperature change in this part is more and temperature change in this part is less. But let me tell you the same thing happens in a fluidic system as well. If we have a gas assembly which has higher temperature in one side and lower temperature in the other side, what happens is gas molecules starts flowing from the hot end to the cold end taking excess energy from the hot part to the cold part.

So, that eventually an equilibrium is established, so that is where we come up with this definition of thermal conductivity, that is thermal energy. So, basically the thermal conductivity is the coefficient that we are going to measure from the fundamental principle. So, the definition of thermal conductivity is the thermal energy flowing per unit area per unit temperature gradient among the system.

So, if we have Q being the total amount of heat that is being transferred then that is equal to -k or sometimes it is written with a Latin letter kappa minus k dA dT dz which is called the Fourier's law of heat transfer. When dT dz once again is the temperature gradient along positive z direction. Once again we are talking about only one direction but it can be very well be generalized to 3 dimensions.

And lastly we have diffusion which is actually the transport of matter. What is diffusion? If we have instead of a temperature gradient, if we have a concentration gradient what do we mean by concentration gradient? Let us say I think in the very first day of discussion of mean free path I have given you 1 example where I have a small perfume bottle, I open it up and eventually the smell propagates among along this room, everywhere in the room you could you can smell it.

That happens when in your household some Puja is going on or some fragrant sticks are being let, so eventually the entire house is full of that smell and that is very obvious. And that is where the concentration gradient comes into picture, when I open the perfume bottle at this point in close to the face of the bottle or mouth of the bottle then the concentration of those gas particles, the fragrance particles are very high whereas outside that or otherwise it is almost 0.

So, now what happens? It starts propagating in an attempt to equilibrate the situation; every system wants to drive itself towards equilibrium. So, high concentration always wants to propagate along in different directions, so that the concentration gradient is does no longer exist. So, that is the basic nature we have or for any natural system this is the nature and this is why we have something called the diffusion.

There is a law Flick's law of diffusion which says J the diffusion current is equal to minus D dA dn dz, where d is the elemental area once again, dn dz is the density gradient. dn just a minute, I do not know what happened all of a sudden everything is gone, I do not know. Anyway I think you already got this something is going on here, some software issue, anyway. So, let me try to load this once again and then I will show you clipless, fantastic, nothing is lost, everything is there.

So, once again the definition of diffusion is the molecular flux flowing or the diffusion current molecular flux flowing per unit area per unit density gradient is the coefficient of diffusion which is marked with D here. So, all the coefficients are marked with red let it be eta, let it be k oh! what is happening? I do not know, something is not right, fine it is there. So, let it be eta, let it be kappa and let it be D.

Now you notice that all the definition there is a negative sign, in here there is a negative sign, in here there is a negative sign, here there is a negative sign, now what does it mean? This negative sign means the flow is opposite to the gradient, so that means if the temperature is increasing in this direction. So, let us assume this is my direction of temperature gradient, so T is T 2 here and we have T 1 here, so that T 2 is greater than T 1. So, heat is flowing opposite to the temperature gradient.

So, of course you realize that dT dz if this is my z direction, so dT dz is positive along positive z direction, whereas the temperature is flowing in the opposite direction. So, that is why this is the origin of this negative sign. So, now it is time that we formally look into this 3 phenomena and try to formalize it, what do I mean by formalizing? I want to start from the fundamentals of mean free path and finally come up with expression of these 3 gradients that is d eta and k, so that we can express this quantity in terms of the fundamental parameters of the system. So, let us start.

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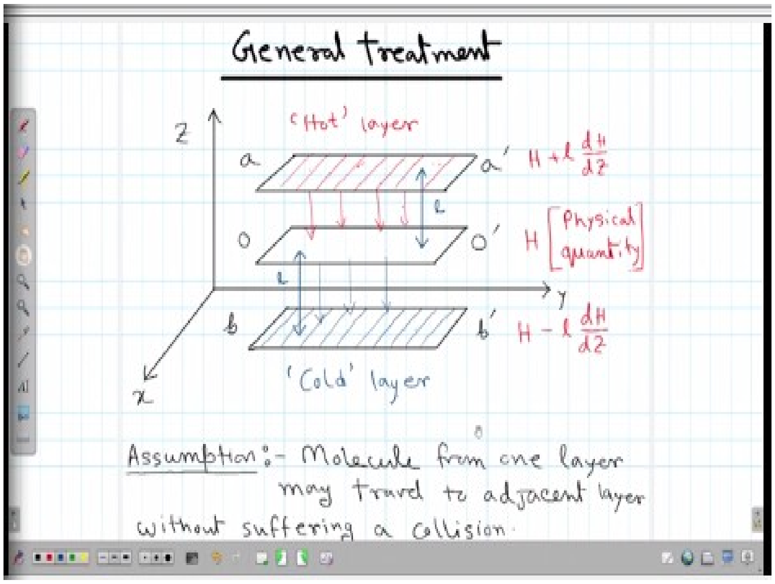
General treatment

Assumption:- Molecule from one layer may travel to adjacent layer without suffering a collision.

So, I would take some take some time to explain this particular figure here. Here what we have is a xyz coordinate system, now let us focus on this middle part, we have a layer which is leveled by O O prime. On top of that we have 1 layer which is a-a prime and at the bottom there is one layer which is b-b prime. So, basically what I did here? It is kind of a semi empirical kind of treatment, it is not exactly molecular level like we did for the kinetic theory of gases so far.

But it is more of a semi I mean kind of an empirical view I mean it is not exactly a macroscopic view, not exactly a microscopic view. What we have in mind is that these 3 layers represents 3 distinct regions in a fluid assembly or gas assembly let us call it. Now each of this, inside each of this layer there is no gradient. So, the density, the velocity of gas molecules everything is uniform in this layer.

Upper layer is our hot layer, what do we mean by hot layer? We talk about a physical quantity which is H, H could be anything, H could be density, H could be temperature, a H could be velocity, anything. At present we just make it very general, we call it H. So, that H this layer O-O prime has the value of that particular quantity as H, whereas the upper layer a-a prime has a value of this particular quantity as H plus l dH dz, this is the z axis.

So, the gradient is along z axis, so l being the perpendicular separation or the shortest distance between these 2 planes. And similarly there is a plane below that is the cold layer, whatever quantity we are talking about. Let us think about in terms of temperature, so if this temperature is T, this temperature is T plus l dT dz and this temperature is T minus l into dT dz, so this temperature is less compared to the upper layer.

So, that means the velocity gradient is along this direction upwards and this has been our assumption while writing all this expression. For example when we are writing D, we are assuming dn dz is a density gradient along plus z direction. So, we have these 3 layers and how does this transport takes place? The molecules from one layer may travel to the other layer or the adjacent layer without suffering a collision.

So, this is the assumption that the molecules or this layers the separation between these 2 layers are such that one layer but the molecules from one of this layer let us assume from this layer or let us say from this layer, so they start traveling towards this layer and they reach there without suffering a condition. Similarly cold molecules from this layer travels or rather I would say they always travel from this to this.

So, the molecules from this layer travel in this direction without suffering a condition. So, thus once again this is kind of a semi classical picture semi macroscopic picture, we are not exactly considering microscopic level, no, we are not exactly considering macroscopic level, we are considering a mixture of both. So, now in order to such phenomena to take place that means the molecules from each layer traveling to the adjacent layer without suffering a condition.

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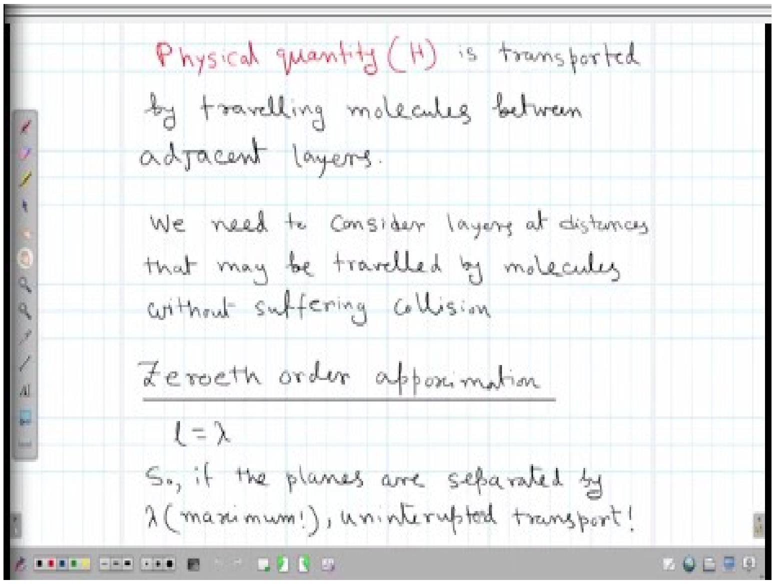

Physical quantity (H) is transported by travelling molecules between adjacent layers.

We need to consider layers at distances that may be travelled by molecules without suffering collision

Zeroeth order approximation

$l = \lambda$

So, if the planes are separated by λ (maximum!), uninterrupted transport!

There has to be a limit or there has to be an estimate of this length scale l, we will do that in a moment. But first let us quickly understand that physical quantity H is transported, so whatever I have said is written here is transported by molecules between adjacent layers. Now the zeroth order approximation for that would be if we just take l is equal to z, so that the planes are separated by a maximum of lambda in.

So, that we can have uninterrupted transport of molecules between 2 adjacent layers, so this is our zeroth order approximation. This which is also once again we have seen that previously that zeroth order approximations are good, it gives you something reasonable. But let us do it in a slightly elaborate way which we call the 1st order approximation.

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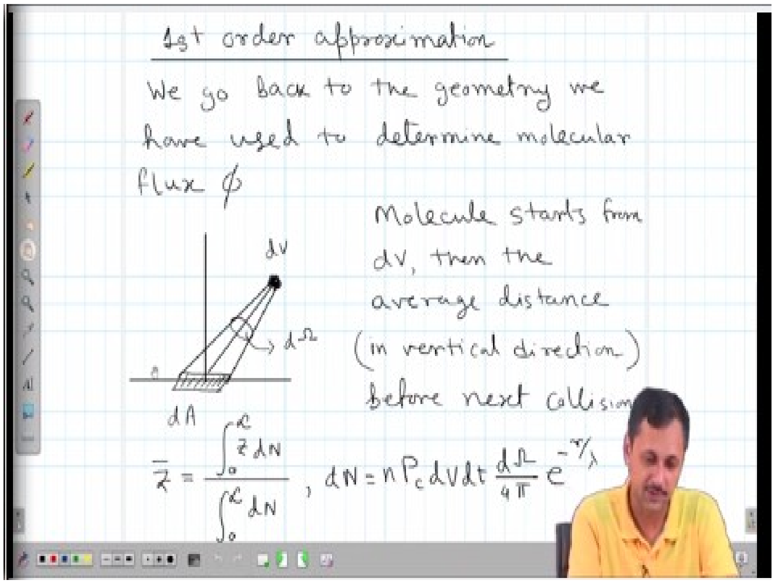

1st order approximation

We go back to the geometry we have used to determine molecular flux ϕ

Molecule starts from dV, then the average distance (in vertical direction) before next collision

$$\bar{z} = \frac{\int_0^\infty z\,dN}{\int_0^\infty dN}, \quad dN = nP_c\,dV\,dt\,\frac{d\Omega}{4\pi}e^{-r/\lambda}$$

You remember that we had this particular geometry when we have a small elemental volume dv which is hitting a small surface area dA on the container wall when we calculated molecular flux. And we use exactly the same geometry to calculate the average distance along z direction. So, our average distance z bar is given by integration 0 to infinity z dN divided by integration 0 to infinity dN, where dN is that number of molecule that will start from this elemental volume dv and end up on this elemental surface area dA. And if you remember we already have an expression for the same which is given by n P c dv dt d omega by 4 pi e to the power minus r by lambda.

Now this expression we have already derived, so I am not going to explain this to you over and over again. But you can always look back to lecture number 9 where this derivation was met in details. So, all we have to do is we need to compute this integration. Now in order to compute this integration we have to write dv in terms of the elemental volume in a three dimensional spherical polar coordinate system which is r square d r sine theta d theta d phi and we write z is equal to r cos theta.

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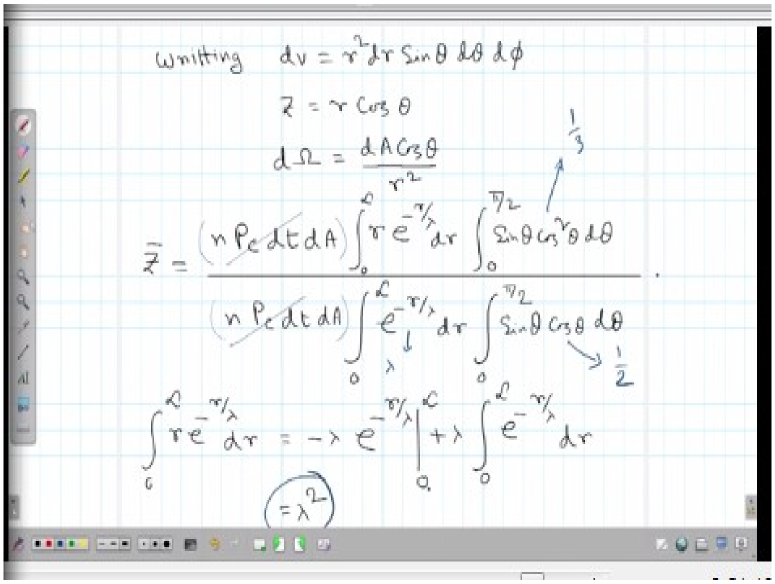

Writting $dv = r^2 dr \sin\theta \, d\theta \, d\phi$

$$z = r\cos\theta$$

$$d\Omega = \frac{dA\cos\theta}{r^2}$$

$$\bar{z} = \frac{(n \cancel{P_c} dt\,dA)\int_0^\infty r e^{-r/\lambda} dr \int_0^{\pi/2} \underbrace{\sin\theta\cos^2\theta\, d\theta}_{\frac{1}{3}}}{(n \cancel{P_c} dt\,dA)\int_0^\infty \underbrace{e^{-r/\lambda}}_{\lambda} dr \int_0^{\pi/2} \underbrace{\sin\theta\cos\theta\, d\theta}_{\frac{1}{2}}} \; .$$

$$\int_0^\infty r e^{-r/\lambda} dr = -\lambda e^{-r/\lambda}\Big|_0^\infty + \lambda\int_0^\infty e^{-r/\lambda} dr$$

$$\boxed{= \lambda^2}$$

So, once we do that and of course d omega, I forgot d omega has to be written as dA cos theta divided by r square. Now once we do that we see that many terms from the numerator and the denominator are in common because both will have n P c dt and the dA and divided by 4 pi which will be cancelling out. So, I just kept this in order to show you but these terms are nicely cancelled out, leaving behind 0 to infinity r e to the power minus r by lambda dr.

And 0 to pi by 2 sine theta cos square theta d theta in the numerator and in the denominator 0 to infinity e to the power minus r by lambda dr because r is not there in the denominator. And of course there is 1 cos theta which is missing because of this, we have z is equal to r cos theta in the numerator and which is not present in the denominator. So, we have sine theta cos theta d theta.

Now we have already computed these integrals, this integral if you remember will give us what will it? That what will give us? It will give you one third, this integral will give you half, it was all derived in the previous lecture, this integration will give you lambda and this is a new integration I just did it by integration by parts. And you see that the first term cancels out nicely and the second term leaves you lambda square. So, this is what we have in hand.

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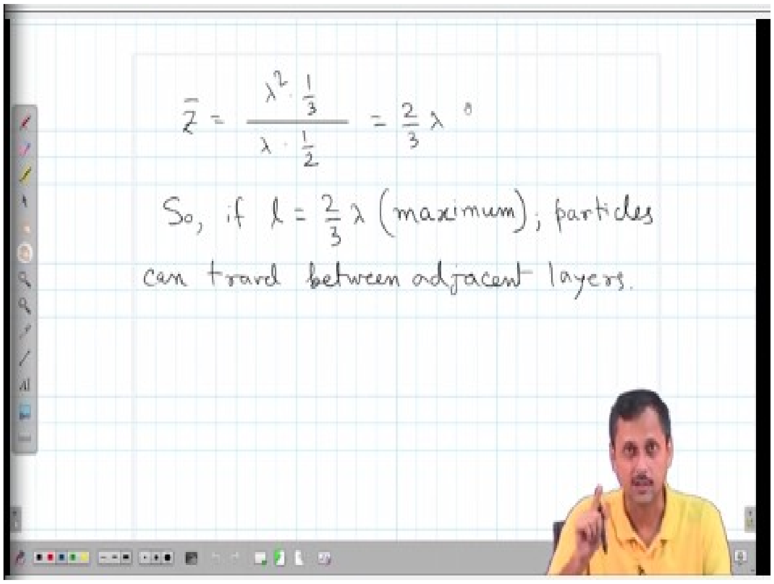

$$\bar{z} = \frac{\lambda^2 \cdot \frac{1}{3}}{\lambda \cdot \frac{1}{2}} = \frac{2}{3}\lambda$$

So, if $l = \frac{2}{3}\lambda$ (maximum); particles can travel between adjacent layers.

Now if we put all these values we see that z bar is equal to lambda square times one third divided by lambda by one third which is two third lambda. So, if l is equal to a maximum of two third lambda then particles can travel between adjacent layers without suffering a condition. So, let us go back to this picture once again.

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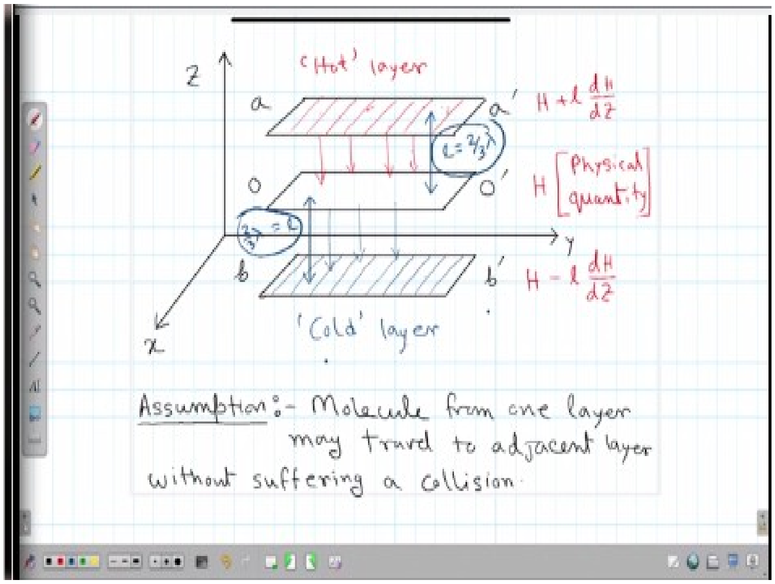

Assumption:- Molecule from one layer may travel to adjacent layer without suffering a collision.

If we have a distance l is equal to two third lambda and l is already here l is equal to this is equal to two third lambda, then and only then we have uninterrupted transport between these 2 layers. But once again please remember that this is an averaged out phenomena, we can have even if we have 2 adjacent layers taken at two third lambda distance, many particles might not reach and many particles and even if we have a length greater than l is equal to two third lambda many particle might still reach.

But on an average the probability of maximum number of particles will reaching between these 2 layers will be uninterrupted or rather I would say the maximum distance for which maximum number of particles will reach between one layer to the other uninterrupted is given by this two third lambda value. So, this comes out from the 1st order approximation what that we have done. So, we will take it up from here in the next lecture and we will try to compute the coefficients of viscosity, diffusion and thermal conductivity starting from this point, thank you.