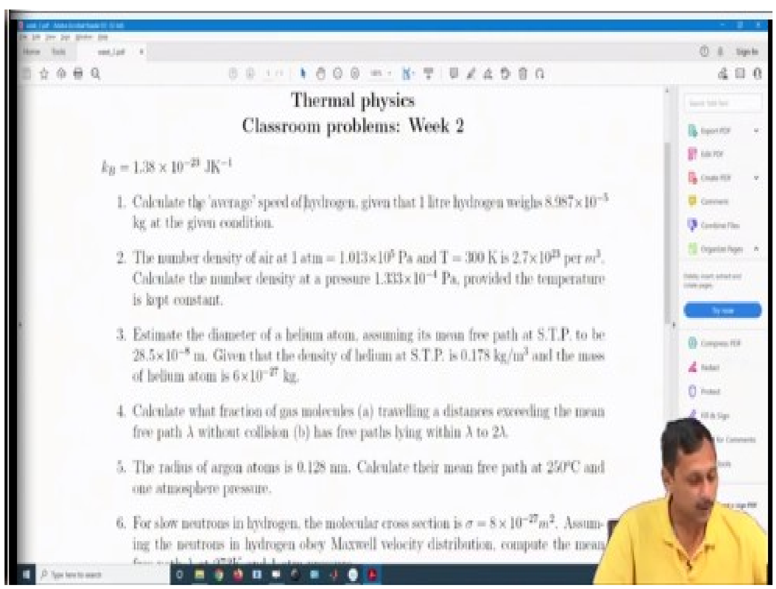
Thermal Physics
Prof. Debamalya Banerjee
Department of Physics
Indian Institute of Technology-Kharagpur

Lecture-10
Topic-Problems on Mean Free Path

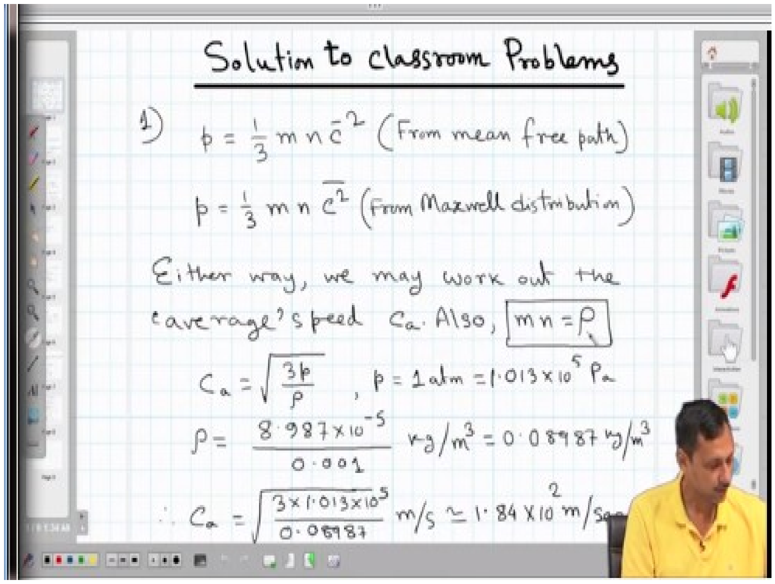
Hello and welcome back to another lecture on this NPTEL course of thermal physics. So, this is the last lecture of week 2 and in today's lecture we will be discussing mostly problems. So, so far whatever we have discussed in this particular week is the concept of mean free path, the expression for mean free path, collision cross section, collision frequency, survival equation and towards the end in the last lecture we have discussed about the pressure expression from mean free path. Also we have found determined an expression for the molecular flux but we will not be using that for this week's problems.

(Refer Slide Time: 01:09)



So, we have a problem set ready, I hope you already have this problem set in hand in a PDF file. So, up here the value of Boltzmann constant is specified which will be needed for many of the problems. The first problem is calculate the average speed of hydrogen, hydrogen atom basic hydrogen molecule given that 1 liter of hydrogen weighs 8.987 into 10 to the power -5 kg at the given condition. So, only the pressure is given, nothing else and sorry the density is given.

(Refer Slide Time: 01:56)

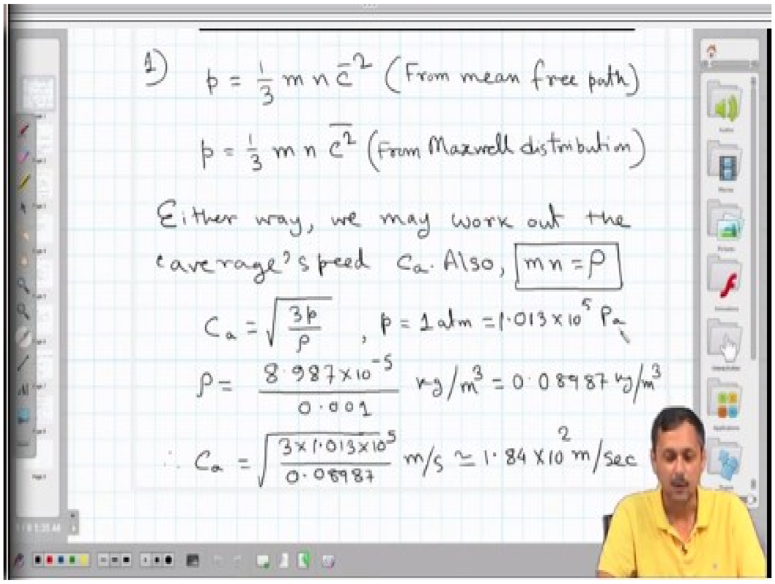


Now in order to solve this problem we note that we have these 2 pressure expression, p is equal to one third m n c bar square which is from the mean free path concept and p is equal to one third m n c square bar from Maxwell's distribution. Now in order to avoid the controversy between or I should not say controversy rather say the confusion between which one is the proper expression, I have used the term average speed.

Because both I mean speed is also an average speed RMS speed is also an average speed, only thing is the averaging algorithms are slightly different, so that is why I just kept the word average here. So, let us take any of the expression, so what we will do is we will just call it the average speed C a because both the expressions are otherwise identical. So, if we come compute C a we can call it one of either of the average speed, either mean speed or the average speed.

Now we have to note in order to solve this problem that m n is equal to rho, right. So, sorry there is a small mistake here, I will correct it in the final version, it is actually one atmosphere the pressure has to be mentioned the pressure is one atmosphere. So, in one atmosphere, under one atmosphere pressure the density is given. So, if we substitute for m n is equal to rho what do we have here?

(Refer Slide Time: 03:41)

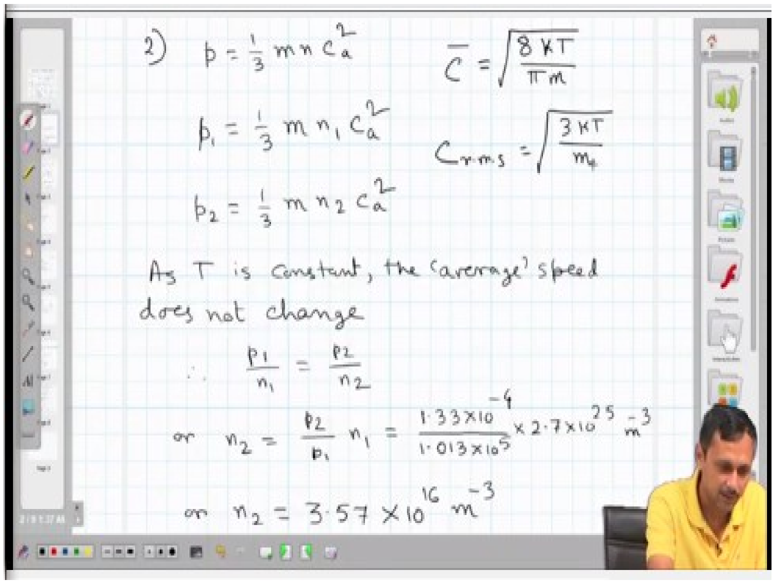


We have Ca that is the speed of interest is equal to root over of three p by rho. Now p is equal to 1 atmosphere which is given let us assume it is given I will correct it, so this is 1.013 into 10 the power 45 Pascal, this is another number you have to keep in mind. Whenever there is one atmosphere we have to convert it into SI unit for most of our applications and please remember this is 1.013 into 10 to the power five Pascals, please keep this in mind.

So, now rho is given or their weight is given for one liter of hydrogen, now one liter is equal to 0.001 meter cube, so in the units of kg per meter cube we have 0.08987 kg per meter cube that is the density. So, if we just plug in these numbers for rho and p that is exactly what we did here, we get the value of 1.84 into 10 to the power 2 meters per second, so basically this is 184 meters per second which sounds reasonable with the number.

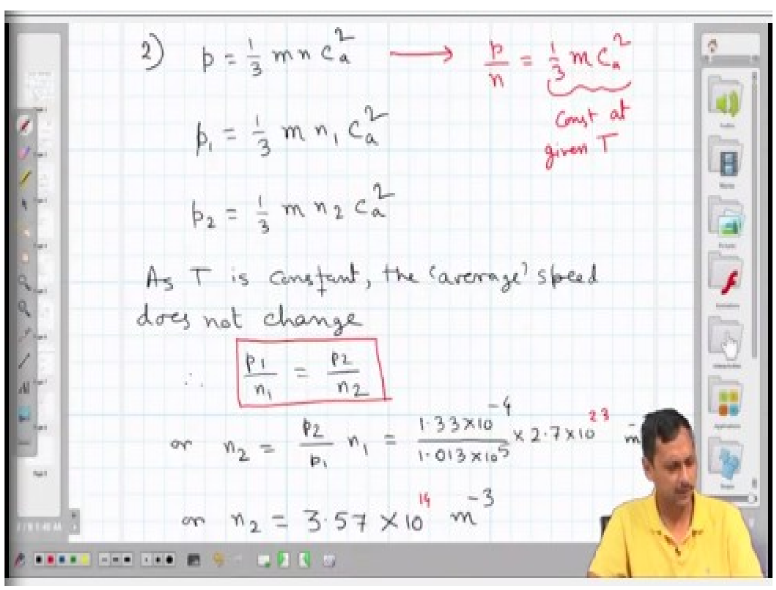
So, for the next problem we have the number density of air at one atmosphere and T is equal to 300 Kelvin is given as 2.7 into 10 to the power 23 per meter cube. Calculate the number density at a pressure 1.33 into 10 to the power minus 4 Pascal provided the temperature is kept constant. So, what is given here? We have the number density at a given temperature and pressure. So, this is the pressure that is given, once again this is the atmospheric pressure and we have to compute the number density in another lower pressure which is almost 9 orders of magnitude lower as compared to the given pressure.

(Refer Slide Time: 05:57)



So, it should not be that difficult and here it does not matter whether we stack the mean speed or the rms speed. Because both of them, so for example mean speed or yeah mean speed has an expression of root 8 KT by pi m and average or rms speed has an expression of root over 3 KT by m. But either way they both depend only on m and T and K is a constant anyway, K and pi they are constants anyway. So, for the same molecule if the temperature remains constant the speed does not change. So, it does not matter whether we take mean speed or rms speed for this particular problem because this will not affect the solution by any means.

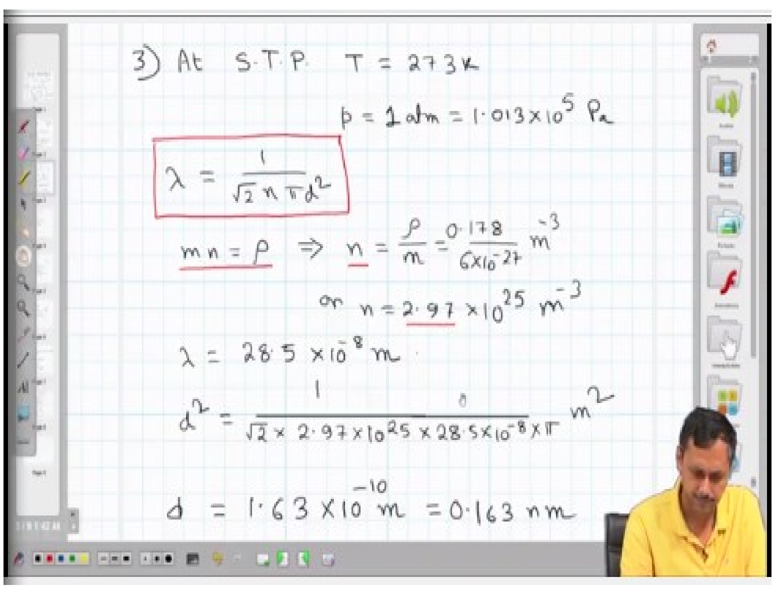
(Refer Slide Time: 07:00)



Always see here is when we rewrite this any of this expression we see p by n is equal to one third m C a square which is a constant at given T. So, we can write n 1 p 1 is equal to n 2 p 2. So, n 2 is given by p 2 by p 1 times n 1, so the ratios are given and we just have to multiply it with the total, sorry there is a mistake here it should be 23, sorry a small mistake, should be 23 here. So, instead of 16 we have 14 so and this is very obvious because 9 orders of change in pressure magnitude change in pressure results to 9 orders of magnitude change in the density also, fantastic.

Third problem, now estimate the diameter of helium atom assuming it is mean free path at S.T.P is to be 28.5 into 10 to the power minus 8 meter. Given that the density of helium at S.T.P is 0.178 kg per meter cube and the mass of helium atom is 6 into 10 to the power minus 27 kg. So, we have to estimate the diameter of helium atom, what is given here S.T.P means standard pressure and temperature which corresponds to 0 degree centigrade temperature and one atmosphere pressure. Mean free path is given lambda and density is given, mass is given, so almost everything is given.

(Refer Slide Time: 09:19)



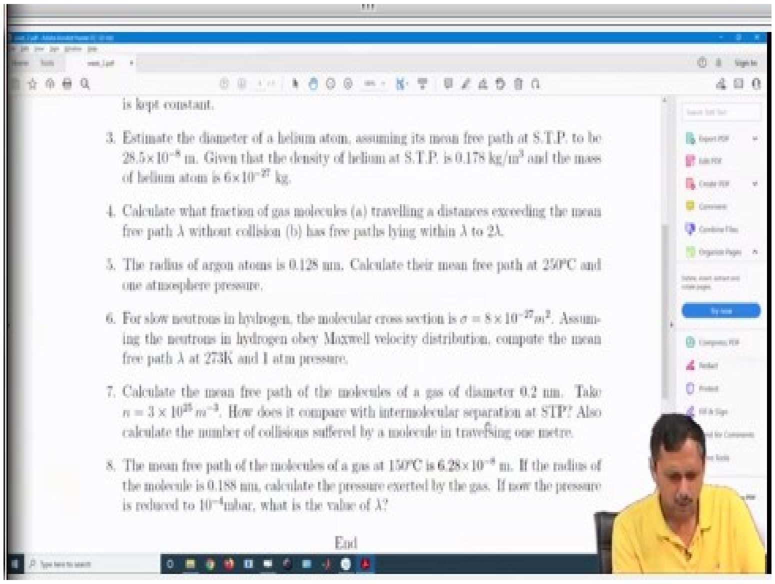
What we need to know here is we have to apply this relation here. So, many parameters given, we have to figure out which one of these are important parameters, what is given is density. Now if we look at the expression for mean free path, we have lambda is equal to 1 divided by root 2 n pi d square. Basically it is sigma, so sigma is the collision cross section, we just write this as pi d square.

Now, we do not need the temperature data at all but the pressure is something that we might need, we will look into it. But what is given here is the density rho and what we need is the number density n, so in order to figure out the relation between n and rho we remember that m n is equal to rho, so n is given by rho by m. So, rho is given which is 0.178 and m is given which is 6 into 10 to the power minus 27 and finally we get n to be equal to 2.97 into 10 to the power 25 per meter cube, lambda is also given.

Now all we have to do is we have to compute this square from this following relation, plugging into this number and we get a d is equal to 1.63 into 10 to the power minu 10 meters. So, this is what our final answer is. What is important here is if you go back to this problem and look you see S.T.P condition is given which is not of any use to us. So, even if we do not know what is the pressure and temperature of that assembly all we care about if the mean free path is given along with all other 2 necessary parameters, we can compute the problem.

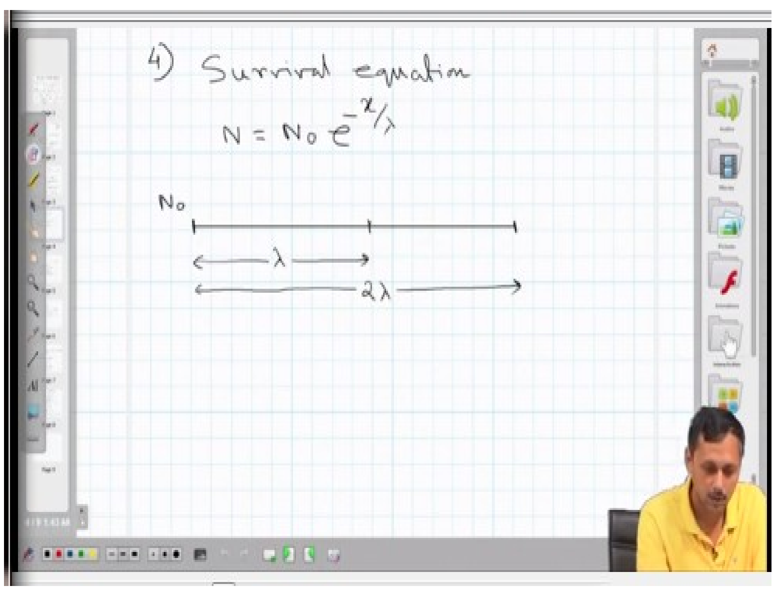
That we have to keep in mind because sometimes in a problem when you are solving a problem in the book sometimes the redundant information is given, you might think that these informations are necessary. But finally sometimes it happens that those informations are there just to make sure that basically to make you think that you want to see why this is important, why this is given? So, you have to keep this in mind, sometimes the redundant information is there.

(Refer Slide Time: 12:05)

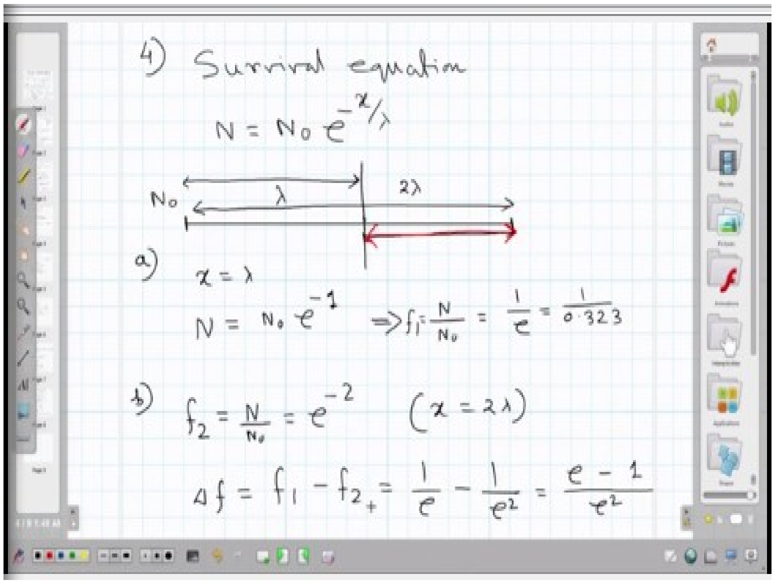


So, with this let us move to next to the next problem which is the problem number 4. Calculate what fraction of gas molecules travelling a distance exceeding the mean free path lambda without collision and has free paths lying between lambda and 2 lambda. So, in order to solve this problem we have to look into the survival equation.

(Refer Slide Time: 12:28)



(Refer Slide Time: 12:34)



Just use the upper part actually, let us say this is my length, so I start with N 0 number of molecules and after travelling a distance lambda or travelling a distance 2 lambda what are the numbers that remains. So, that is given by the survival equation. So, for part a what we have to do is? We have to just put x is equal to lambda, so basically survival equation gives you the number as estimate of molecules that remains without suffering a collision over a distance x.

So, for the first part all we have to do is we have to put x is equal to lambda and N is N 0 e to the power minus one, this gives you the ratio the fraction we are looking for is basically N by N 0 this is one by e. I do not know the remember the value of N but it will be something like 0.323 something like this, so do not quote me on this but just compute, take a calculator and compute this number. Now for part b, it is slightly tricky, let us look at it once again, has free paths lying within lambda and 2 lambda.

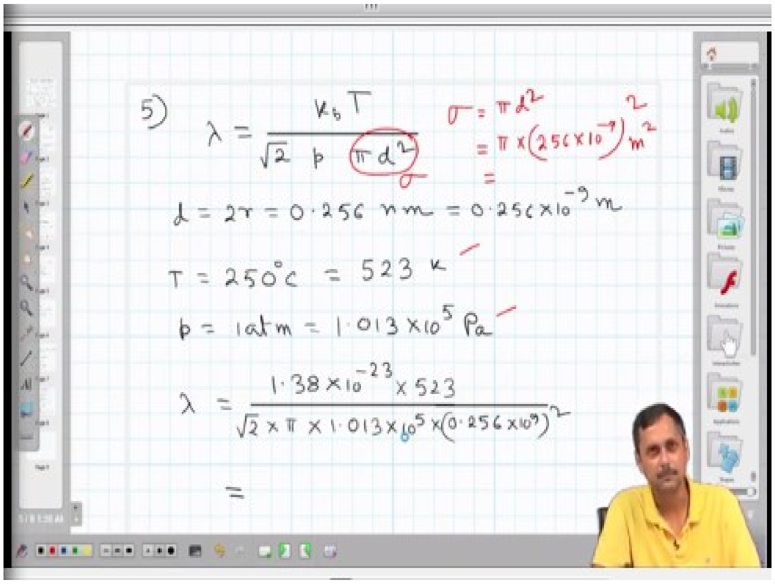
So, we have to basically compute the fraction of molecules that have not that will suffer a collision between on an average of course because these are all average overall ensemble average values. So, that will suffer a collision travelling from lambda to 2 lambda, now how will you do that? We already know what are the fractions that will survive till lambda. Next what do we do? We compute another fraction let us call it f 1, let us compute f 2 which is the number of molecules that will survive beyond a distance 2 lambda.

So, f 2 will be given by N by N 0 equal to e to the power minus 2, so basically we put x is equal to lambda. So, basically this is 1 by e square and the fraction we are looking for is the difference between f 2 and f 1 or f 1 and f 2, f 1 will be more f 2 will be less. So, our delta f will be f 1 - f 2 which is 1 over e minus 1 over e square which is e square times e minus 1, so this is the number. Once again I do not know what number it will be but as far as I remember it will be something like 0.22 something. I wanted to compute this in front of you, so that you get a better feel on what calculations, what is the logic behind this calculation otherwise the calculation is super simple.

So, once again I have computed for the first part I have computed the number of particles that will survive without a collision after traveling this distance lambda. And for part 2 I have computed the fraction that has a probability of suffering a collision between lambda and lambda to 2 lambda which is basically the subtraction of f 1 minus f 2, I hope it is clear. So, let us move to the next problem which is problem number 5, simple.

The radius of argon atom is given; calculate the mean free path at 250 degree centigrade and 1 atmosphere pressure. And let me tell you there is no redundant information here, all the information is necessary, 1 atmosphere pressure is given which is 1.013 into 10 to the power 5 here temperature is given, radius is given, radius means essentially the collision cross section is given.

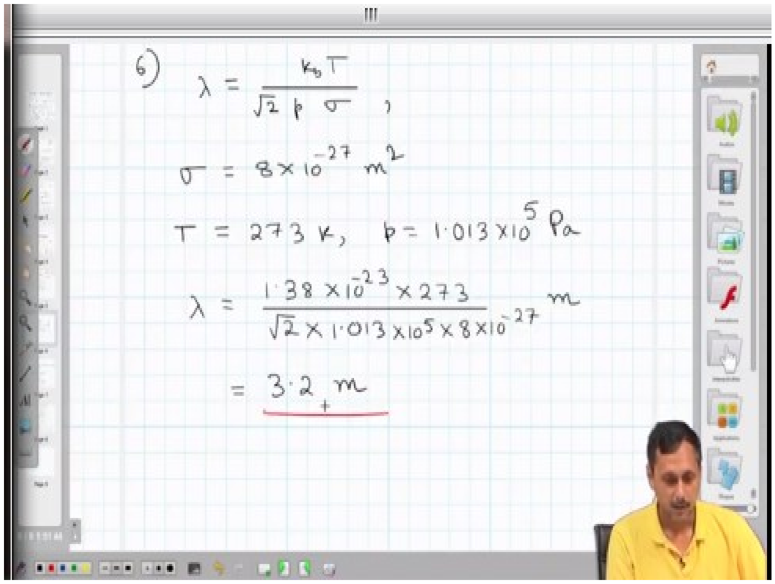
(Refer Slide Time: 17:19)



So, what we need to do is we need to use this particular equation which is K b T by root 2 p pi d square, d is equal to 2r which is 0.256 nanometer which is equal to 0.256 into 10 to the power minus 9 meter. Alternatively what we can do is we can simply compute the collision cross section which is basically this quantity sigma, so sigma is equal to pi d square and this will be given by pi into 0.256 into 10 to the power minus 9 whole square, so it will be I do not remember offhand but it will be something like, so it will be 18 and then point yes.

So, something of the order of 10 to the power minus 19 meters 1 by meter square not 1 meter square. T is given, p is given, all we have to do is we have to plug in these numbers and unfortunately I could not finish the calculation I do not remember what number it will come but I think you can do it for you, you can do it, I will provide the this one, the answer in the forum.

(Refer Slide Time: 19:03)

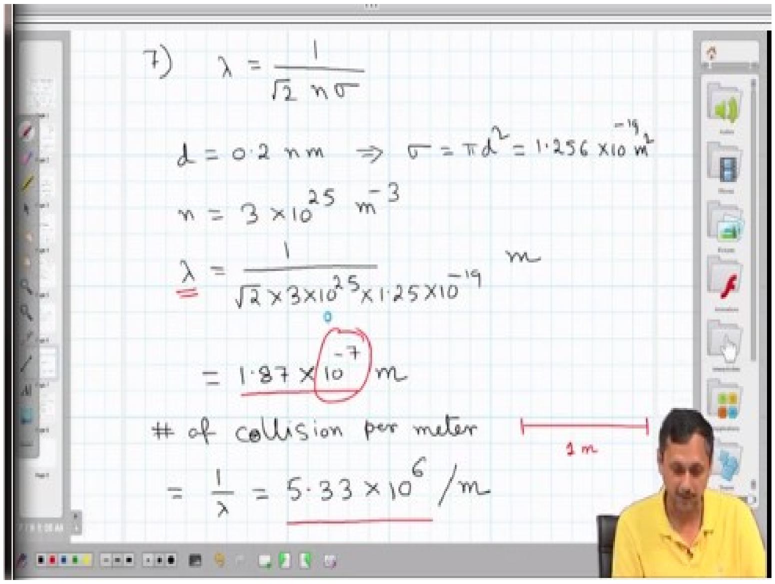


Or rather I will provide it with the final version of this. Now problem number 6, for slow neutron in hydrogen the molecular cross section is given as sigma is equal to 8 into 10 to the power minus 27 meter square. Assuming neutrons in hydrogen over a Maxwell's velocity distribution, compute the mean free path at 273 Kelvin and 1 atmosphere pressure. Once again this is a very straight forward problem, here the sigma itself is given, p is given.

So, we just have to use for the both the problems we have to use this expression for lambda. You remember N, it was 1 by root 2 N sigma, we have to substitute N with K b T by p, we did that in the class. So, we just use the exact same expression for problem number 5 and 6, problem number 6 is super simple, putting these values we get 3.2 meters. Now 3.2 meters is a relatively large number for a mean free path value.

And this is happening because look at the collision cross section, collision cross section is 10 to the power minus 27. So, 10 to the power minus 27 is a relatively small collision cross section that is why it gives you such a large mean free path. Problem number 7 calculate the mean free path of the molecule of a gas of diameter 0.2 nanometer, simple. Take n is equal to 3 into 10 to the power 25 per meter cube, so the number density is given, the diameter is given. So, computing the mean free path is actually very easy.

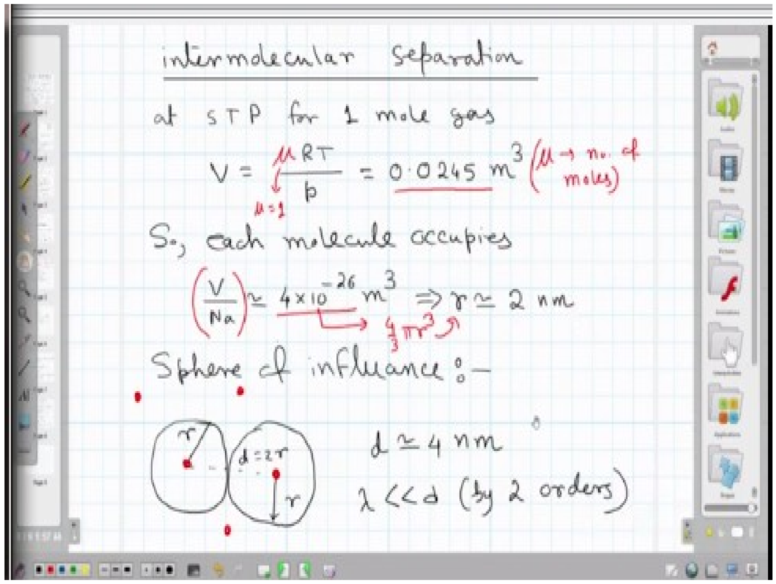
(Refer Slide Time: 20:59)



Once again it is simply put in the numbers, you get 1 by root 2 n sigma, you put d is equal to 0.2 nanometer, so that means sigma is equal to pi d square is equal to 1.2 into 10 to the power minus 19 meter square, n is given, lambda very straightforward it comes out to be 1.87 into 10 to the power minus 7 meters. Now there are 2 more parts to this problem, one is the number of collisions per meter. So, there is how does it compare with the inter molecular separation at S.T.P that is the second part I will come to that in a moment.

Also calculate the number of collisions suffered by a molecule in traversing 1 meter. Now this one is easy, if I have let us say this distance is 1 meter on an average at 1.87 into 10 to the power minus 7 meters or at a distance lambda it suffers a collision. So, for unit length how many collisions it will suffer? It will be simply 1 over lambda which is given by 5.33 into 10 to the power 6 per meter. So, this is the number of collision is nothing but 1 over lambda. And this is nothing difficult, straight forward. There is a third part to the problem how does it compare with the intermolecular separation at S.T.P?

(Refer Slide Time: 22:40)



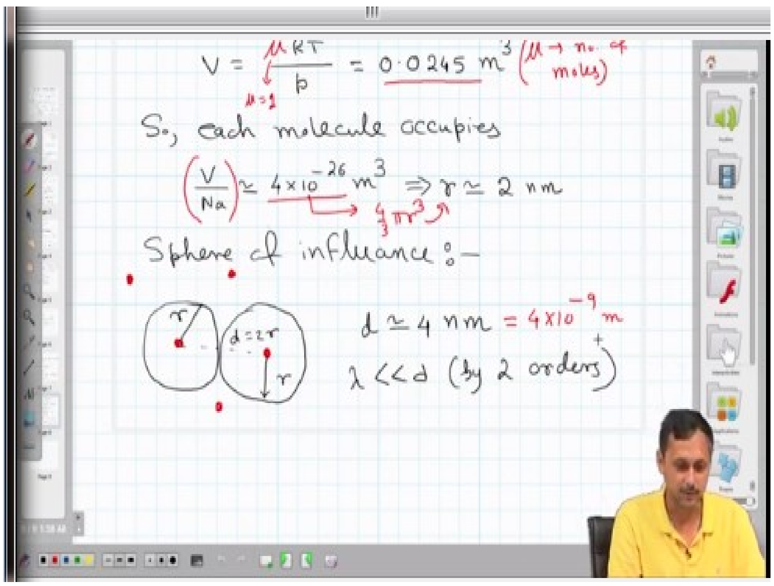
Now how do we compute the intermolecular separation? For S.T.P each any ideal gas has a volume of 0.0245 meter cube, why? Because V is equal to nRT by p from the relation pv is equal to nRT if you put R as the universal gas constant, here actually do not be confused, actually I should write a separate notation mu, mu is the number of moles. So, for 1 mole mu is equal to 1, so in this case mu is equal to 1 and R is equal to the universal gas constant, T is equal to 273 and p is equal to 1 atmospheric pressure which is already given many times. So, we have V is equal to 0.0245 meter cube, this is the standard volume for an ideal gas at S.T.P.

Now each molecule occupies how much volume? V divided by Na, Na is the number of molecules that we have inside 1 mole of an ideal gas. So, per molecule the volume occupied is this which is given by 4 into 10 to the power minus 26 meter cube which if I just apply a four third pi r cubed relation this gives you and r is equal to 2 nanometer roughly. Actually it should be 2.1 nanometer if I am not very wrong but it does not matter 2 is also ok. Now let us look into the sphere of, so basically this is the distance this is basically the measure of average intermolecular distance.

So, let us assume these red dots are my molecules, so each molecule will have a volume of 4 into 10 to the power minus 26 meter cube for itself which corresponds to a radius of 2 nanometer typically. So, the closest the centers or basically the point masses can come together, this is d is equal to 2 times r which is given by 4 nanometer. Please remember what is the molecular diameter typically, the typical molecular diameter is at least 1 order of magnitude less than this, typically I should not say at least typically 1 order of magnitude, 0.2 nanometer, all this data is given, look at this.

Look at problem number 5, the radius of argon atom is 0.128 nanometer and the radius of influence we get is 2 nanometer, so there is a factor of there is 1 order of magnitude between them. And interesting to note is the diameter that we get or the distance that we get is typically 4 nanometer which is much, much less than lambda. What is the value of lambda?

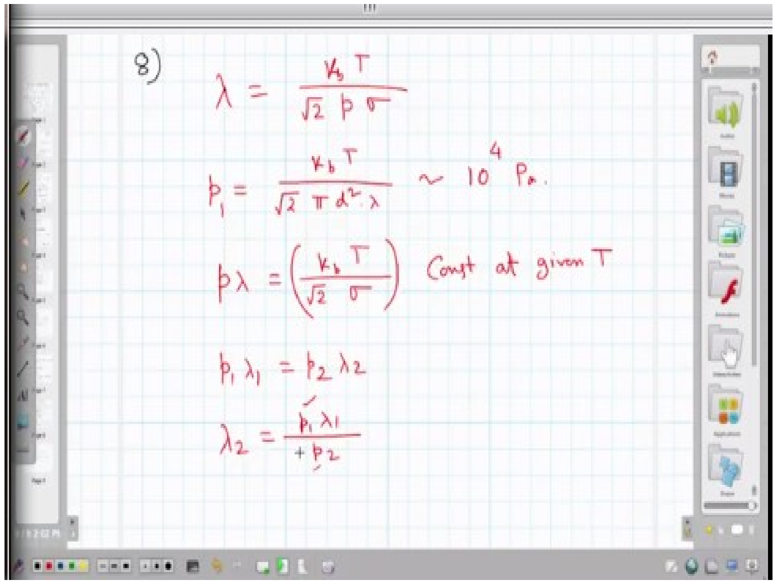
(Refer Slide Time: 26:09)



4 means, this is 4 into 10 to the power minus 9 meters and what is the mean free path we just calculate computed? 1.87 into 10 to the power minus 7 meters, so this is 10 to the power minus 7 as compared to 10 to the power minus 9 of intermolecular separation. Now what is interesting here? That means the mean free path is actually much, much longer as compared to the average intermolecular separation in a gas assembly.

And that actually tells you that collision is a very random and probabilistic phenomena. Just because 2 molecules are coming close to each other does not mean that they are staying close to each other does not mean that they will collide, please keep these numbers in mind, this will be useful for future.

(Refer Slide Time: 27:00)



And then we have a last problem which I have not solved, once again it is a very easy problem. The mean free path of the molecule of a gas at 150 degree centigrade is 6.28 into 10 to the power minus 8 meters, if the radius of the molecule is 0.188 nanometer. Once again you see this is one order of magnitude shorter than the typical intermolecular separation, calculate the pressure exerted by the gas, this we can very easily do.

Now if the pressure is reduced to 10 to the power minus 4 millibar, what is the value of lambda? Now I am not going to solve it for you because I have already solved very similar problems many. Last part I will discuss and for the first part it is super easy, because all we have to do is we have to use this particular relation lambda is equal to k B T by root 2 p sigma. So, p will be equal to k B T divided by root 2, sigma will be pi d is equal to square times lambda.

So, if you put this numbers and let me tell you, you will get a pressure of the order of 10 to the power 4 Pascal. I am not doing it for you, you should try it yourself, it is very simple. I will give you the final answer in the final notes that you get. Calculate the pressure exerted by the gas that is easy. If now the pressure is reduced to 10 to the power minus 4 millibar what is the value of lambda? Now it is 10 to the power 4 and then if I reduce it to 10 to the power minus 4 millibar which is if I am not very wrong 10 to the power minus 2 Pascals, millibar and Pascal there is a conversion please check that.

So, all we have to do in order to solve this, please remember that p lambda is equal to k B T divided by root 2 sigma which is constant at given T. So, if we keep the also that has to be mentioned in the question I will do that. As the temperature is constant the product p lambda remains constant. So, if this is the case then we have la p 2 or rather p 1 lambda 1 is equal to p 2 lambda 2 and lambda 2 is equal to p 1 lambda 1 by p 2, p 1 is given, p 1 is something let us call this p 1, p 2 is already given, lambda 1 is given because what was it?

This is 6.28 into 10 to the power minus 8 meters, all we have to do is we have to compute lambda 2. And without surprise we will find out that lambda 2 will be much, much greater compared to lambda 1 as p 2 is much, much less compared to p 1. So, please finish this problem, it will be a good exercise for you and as I have as promised I will provide you the answer, you can always cross check.

So, this ends the lecture for week 2, for week 3 we will be focusing on the transport properties. And from there we will be talking about properties of gas assembly like diffusion, thermal conductivity and viscosity starting from elementary consideration of mean free path, till then thank you.