Physics of Renewable Energy Systems
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Lecture 12
Betz Criteria for extracting wind power

Hello, so, let us now continue our discussion on this topic of wind power and what is the use of wind power for India and what is the future of this technology for our country. So, in today's lecture, we will be talking about the best criteria that is used to derive a formula which defines the maximum power that can be extracted using a wind turbine.

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So, based on the knowledge which we have attained over the last few lectures starting from continuity equation to Bernoulli's equation, we will move a step forward and we will derive a mathematical formulation which is called as Betz criteria that gives you the capability to extract wind power using the turbine.

You will see that there are n number of factors that actually control the operation of wind turbines and also they can lead to disadvantageous conditions, which can reduce the efficiencies and how to counter those limiting factors. And obviously, as we have been discussing in all the classes, you will see that, wind power is associated with certain advantages and certain limitations.

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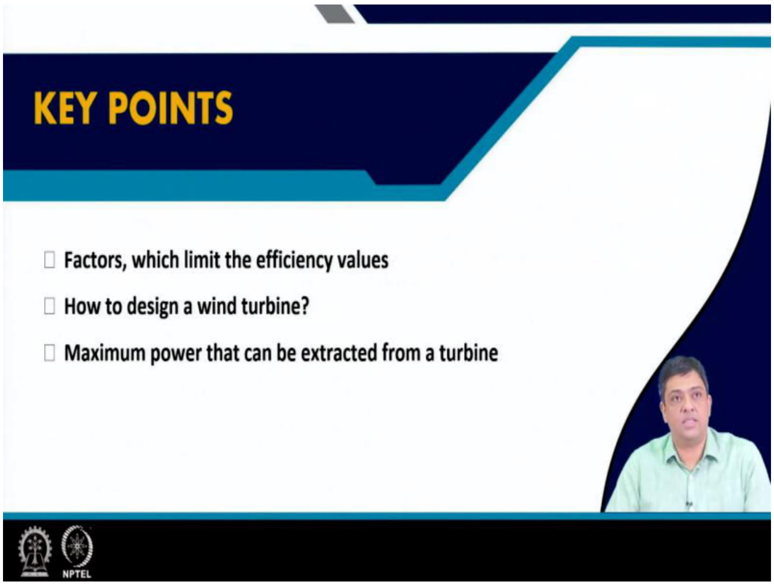

So, let us start and hope that by the time we finish this lecture, you will be able to understand clearly the factors which limit the efficiency values of turbine based systems and how do we design different types of wind turbine blades or what are the factors which control the designing of the wind turbine blades and the whole system.

We will start with that also and you will hopefully understand by the time we finish. And there is a limit to the maximum power that can be extracted from a turbine and how do we reach to that value will also be hopefully clear to you by the time we finish this lecture.

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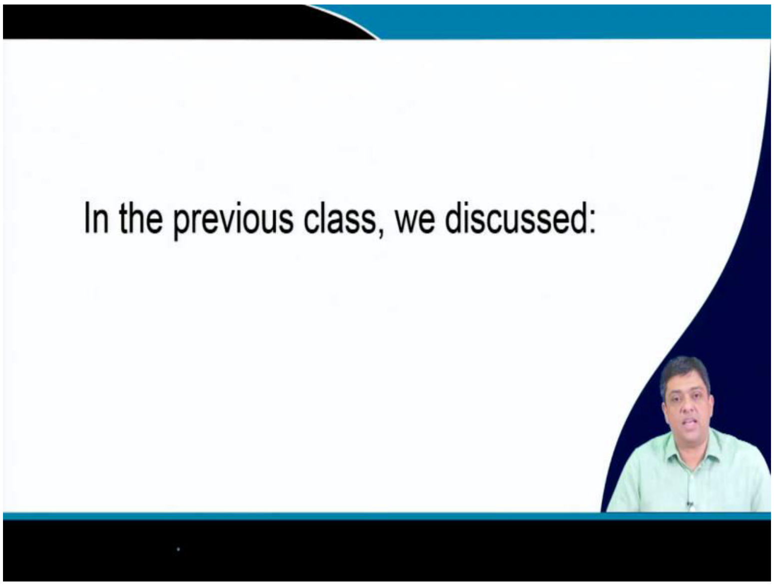

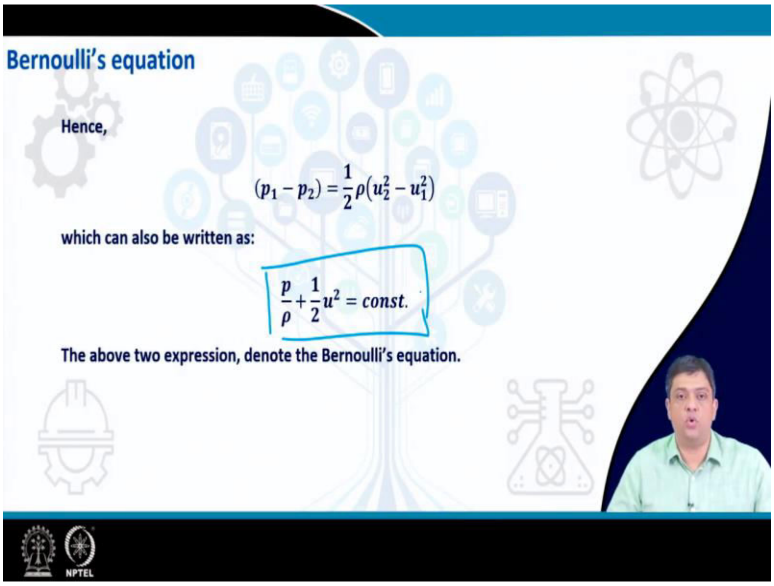

So, what have we actually discussed in the previous class? In the previous class, we have derived the expressions to reach a condition that is termed as Bernoulli's equation and the equation basically relates the pressure and velocity and if there is a change of pressure and velocity in two regions, then how do we relate them and what are the consequences of those varying factors or the values of those two parameters.

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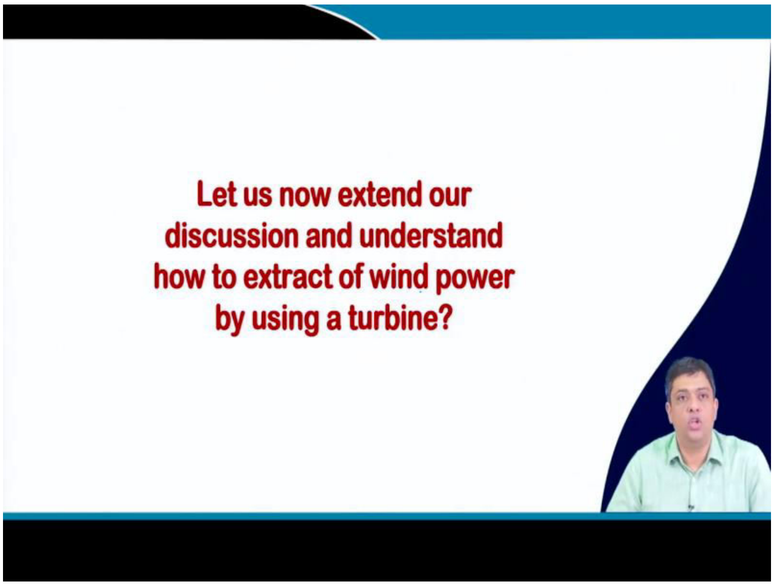

And we will start using Bernoulli's equation and try to move a step forward and see how that becomes useful to understand the mechanism by which we can extract wind power using a turbine.

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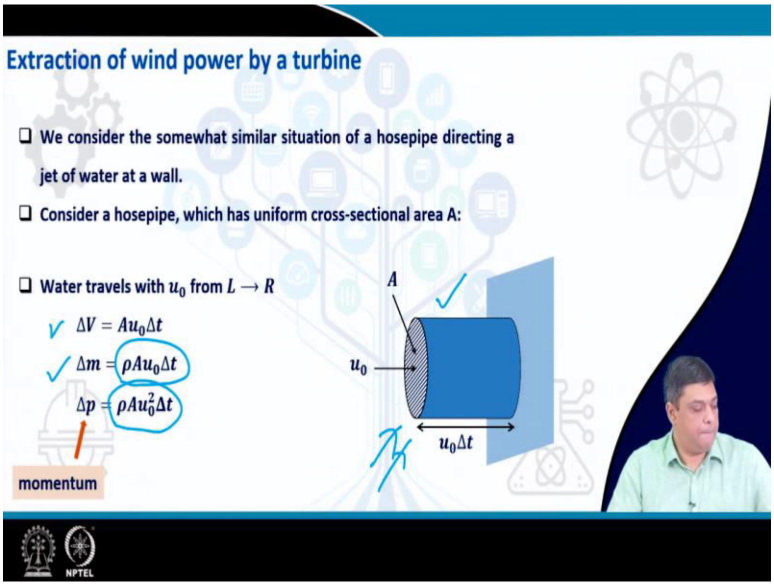

Very similar to what we did in the previous class where we were taking a fluid passing through a hosepipe and then we were trying to find out what is the volume of the flowing fluid which will enter and what is the volume of the flowing fluid which will exit if you had a varying diameter of the hosepipe and then we found out the conditions. That was the condition which was derived using a flowing ideal fluid in a continuously way reducing or increasing hosepipe and we reach the continuity equation.

In today's class, let us start with a very similar condition. We will take a hosepipe and there we had not defined the what fluid we were taking; we were just calling it as an ideal fluid. Here, let us talk and simplify the problem by using water as the one which is fluid which is flowing through the hosepipe. So, let us consider a hosepipe where water is flowing and the cross sectional area of the hosepipe is A.

And water travels from my left to my right that is what we will assume at and on the right hand side, you are fixing this hosepipe to a wall. So, what happens, we are now the water is not flowing through but it is going to hit the wall. So, wall is the obstacle this water is going to hit, please remember where are we moving to, we are moving towards the condition where we are saying that if there is a flowing fluid it hits the obstacle then can we extract and transfer certain amount of power from this fluid to the obstacle?

And just in the back of your mind please keep this pointer clear that we are trying to simulate a condition where a flowing wind is going to hit the turbine blade and then we are trying to extract energy from this flowing wind. So, that is where we are heading towards. So, what is the velocity delta V or volume delta V that is given as Au0 delta t. So, the amount of water which will be flowing through this cross sectional area in time delta t is Au0 that is the velocity at this point and in that time delta t.

Then what is the mass which will travel? You just multiply it by the density. So, you get rho Au0 delta t and if you want to write the momentum which is being transferred or which you are having the change amount of momentum which is being transferred from the point where the water is entering this hosepipe then you get rho Au0 square delta t, that is what we can straight away write. And so that what is the distance which has been travelled, you are considering u0 delta t. So, you are traveling this amount of area.

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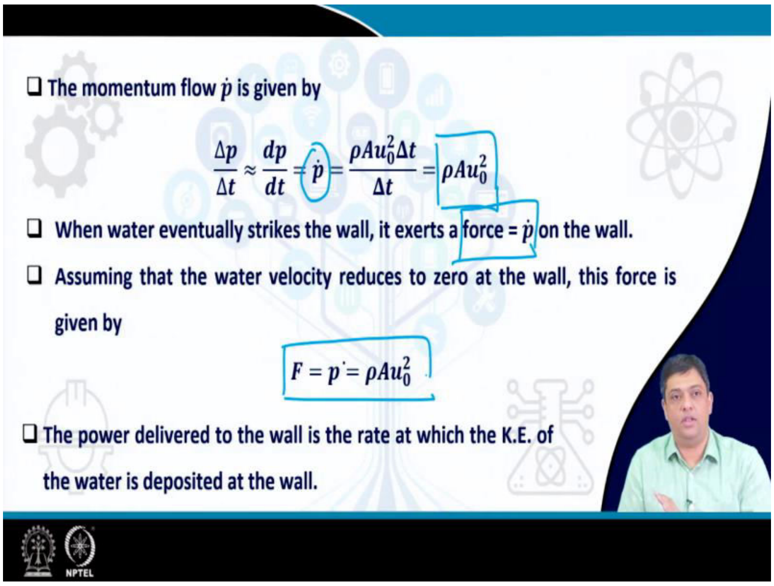

Now, we have derived momentum. So, can we talk about the momentum flow. So, what is the momentum flow you take dp by dt and you reach to a condition of rho Au0 square. This is what the momentum is being transferred or what is the momentum flow. Now, what did we discuss, we have discussed that the water is going to hit the wall. What is the wall? It is basically the obstacle. And when the water eventually strikes the wall it exerts a force, force equals to p dot, so it exerts a force equals to p dot on the wall.

Let us assume that when water hits the wall, its velocity reduces to 0. This is slightly different from what we will see when we start using wind, in wind you cannot believe that the wind velocity is reduced to 0 but to start our discussion and building our knowledge towards extracting power from wind, we are talking about a condition where a fluid that is water when it strikes the wall the velocity reduces to 0 at the wall.

And therefore, what do we get we have the relation that force is equal to p dot that is equal to rho Au0 square. So, the power delivered to the wall is the rate at which the kinetic energy of the water is deposited at the wall.

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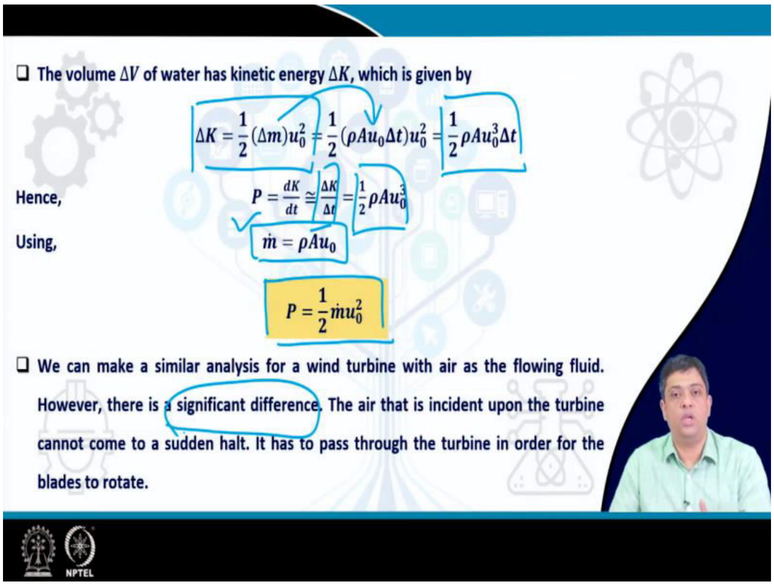

So, let us see what happens, if you consider delta V as the volume of water, which has the kinetic energy delta k, then we know delta k is equal to half delta m u square, k is equal to half mv square, that is a relation we are using. So, what is the value of delta k it will be equal to half delta m u0 square. We know what is the value of delta m. So, we introduce it, so what do we get, then we get half rho Au0 delta t bracket multiplied by u0 square and the following relation becomes equals to half rho Au0 cube into delta t.

Now, P is equal to what dK by dt and therefore, we can write for very small elements, we or time which is taken is quite small or for small elements depending upon which consideration you are taking you can write dK by dt is approximately equal to del K by del t and the moment we write it, we get p is equal to half rho Au0 cube.

Using the relation m dot is equal to rho Au0 we get P is equal to half m dot u0 square. So, now, we have reached to a relation which can be used to understand the flow of wind and then how we can use the same relation in understanding the extent of power which will be transferred to the wind turbine plates. So, we can use a similar analysis to understand the wind turbine interacting with the air that is considered as the flowing fluid.

But, there will be a significant difference as I mentioned in the previous slide, the air that is incident upon the turbine cannot come to a sudden halt and it has to therefore, pass through the turbine in order for the blades to rotate. So, you cannot say that air will just stop, if you stop and there is a significant change in the flow directions, then what will happen, you can actually induce the conditions of turbulence and then what will happen is the overall scenario will change and your turbine may stop working because of the damage that a turbulence can lead to in the mechanical and the operation of the turbines.

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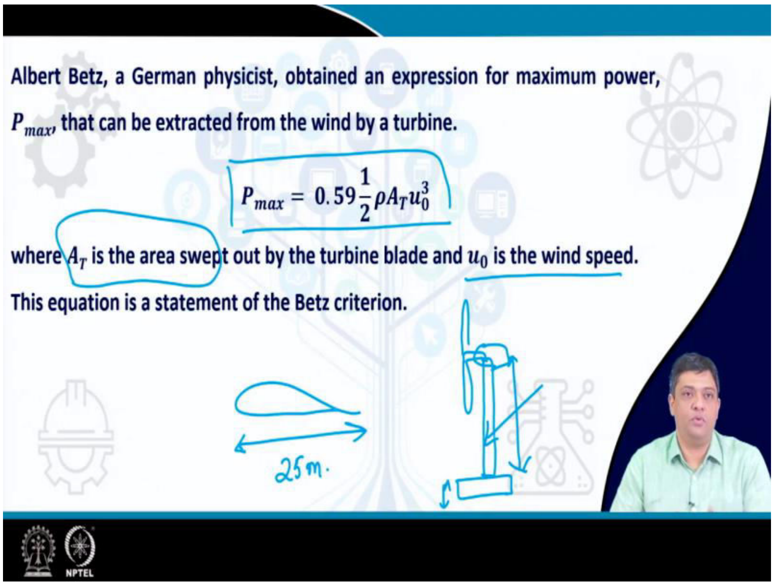

And to what I just said may not have become very clear because those were just words. So, let us try to mathematically explain the same thing and then the two explanations will come together and I hope things will become very clear. It was Albert Betz, who was a German physicist who looked into the various factors and the mathematical formulations and came up with an expression for maximum power that can be extracted from the wind by a turbine which was being placed in the path of the flowing wind.

And he showed that the maximum power that could be extracted was p max, which was equal to 0.59 into half rho ATu0 cube, where AT is the area swept out by the turbine blades and u0 is the wind speed. You will immediately see two things the maximum power is clearly dependent on the area being swept out by the blade and the wind speed.

So, if the speeds are very high this power which, value which you are extracting will change and also if the size of the turbines are very large or they are made smaller than the maximum power you can extract will be different, please understand that there is a limit in the size of the turbine blades which can be fabricated in use as of today.

Because there are mechanical issues which come into picture that limits the size of the wind turbine blades that can be operated. You may just for an example let us take, so, I want to install a wind turbine and then somebody tells me okay please use the wind turbine blade which is 25 meters. And then if I install a turbine blade which is 25 meters then I, if the, what will be the height of the tower where you will be installing this blade? It has to be at least 25 meters plus additional 5 to 10 meters.

Why? Because you have to make a base you have to ensure that when the turbine is actually, blade is actually rotating it does not hit anything which is beneath it otherwise a person is walking it will be hit by the rotating blades. So, additional 2 meters, the base will be 2 meters or more than you have additional necessarily or other factors which you will see in later slides.

So, the height, if you increase indefinitely the size of these blades, the height of the towers will also increase. If the height of the towers increased the mechanical and the civil engineering considerations will come into picture. So, these are all related factors which as of today clearly limits the size of the wind plate and subsequently the area that can be swept out by these wind blades.

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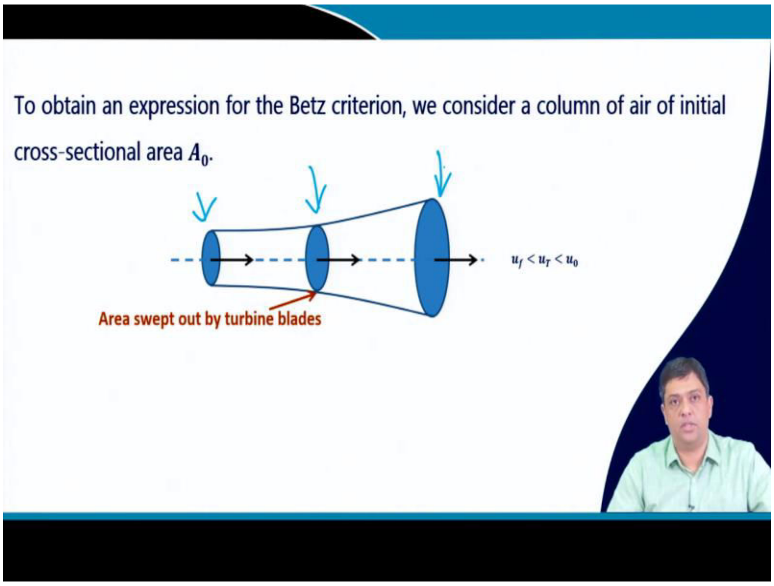

So, to obtain the expression for Betz criterion, let us consider a column of air which is entering the initial area A0, this is the area being swept by the turbine blades and then they are moving out of the turbine blades or exiting from the turbine blades.

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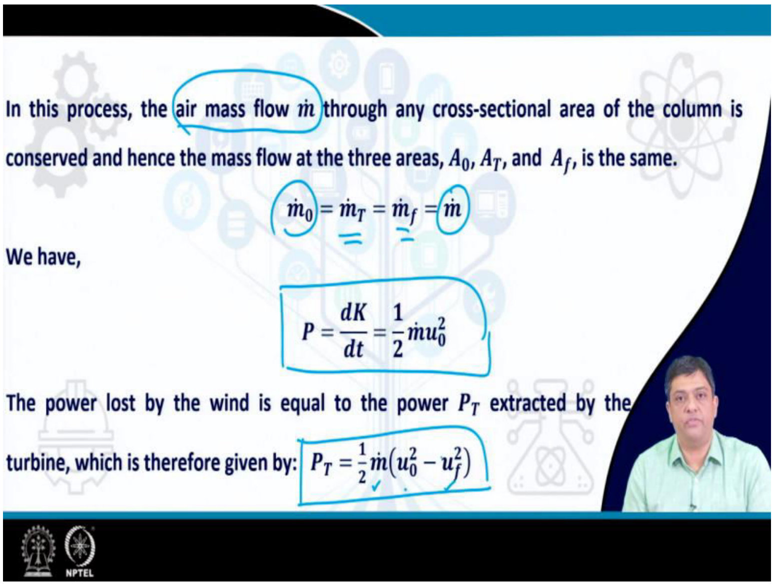

Similarly, to what we have discussed earlier we have the air mass flow value which will be flowing through A0, AT and the final cross sectional area Af and we will be writing using the continuity equations that m0 dot is equal to m dot and they are also equal to mT dot and mf dot. So, if we know this and we have seen what is the power being transferred to an obstacle we have power is dK by dt that is equal to half m dot u0 square.

What will be the power lost by the wind to the turbines? This will be the power equal to the value that will be extracted by the turbine. And therefore, we have PT equal to half m dot u0 the velocity at which the wind is hitting the turbine and the velocity uf at which the wind is exiting the turbine blades, the difference of half m dot u0 square and half m dot uf square will give you the power PT that can be extracted by the turbine. And that has been written as PT is equal to half m dot bracket u0 square minus uf square.

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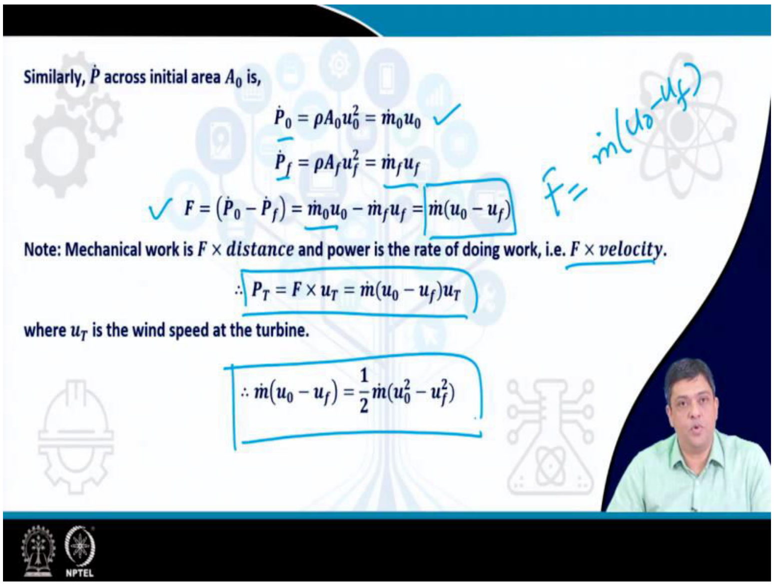

Similarly, we can write P dot across the initial cross sectional area A0 and you can get P0 dot is equal to m 0 dot u0 and Pf dot is equal to m f dot uf. What is the force? Remember the fluid mechanics, we will be talking about Newton's second law. So, and you will be talking about the conservation of various laws. So, what do we get we get forced equals to m 0 dot u0 minus m f dot uf.

And that can be written as, so force is equal to m dot u0 minus uf. This is what we have obtained, please note the mechanical work is given by what? It is force into the distance and the power is defined as the rate of doing work that is equal to force into velocity. And if we know this value, we have, we can write the relation of PT that will be equal to m dot u0 minus uf into uT, where uT is the wind speed at the turbine and we come to a relation which is equal to m dot u0 minus uf equals to half m dot u0 square minus uF square.

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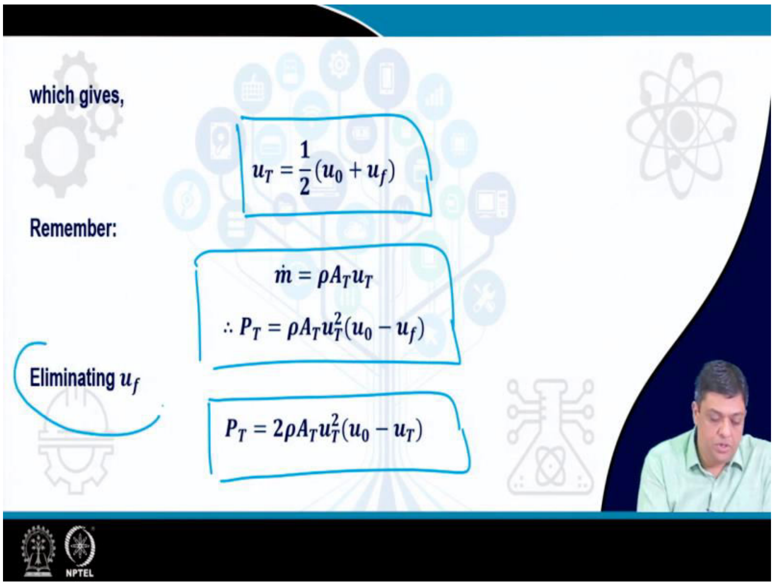

This gives us the relation that uT is equal to half of u0 plus uf. So, it is basically the wind speed at the area being swept out and by the turbine blade or element in the turbine blade is basically an average of the speed which enters the turbine and the speed which is exiting the turbine blades. And you should again remember that these two relations have been derived earlier that m dot is equal to, so mass flow is equal to rho AT uT.

And eliminating uf from the above equations, what do we get we get the value of PT that is the power being extracted using a turbine that comes out to be 2 rho AT uT square into u0 minus uT. So, these are by obtained by eliminating the value of uf.

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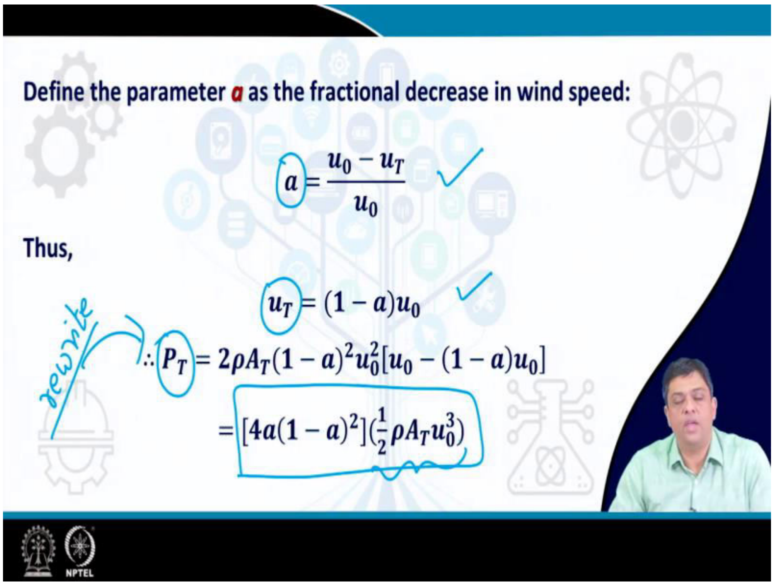

Let us define a coordinate a which is given as u0 minus uT divided by u0. Therefore, we can write that uT is equal to 1 minus a into u0. If you have these two parameters, then you can rewrite the value of PT and you will reach to a relation which is 4a into 1 minus a square multiplied by half rho AT u0 cube.

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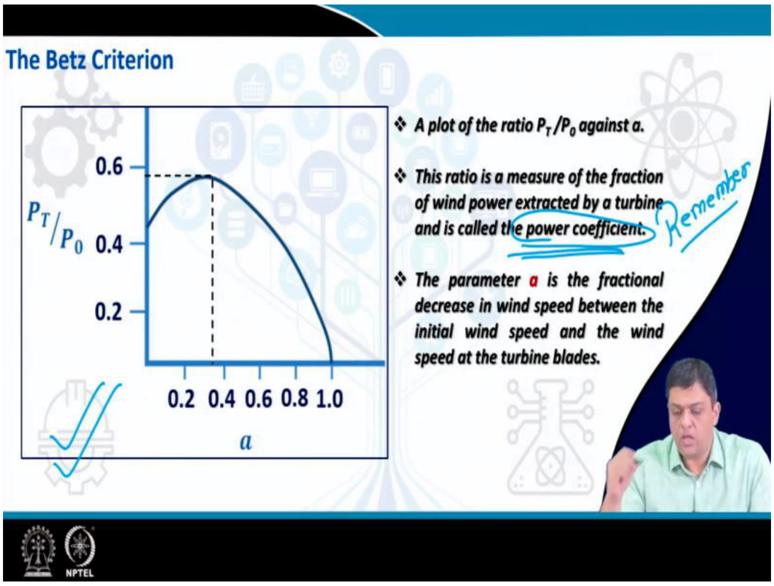

And if you plot the real values of PT by P0 as a function of this parameter a, what is a? uT minus u0 upon u0, then what do you get you get a curve which is shown in this graph. The parameter is a is basically defining the fractional decrease in the wind speed between the initial wind speed that is entering the turbine blade and the wind speed that is exiting the turbine. So, what is the fractional decrease?

And the ratio PT by P0 is the measure of the fraction of the wind power extracted by the turbine and is called the power coefficient, please remember, this is what is called as power coefficient.

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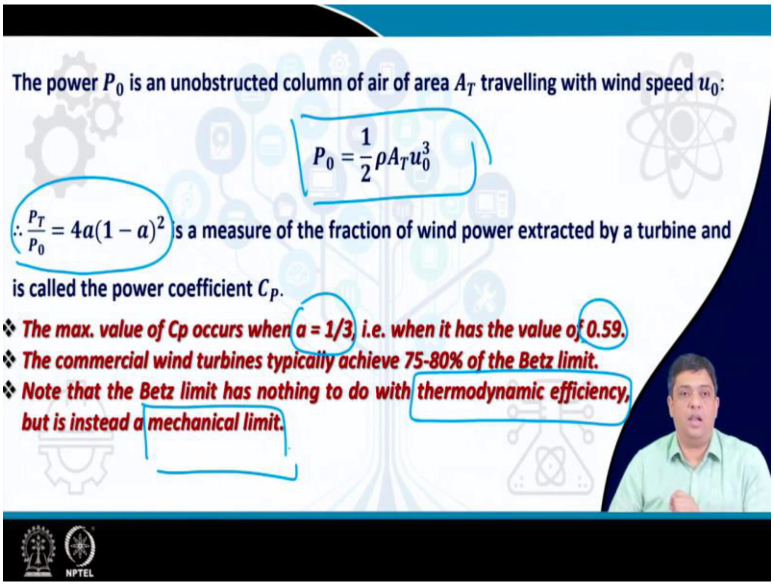

And if there is no obstacle in the path of the wind, then the power P0 is an (obstacle), for a column of air which is flowing without any obstacles in its path is simply given by P0 that is equal to half rho AT u0 cube. We have seen that PT by P0 is equal to 4a 1 minus a whole square and this is the measure of the fraction of wind power extracted by the turbine. What is the maximum power of the power coefficient that you get? a is the value that is obtained at the condition that a is equal to 1 by 3.

So, the power coefficient Cp a gives the value highest when a is equal to 1 by 3 and this happens for a value of 0.59 and that is where the Betz criteria was actually obtained. And you will see that the commercial wind turbines typically achieve not the full capacity to extract power from the wind, they actually have a capacity to extract power which is 75 percent to 80 percent of the Betz limit.

So, Betz limit gives what is the maximum power which you can extract and out of that power you have the turbine blades, which can extract 75 to maximum 80 percent of the value. Please note as I had also indicated earlier that Betz limit has nothing to do with thermodynamic efficiency, it is basically coming in because of the mechanical limit. So, you have other factors which are limiting the final value of power that can be extracted.

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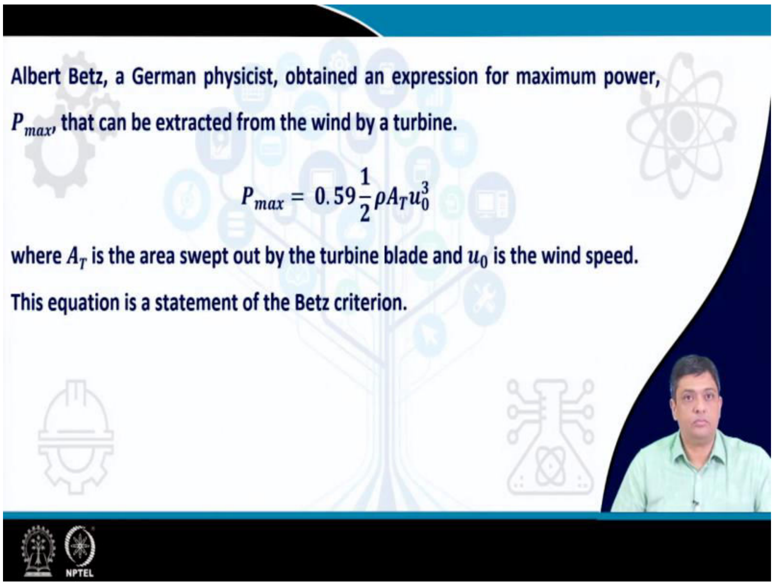

So, with the discussion we had till now, I hope it is clear to you that what is Betz criteria and how do we obtain the relation P max is equal to 0.59 half rho AT u0 cube. So, the origin of this term or the factor 0.59 should be clear to you by now and remember this equation is the statement of Betz criteria.

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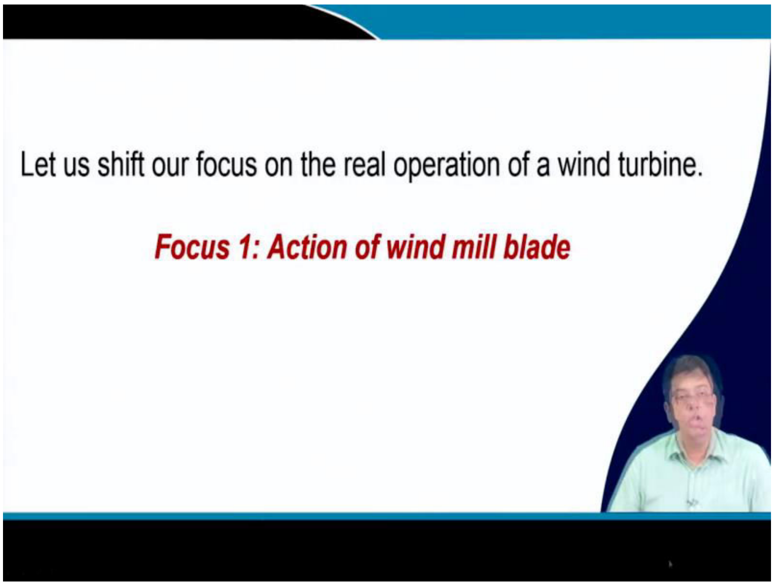

So, after understanding Betz criteria, let us now shift our focus and give you a brief introduction as to what happens in the real operation of a wind turbine. And then in subsequent lectures, you will find that I will be giving a detailed description on the operation of the wind turbine and how these concepts of continuity equation, Bernoulli's equation and Betz criteria is used to define the maximum power that can be extracted using a wind turbine.

So, let us now focus on the way this wind interacts with the wind turbine blade or the windmill blades.

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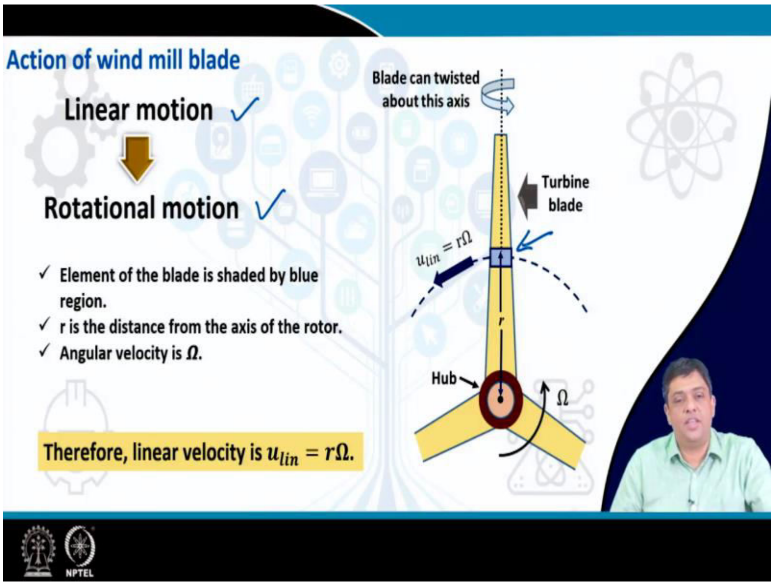

So, if you consider the blade, there are two motions which you will see one will be linear motion and the rotational motion. So, you will have rotational motion and also the linear motion of the blades along the axis. So, if you see that the element of the blade, which is shaded in blue and that element is, we are considering that element that is rotating and we want to see what action does this element actually does.

And we will try to then correlate it with our earlier equations and see, can we extract the value of power which can be extracted using this element and then if we can integrate over the whole area or the length or the area of the blade can we get the maximum power value, r is this distance of this element from the axis of the rotor and the angular velocity of this element is given by omega. Therefore, what is the linear velocity? It is simply r omega.

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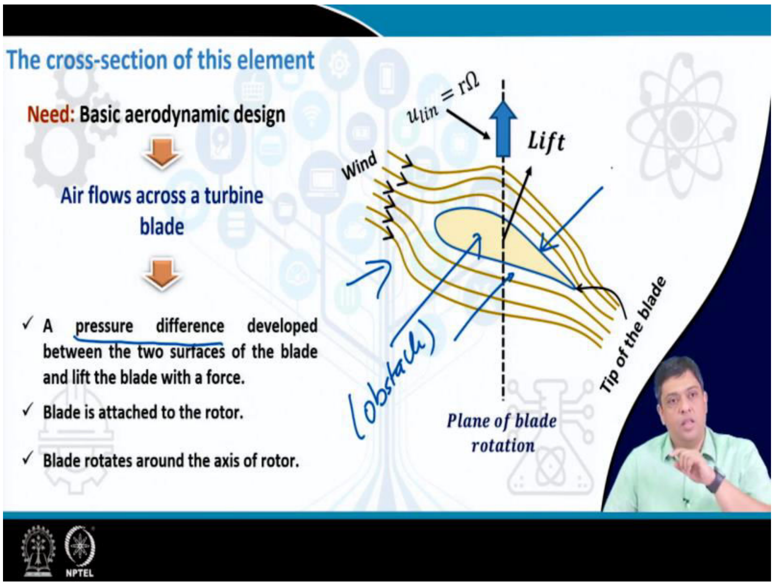

Now, if you see the design of the blade is such that when the wind which we are considering as a streamline flow, so, the wind is actually hitting this what you can call it an obstacle and in this case, it is basically the turbine blade. Now, you see what had been discussed when you have a streamline flow, then when it hits an obstacle above the obstacle the pressure increases and then when you go down then the again the layers become separated out and then they again flow out as streamline flow.

But, there is a pressure difference, which is developed between the two sides of the surface of the blade, because you can see that the bottom surface and the top surface are slightly different and therefore, the way this wind will be flowing through the turbine blade will be slightly different.

This is also termed sometimes as the aerodynamics of the turbine blade, how you design the way the wind will be flowing on the, through the turbine blade that is above the turbine blade and the bottom side of the turbine blade. And this pressure difference, remember Bernoulli's equation that leads to the lift of the blade, which is developed because of this pressure difference. The blade is attached to the rotor and the blade rotates around the axis of the rotor.

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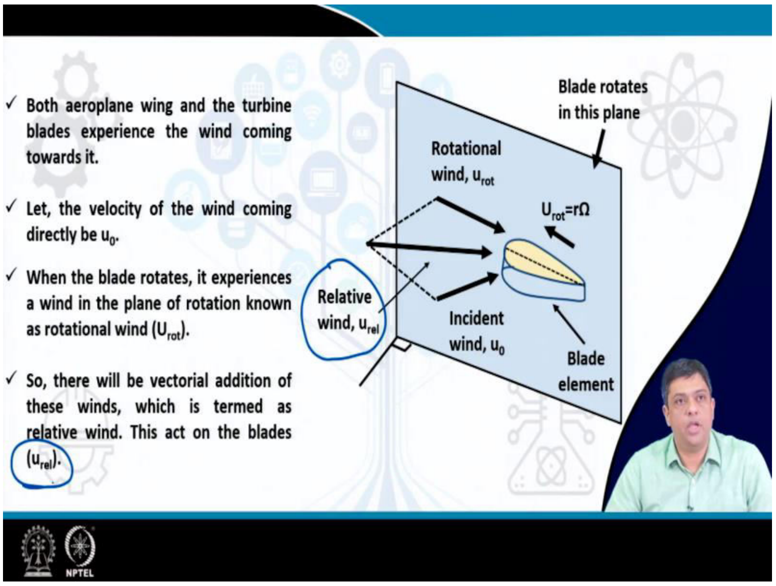

So, you can talk about the same thing in another way and this is what you can write that the relative wind is u relative which is simply obtained by vectorial addition of the rotational speed and the incident speed and this is called as the relative wind. And that is what will become important for our discussions.

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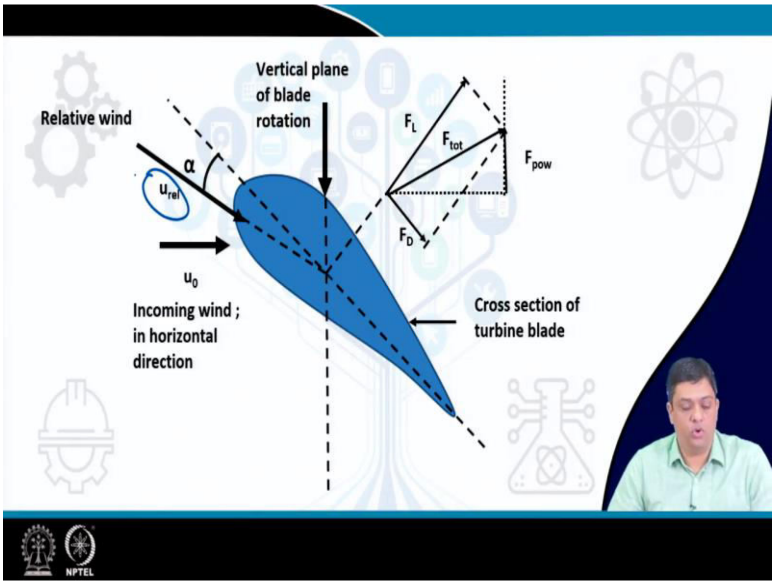

So, the whole thing can be looked once again why am I am repeating this slide, you should be very clear what is happening now, you have a blade that is rotating and you have two things which can occur; wind speed and the rotation of the plate. So, effectively what is the wind which is leading to the generation of force that is driving the whole process of the motion of the plate. So, you will see that you have a vectorial addition of u0 and the blade speed and you get the value u relative.

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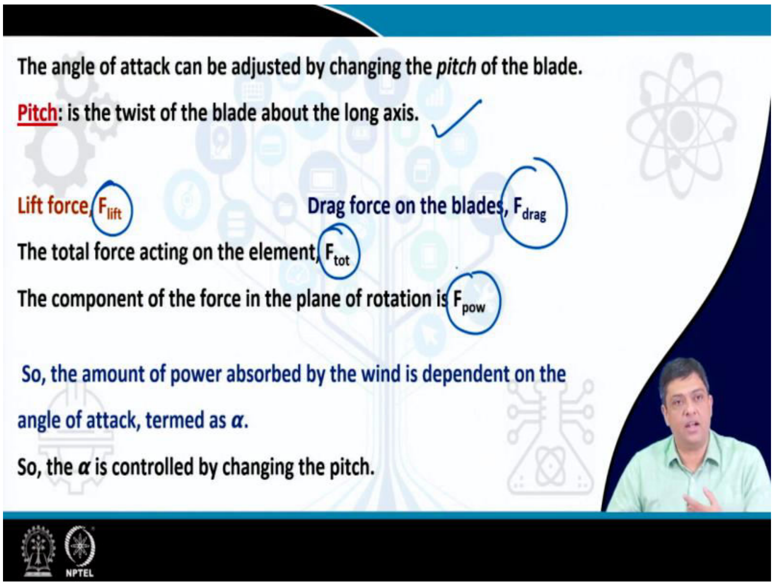

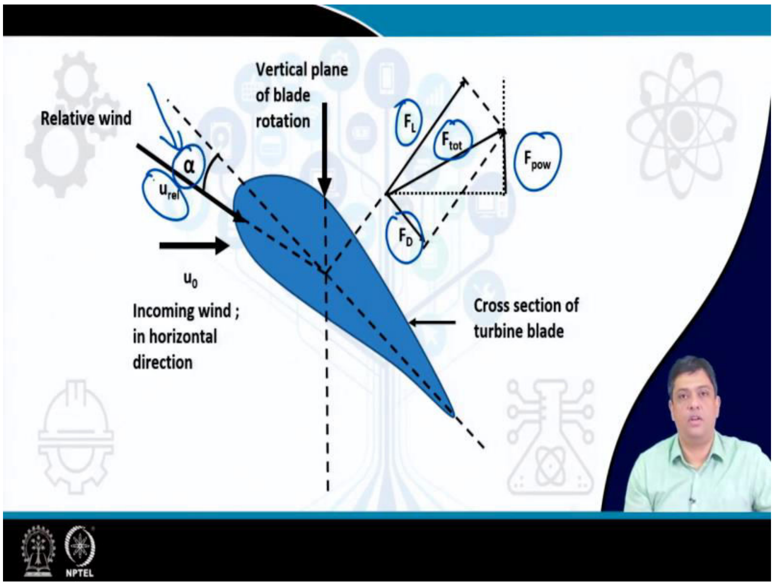

The angle of attack can be adjusted by changing the pitch of the plate. And what is pitch it is the twist of the blade about the axis and there are two more terms which you should understand; that is one is the lift force and the other is the drag force on the plates. And the total force is basically written as F total.

And the component of the force in the plane of rotation is given as F power. So, the total amount of power absorbed by the wind is dependent on the angle of attack and is termed as alpha. Now, these things will become very clear to you, what how do we obtain F total? What is the value of F power, if you write FL and FD and what is this alpha relating to?

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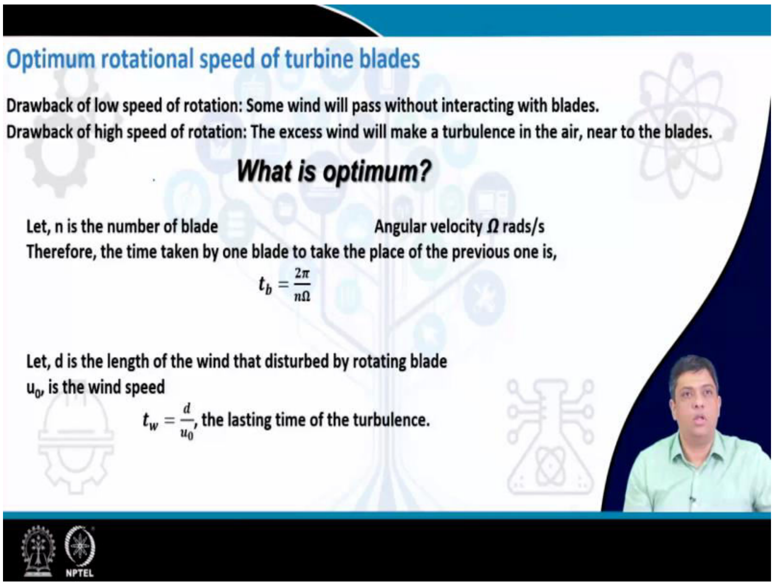

So, obviously, as we discussed earlier, you cannot just go on increasing the wind speed and the rotation speed of the blade, because there are other factors which will come into picture. So, what is the optimum rotational speed of turbine blades. If you have very low speeds, then some wind will pass without interacting with the blades. So, you will not have the transfer of power.

If it is very highs wind speed you are talking about, then you will lead to conditions of turbulence and then the whole system can lead to a mechanical failure and then you can lead to the damage of the turbines. So, what is optimum, what is the optimum value which you should consider while you are allowing the wind turbine to operate?

So, let us consider certain assumptions and then try to reach to a condition. Let n be the number of blades which are there in a turbine, the angular velocity is omega radians per second. So, what is the time taken by 1 blade to have the rotation tb is equals to 2 pi by n omega. Let d be the length of the wind that is distributed by the rotating blade u0, rotating blades. Okay, can we repeat this slide, there are two things and just repeat this, the whole slide.

So, as we discussed earlier, you cannot have the speeds which are very high, which speeds are we talking, we are talking about wind speeds, which are very high because that will lead to the condition of turbulence and that can cause mechanical failures. Then, if somebody says please consider the wind speeds which are very low, then it has its own limitation, in that condition you will have certain fraction of wind which will pass through with the turbine blades without interacting with it.

So, what is the optimum value? The optimum value can be obtained by considering certain initial conditions and assumptions and you will see what is the value that is giving us the optimum value. Let n be the number of blades, the angular velocity is omega radians per second, the time taken by 1 blade as discussed in the previous section, the time taken by 1 blade to complete the rotation is two pi by n omega.

Let d be the length of the wind that is distributed by the rotating blade and u0 as the wind speed. Then tw will be equal to d by u0 where tw is that lasting time of turbulence. So, turbulence is the time which is actually leading to turbulence and what is the magnitude. So, if the speeds and distances are different, you will have the lasting time of turbulence which will be different.

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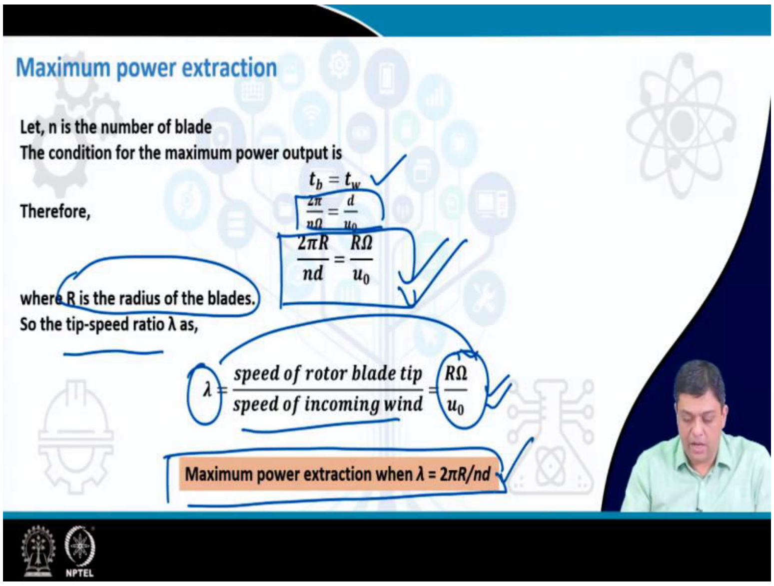

So, what is the maximum power that can be extracted? So, let there be n number of plates, we have seen tb is what, that is the time taken for the first blade to move to a place which was being occupied by the previous plate. So, the time taken for the blade to move from condition, place 1 to place 2, where place 2 was being occupied by the blade number 2. So, for maximum power transfer, we should have tb that equals to tw.

What is tw, the time duration up to which the turbulence is lasting. Therefore, we get a condition 2 pi by n omega is equal to d by u0 and if you know the radius of the blade, which is given by R, you can reach to a condition for maximum power that is equal to 2 pi R by n d is equal to R omega by u0. So, we have reached to a condition of maximum power extraction.

And if you define lambda as the tip speed ratio that is speed of the rotor blade tip and divided by speed of the incoming wind, then lambda is equal to R omega by u0, then you can write that the maximum power extraction will be taking place when lambda is equal to 2 pi R by n d.

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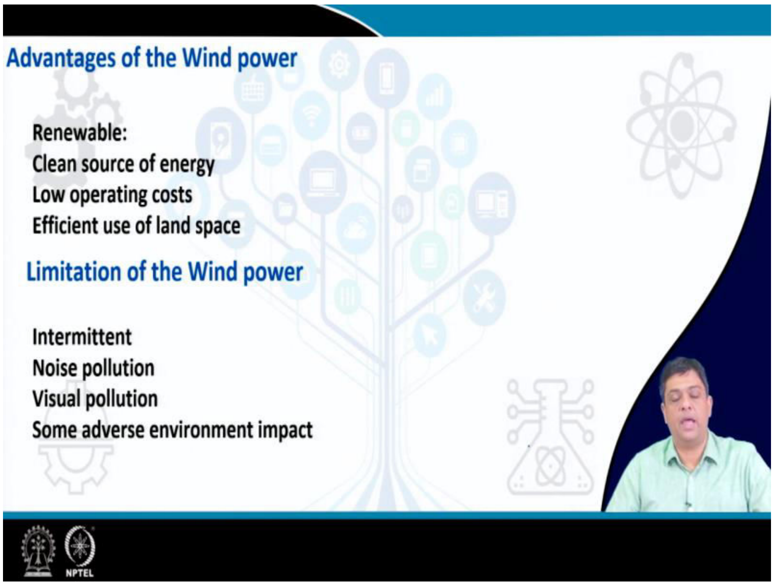

So, we have obtained the condition which should be satisfied or which should lead to the optimum extraction of power from wind. As I said, each technology has its own advantage and disadvantages. You will see in subsequent lectures also that the advantages associated with wind power is it is a typical renewable system, it is a clean source and once installed, it has low operating costs and it also allows efficient use of land space.

But, as expected or it is known that renewables will be associated mostly with a term that is intermittent nature, wind is no different and wind power is also affected because of the intermittent or an additional term variable nature of the source. While operating the turbines you can lead to noise pollution and also visual pollution that we had discussed in the previous class.

We have also seen that while operating the turbine blades, you can have conditions which can lead to adverse environmental impacts, because the, these blades can injure birds or they can lead to certain health hazards for insects.

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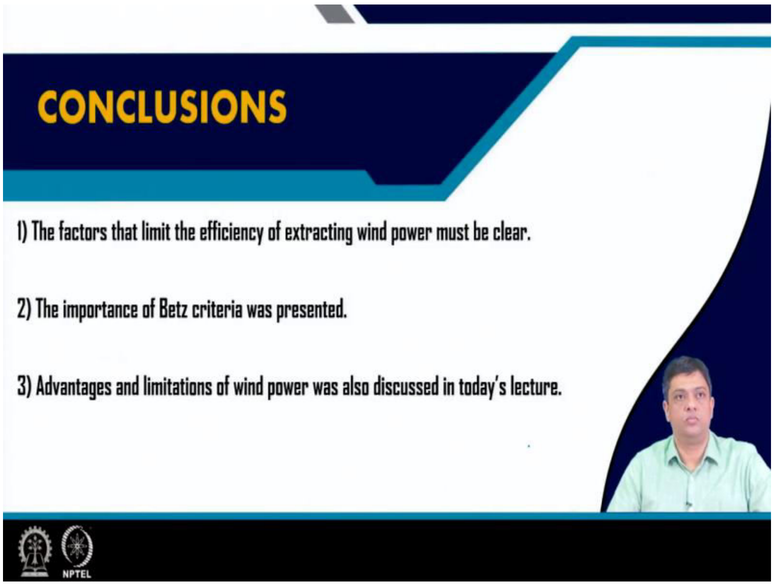

So, let us conclude today's lecture and I hope that it must be clear that there are factors that limit the efficiency of extracting wind power and what is actually the maximum power that can be extracted from the wind is given by Betz criteria and also that the wind turbine or the wind based systems which are used, they actually operate at 75 percent to 80 percent of the Betz criteria. And there are advantages and disadvantages associated with wind.

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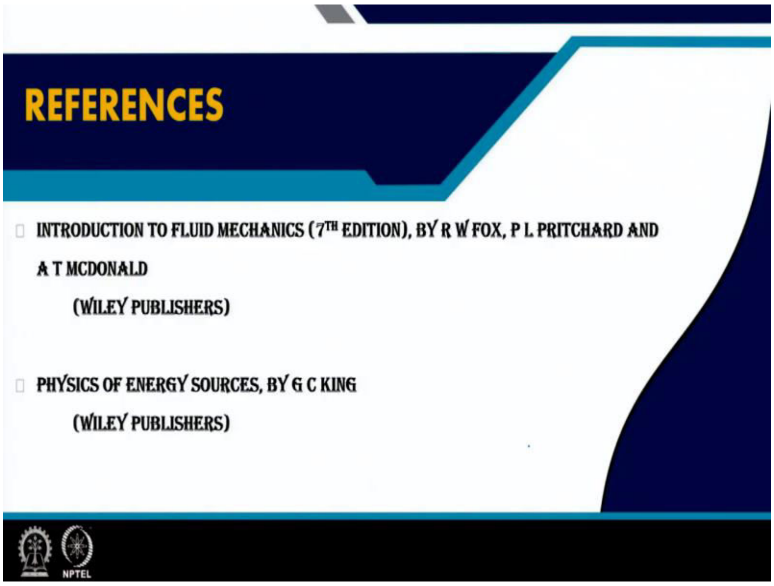

These are the two books which we have used in today's lecture. And I thank you for attending this lecture. From the next class we will build upon this knowledge and we will start talking about the real wind turbine construction and how it operates. I thank you once again for attending today's lecture.