Physics of Renewable Energy Systems
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Lecture 11
Continuity Equation and its applications

Hello, so, let us start our discussion on the next topic of Module 3 and that is the continuity equation and its applications. In the earlier lecture we discussed on the origin of wind, we discussed the basics of fluid mechanics which are going to be used in the wind energy extraction, and we also discussed on the concept of laminar and turbulent flow, although we discussed it very briefly. And in today's lecture, we will build upon those basic topics which were introduced to you in the earlier lecture.

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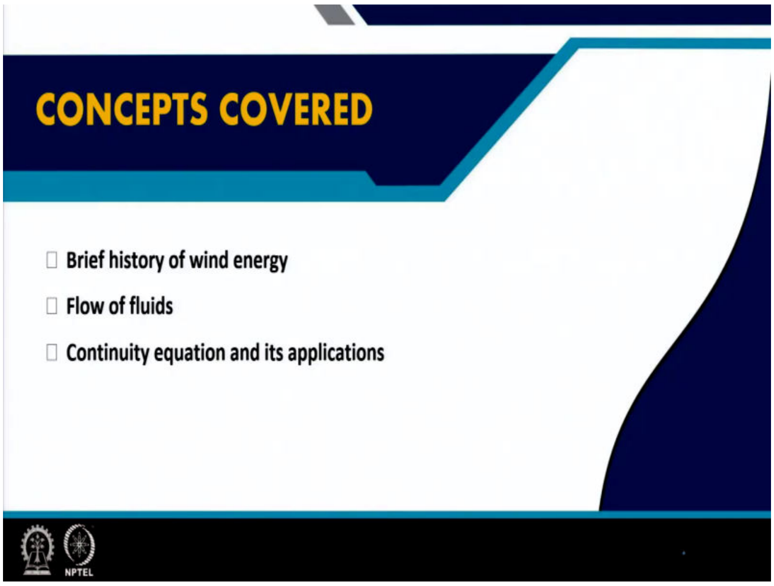

So, let us start with the topics that would be covered in today's lecture, I will give you a brief history of wind energy. Why I would like to talk about history will become clear to you in few minutes. We will also talk about the flow of fluids and how to model it. And we will try to reach to the point where we are able to write the continuity equation for the flow of ideal fluids and how this equation can be used in deriving certain other mathematical formulations, which would be extremely useful or utilized in wind energy.

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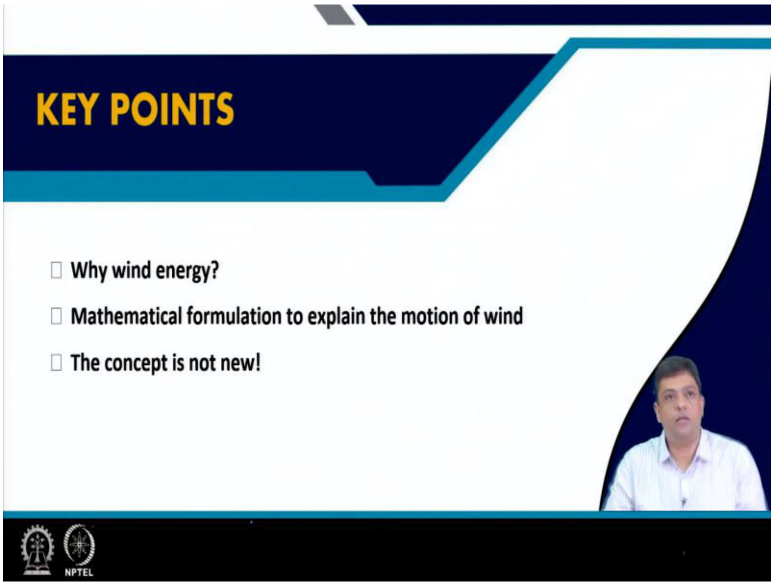

After hearing today's lecture, I hope it will become clear to you that why wind energy is useful to us, what is the mathematical formulation used to explain the motion of wind and how that mathematical formulation is actually derived. And one should be very clear and I hope that you will take back this key point that the concept is not new.

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So, let me start today's lecture by discussing the brief history on the use of wind energy by humans. And I start with slightly negative point for a few of you, and that is that it is not a new concept, but we should understand that technology takes a long time to mature before it is at a level that it becomes readily available to everybody.

And we should not believe that every technology we talk about is a new technology. We are contributing in the growth of a technology or a system or any kind of phenomena, which was being understood by our predecessors and we will contribute in its knowledge and then only it will reach to a maturity stage.

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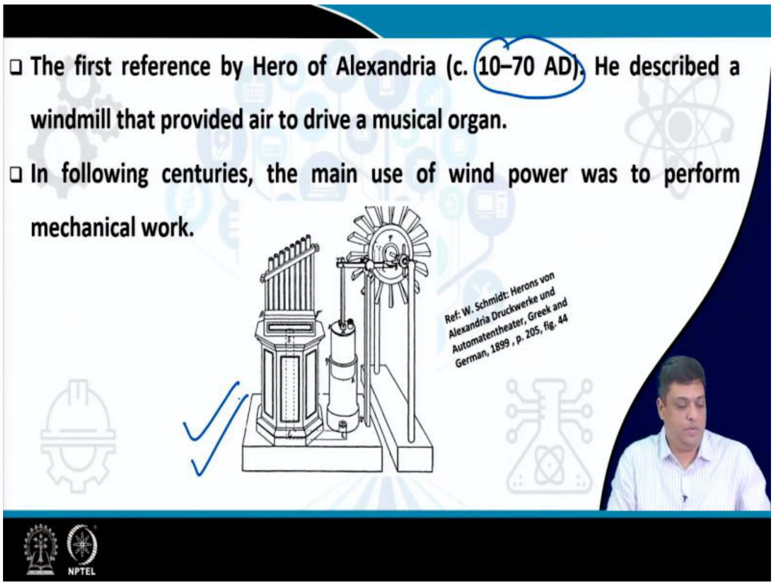

And the first reference of using wind as a source for extracting energy or power goes back to 10 to 70 AD where Hero of Alexandria was found or it has been documented that he used a windmill to provide air which in turn was driving a musical organ. So use of wind to drive a system is not new. And what happened that after these initial application of using wind for designing musical organs, it was significantly in later stages that wind power was being used to perform mechanical work. But initially the use of windmill to drive musical organs are routinely mentioned in the history.

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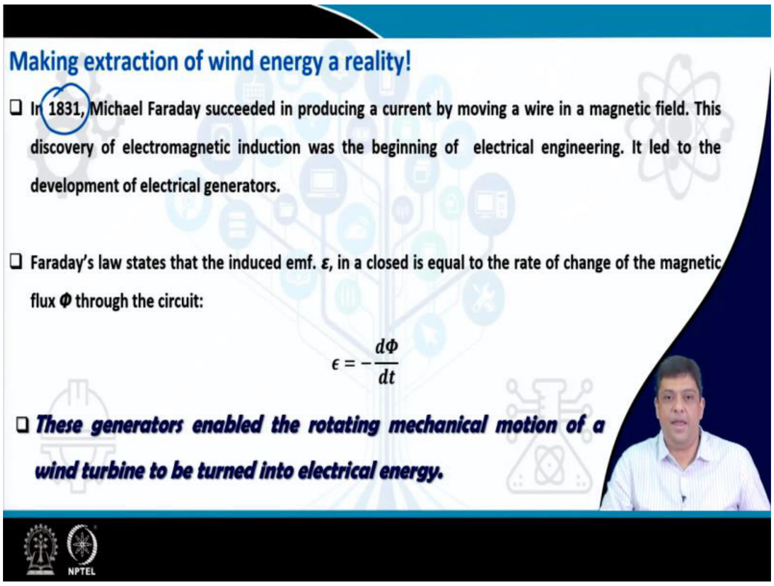

The use of wind energy actually became more prevalent after the discovery of Faraday effect, where the discovery of electromagnetic induction led to the birth of the field of electrical engineering and it led to the development of electrical generators. So, what happened, what was the use of Faraday's law in wind turbine?

It ensure that the generators which were using the rotating mechanical motion was able to convert this energy into electrical energy. And therefore, in 1831, the impact of Faraday's discovery was felt in many other fields than what we actually believe routinely. And one of them was the wind energy.

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But, most probably the significant development came from scientists and engineers at that time working in Denmark and Paul la Cour, in around late 1800s or early 1900s, who was a Danish scientist, he was working extensively on wind power and extracting energy from wind. And he built the first wind tunnels for the purpose of aerodynamic test to determine which was the best shape of the turbine blades. And that was the time where use of wind turbines or windmills became more prevalent in Europe.

And if you have seen many movies in India, many of them have actually been shot in Holland, with windmills in the background. And Holland is a country which is famous for its windmills. And what was this country doing initially? This country was extensively using windmills to pump water out of the lowlands and back into reverse, so that this dry land could be used for farming.

So, because in low lying regions, you have water moves into the land, you have to take that water out of that land and throw it back into the water body so that their land becomes right and you can farm that land and you can have vegetation. Similarly, the concept was used in America, take out water, that is underground water, and then use that water to irrigate the fields.

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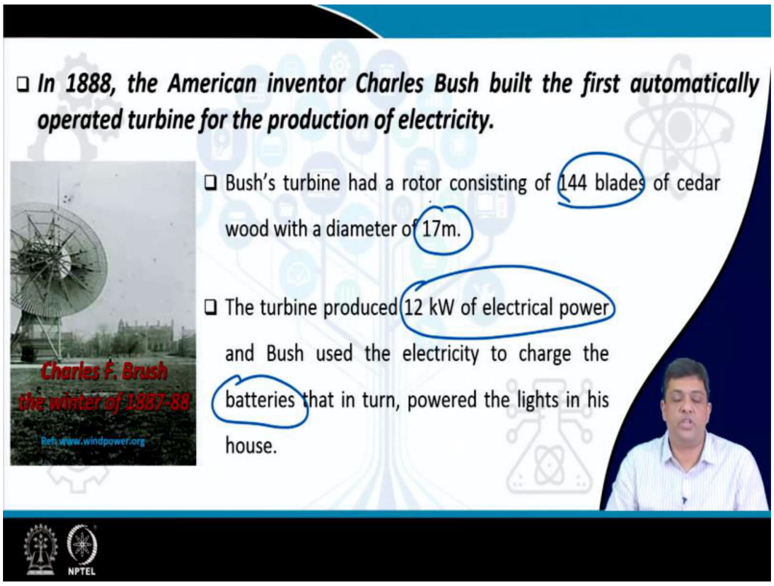

So in 1888, the American inventor, Charles Bush, most probably built the first automatically operated turbine, which was coupled with batteries. That means, way back in 1888, the discoverers, the engineers, the scientists already knew that the use of renewable based energy generation unit will have storage as an integral component, although they did not specifically mention it, but now we clearly understand that renewable based systems will have storage technologies as an integral component.

So, what was this turbine doing? This turbine was generating 12 kilowatts of electrical power using 144 blades with a diameter of 17 meter each. And so, during the time when the wind speeds were sufficient, the turbine was operated, the generated power was stored in batteries, and then Charles used the stored power to light his house or operate the electrical instruments which were present at that time.

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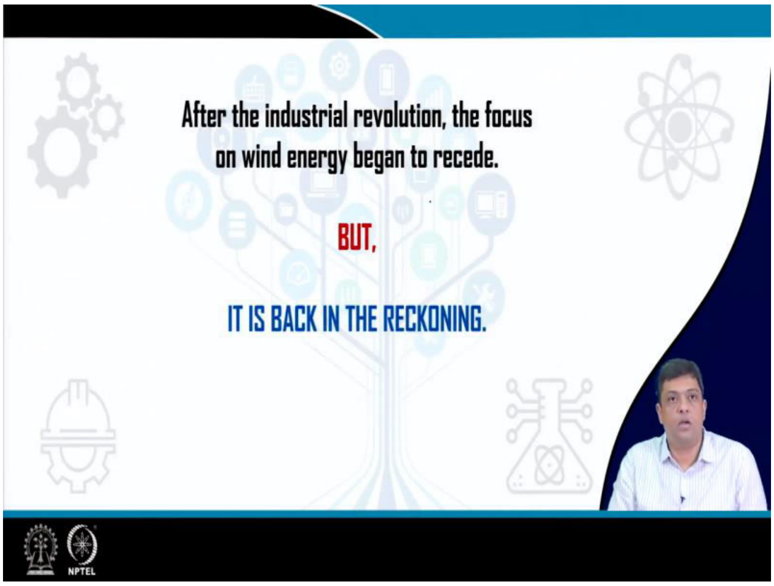

But, it was after the industrial revolution that the focus of wind energy began to recede and newer technologies came in the forefront and then the use of fossil fuel became quite prevalent and then wind energy went to background in most part of the world, there were countries which continuously used their technology, but rest of the world left interest or they had lowered interest in the field of wind energy. But now, wind energy is back in the reckoning as we move towards the renewable based energy landscape.

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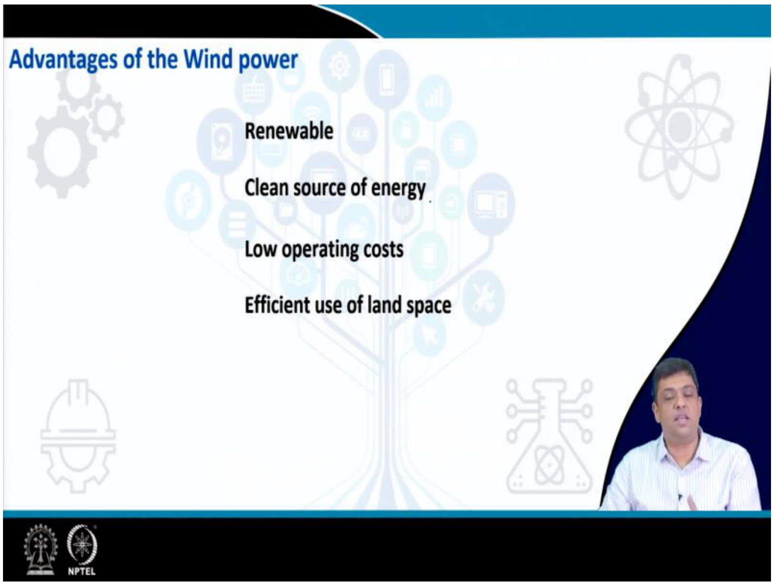

And why are they being used or why this whole concept is making a comeback. Obviously, it is a renewable source, it is a clean energy source. Once installed, it has a low operating cost I you can efficiently use the land space, that is a major advantage for wind based power stations.

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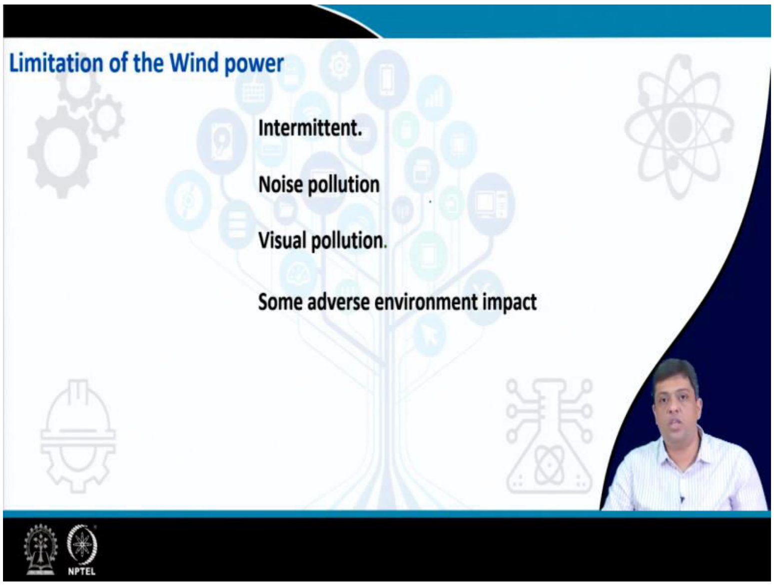

But, as expected, they also have some limitations, which you should understand right at the beginning, they are having a component of the term which goes along with it and that is intermittent, so, the source may be intermittent. The operation of wind turbines can lead to noise pollution and people who are living near that wind farms can actually feel the noise.

Also the landscape changes. So, if you have wind farm installed on a beautiful hill, then the whole visual impact of the hill changes and that can be termed as visual pollution. And because of this continuous operation of wind blades which can be very sharp, they can be very long you can lead to adverse in environmental impacts, which are mostly related with the deaths of birds or insects which are flying near the wind plates.

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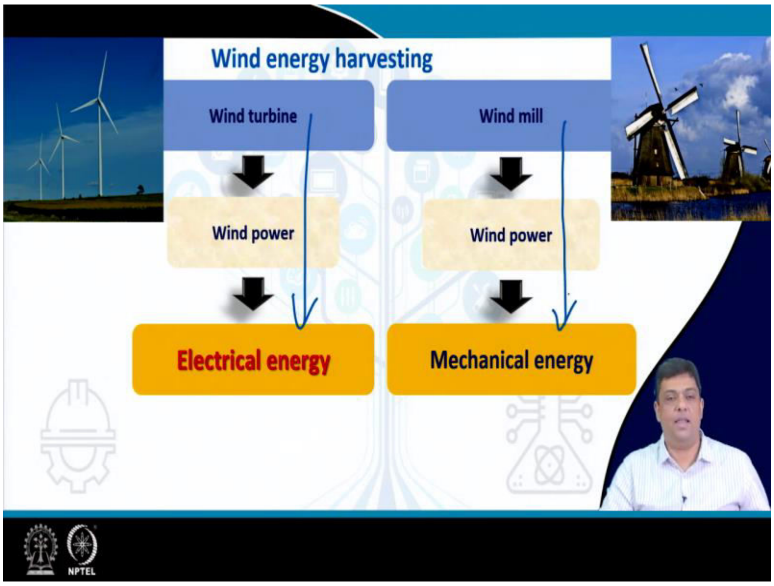

And 2 most common ways of extracting energy are the use of wind turbine or windmills. The basic difference is that by using wind turbines you get electrical energy directly while using windmills, you get mechanical energy.

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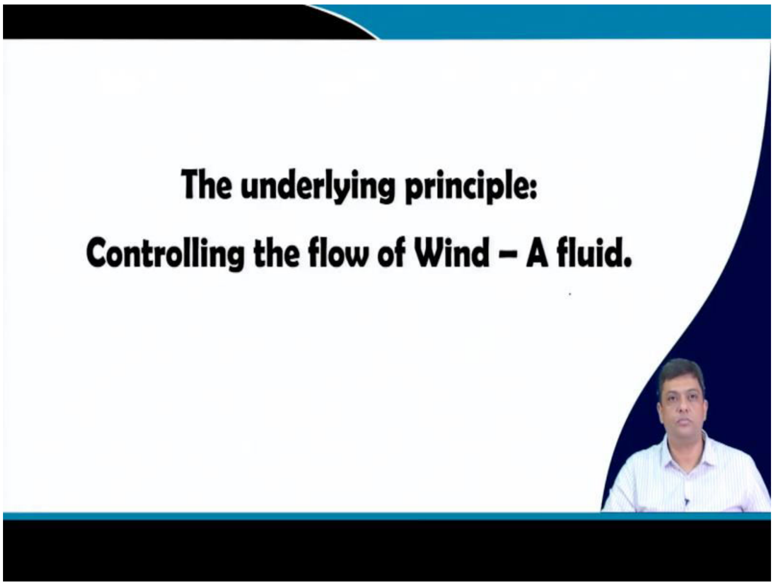

And in the operation of both of them, the underlying principle is the controlling the flow of wind and then extracting energy from there or power from there. So, we should understand how do we actually understand the flow of a fluid.

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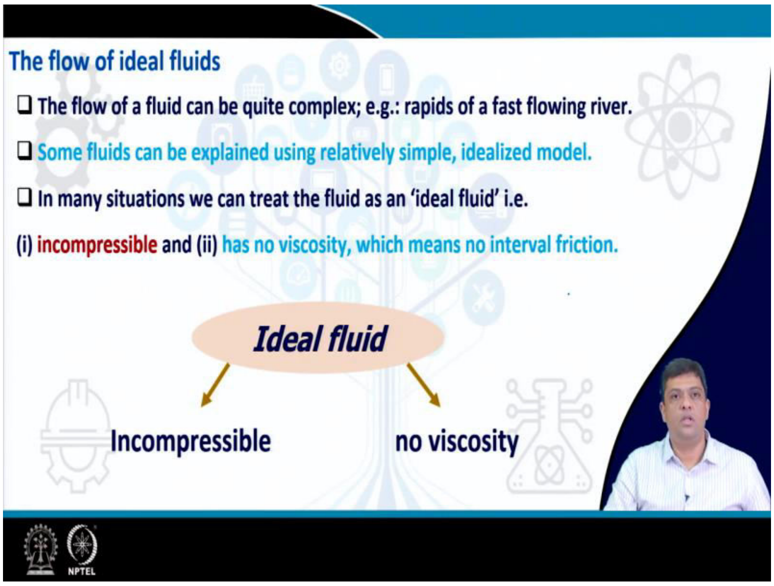

And we will consider, to begin with we will, this slide we will start again. To start we will consider an ideal fluid which are basically incompressible and they do not have viscosity that means no internal friction between the layers.

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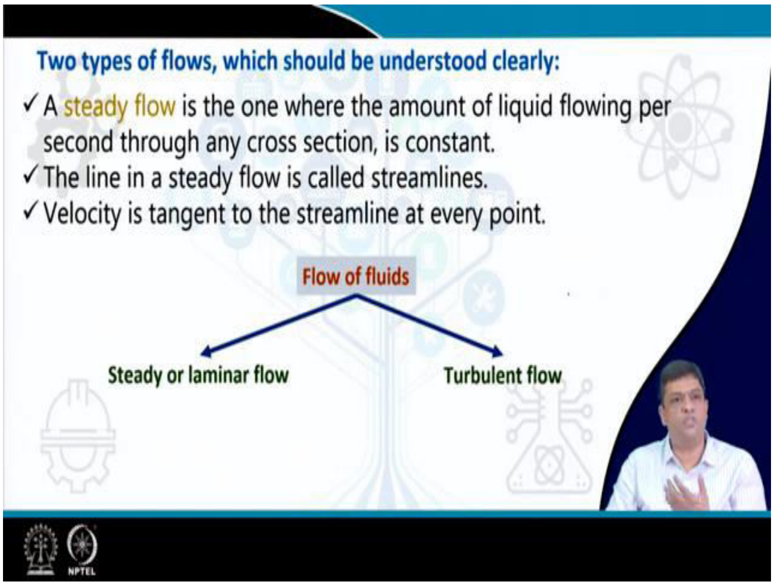

And these types of fluids will have two types of flows either steady or laminar flow or turbulent flow, we will consider to begin with the use of steady or laminar flow and the concept of steady and turbulent flow has already been discussed in earlier lectures.

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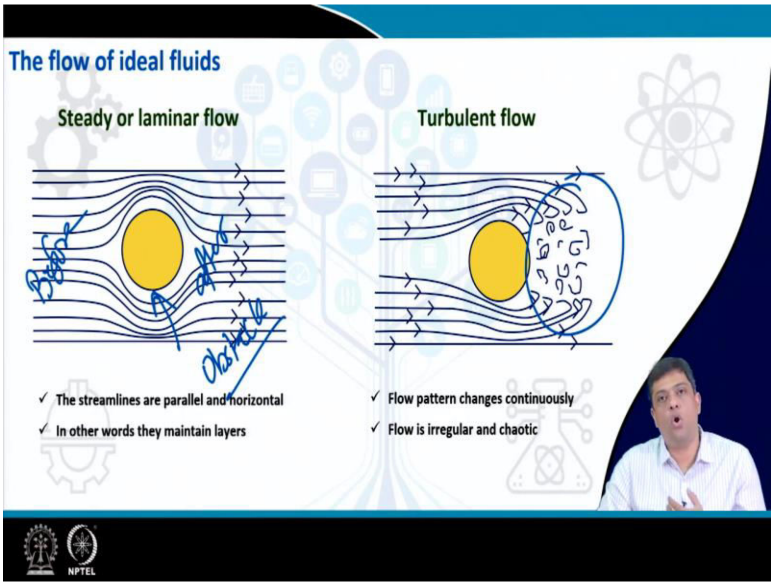

So, I will just put this slide and you will understand schematically, how does the nature of the flow change before and after the obstacle. So, if the flow is steady or laminar type, then before and after the appearance of obstacle in fluid flow, the nature does not change, but if it is turbulent, then the nature becomes irregular or chaotic, this is what you will be understanding by now.

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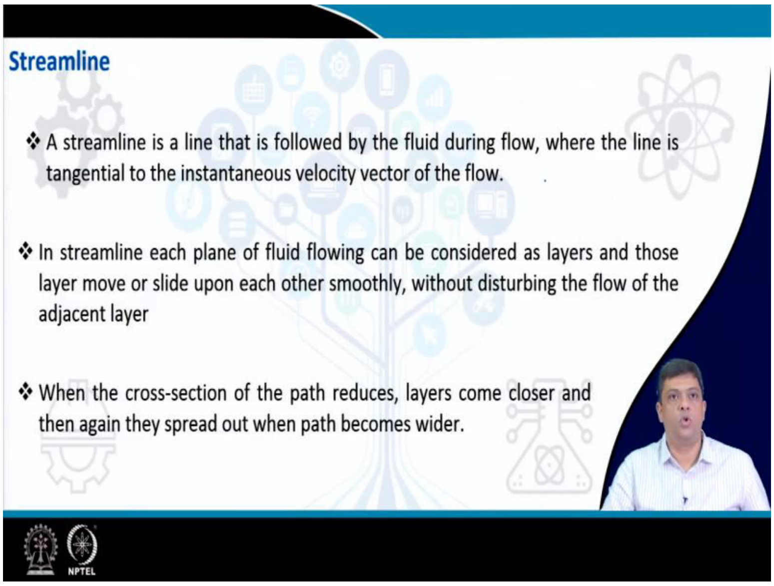

I the definition wise what is happening the point which should be very clear to you is that a streamline flow, what is happening if you consider the layers and these layer if they are sliding over each other, then they do so, very smoothly without disturbing the flow of the adjustment layer.

So, if there is a deformation, the deformation is smooth. Why are we talking about deformation, because the definition of the fluid in the previous lecture that a fluid is such that if there is a shear stress, it will lead to certain deformation. So, if there is anything which is sliding over each other, then it should not impact the layer below if we are talking about an ideal flow.

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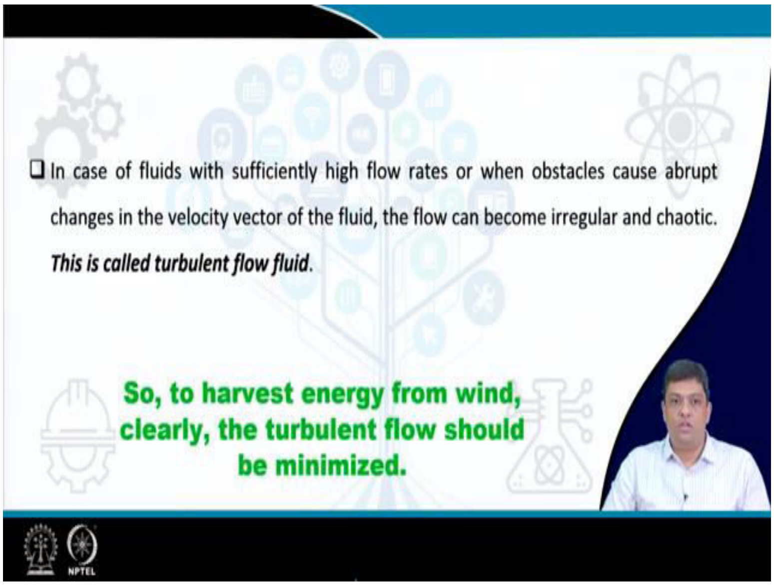

So, in the case of turbulent flow, we know that the flow becomes irregular or chaotic. Therefore, if we want to harvest energy from wind, the turbulent flow should be minimized otherwise, we will not be able to control the motion of the blades or the windmill or the wind turbine, because the flow of the wind will be unpredictable and therefore, the motion of the blade will not be predictable and that will have serious consequences that is why we say that turbulent flow should be minimized.

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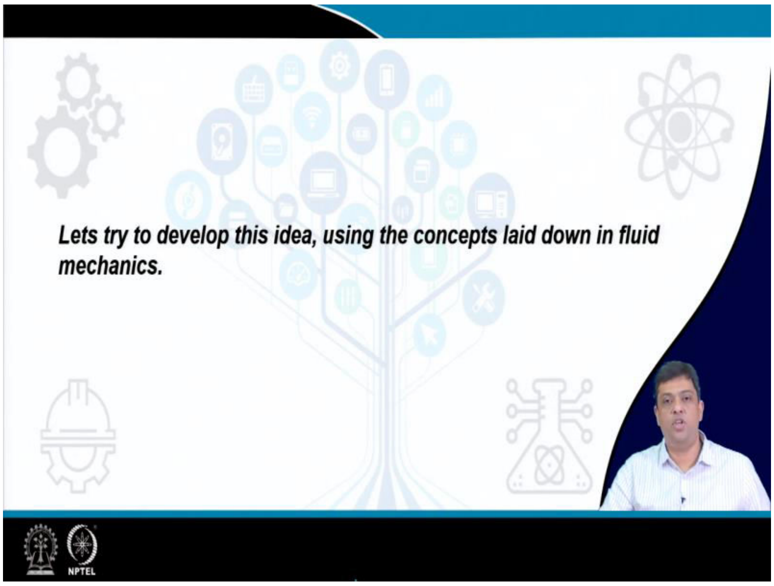

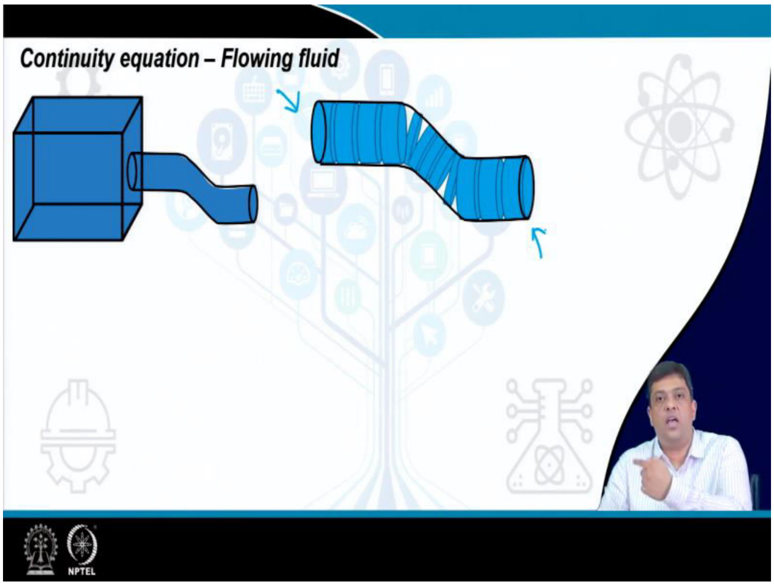

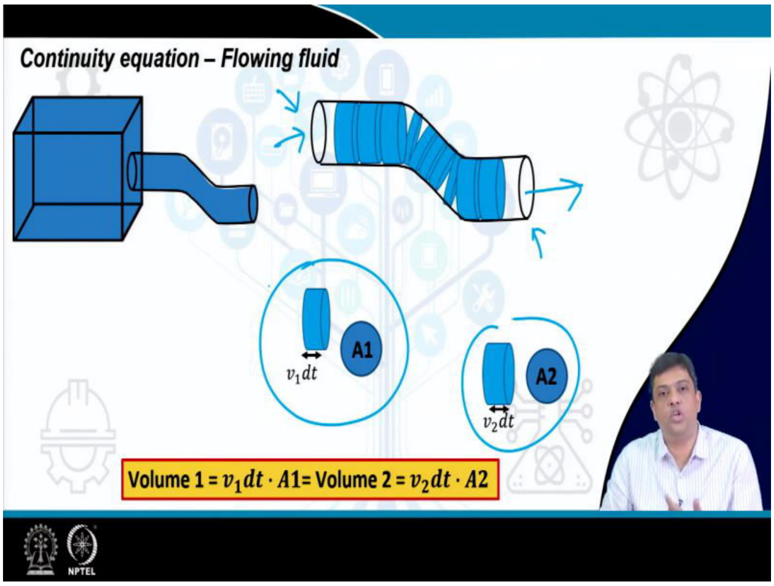

So, let us develop an idea using the concepts which we have laid down and see how that can be used to develop the wind plates. The first thing is the continuity equation what is continuity equation let us look it from the animation first and then I will write each of these steps one by one, so, that you understand this concept clearly.

So, let us consider a pipe which, through which water is or a fluid is flowing and for example, you can take a hose pipe which you have in your garden where with using which you water the plants. So, there is a pipe through which a fluid is flowing. So, we take that as an example. Now, there are two things you have an input and the output. So, what happens if the area is such that at the input you have A1 as the area and at the output you have A2 as the cross sectional area of the pipe through which this fluid is coming out.

So, what you will see if u1 and u2 are the velocities at the entry and the exit. So, you will find that the velocities or the volume which will be covered in time dt would be v1dt into A1 and volume 2 would be v2dt into A2. This is the concept which we are going to use. So, entry exit, what is the volume which is coming out and what was the volume of the fluid which entered. You will see that continuity equation tells me that these two should be equal and the consequences would be immense.

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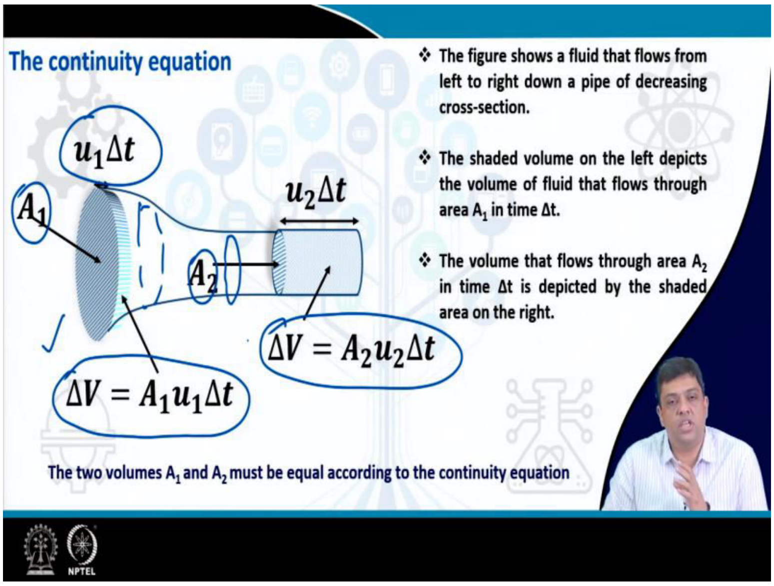

So, let us see, how to derive this continuity equation and the concept which I said earlier that continuity equation tells me that the volume of fluid entering and the volume of fluid exiting in time delta t would be equal. So, let us consider a pipe which has a decreasing cross sectional area, so, you have a cross sectional area which is actually decreasing.

So, at entry let us say you have a cross sectional area as A1 and at exit you have the cross sectional area as A2. The shaded volume on the left that means, here depicts the volume of the fluid that flows through A1 in time delta t. So, you have u1 as the velocity, delta t is the time, so delta V that is the volume which will be flowing in at the entry point of this pipe would be A1u1 delta t.

And the volume which will be exiting from the other end would be A2u2 delta t and why this will be happening, because we are considering an ideal fluid that means it is maintaining its flow field and the stream line flow is being maintained. So, the two volume should be equal that is what we are trying to see. And if that is the case, then what is the relation I obtained.

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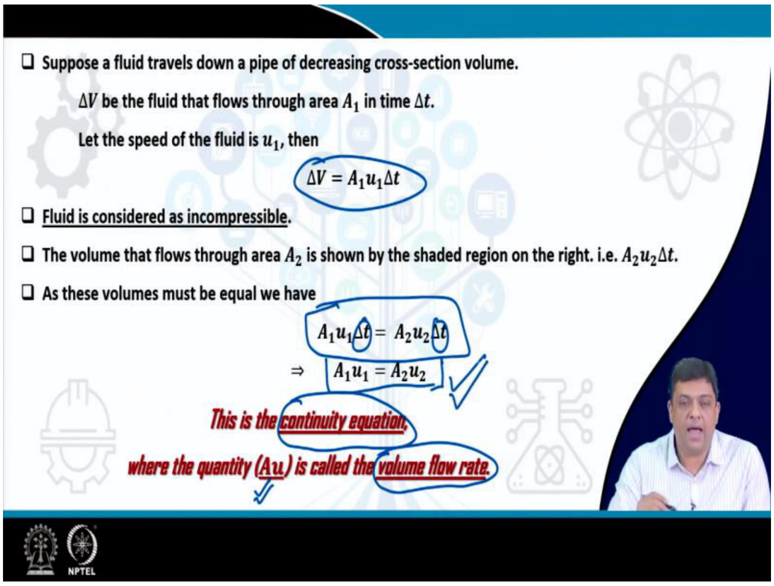

So, suppose a fluid travels down this pipe what is the volume I have obtained? I have obtained the value as A1u1 delta t, as I said it is incompressible, it is also ideal. So, if it is ideal incompressible that means the volume which has entered in time delta t and if I have the same time while I am monitoring the exit, then the two terms should be equal.

And if it is that true, then I get the relation that for an incompressible fluid flowing down pipe which can have a varying cross section area then also the condition A1u1 is equal to A2u2 will hold good and this condition is called the continuity equation and the quantity Au which represents either A1u1 or A2u2 is called the volume flow rate. And this equation has very, very significant consequences.

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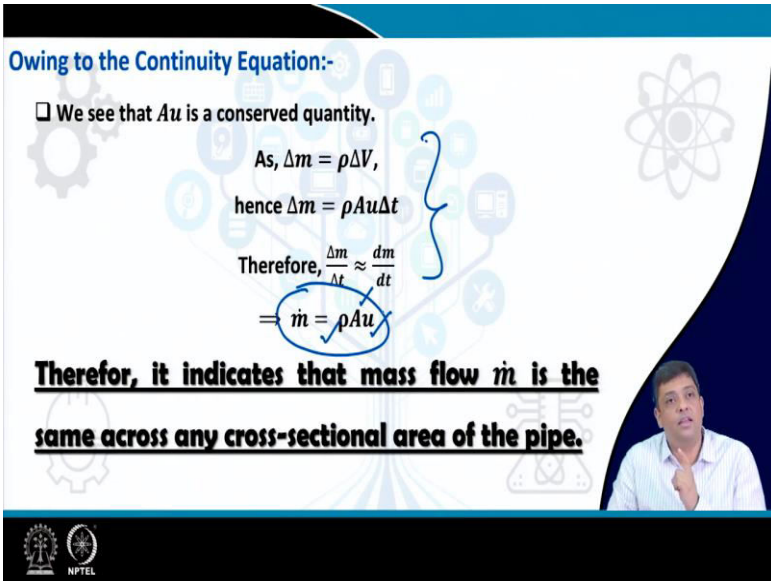

And as I said continuity equation has serious consequences. And the first is that it indicates that the mass flow is the same across any cross sectional area of the pipe and you can derive this equation very easily, that mass flow rate is constant, density is constant, area is known velocity is known. So, the mass flow is same across any cross sectional area that is very critical to understand and the answer is coming in because of the continuity equation.

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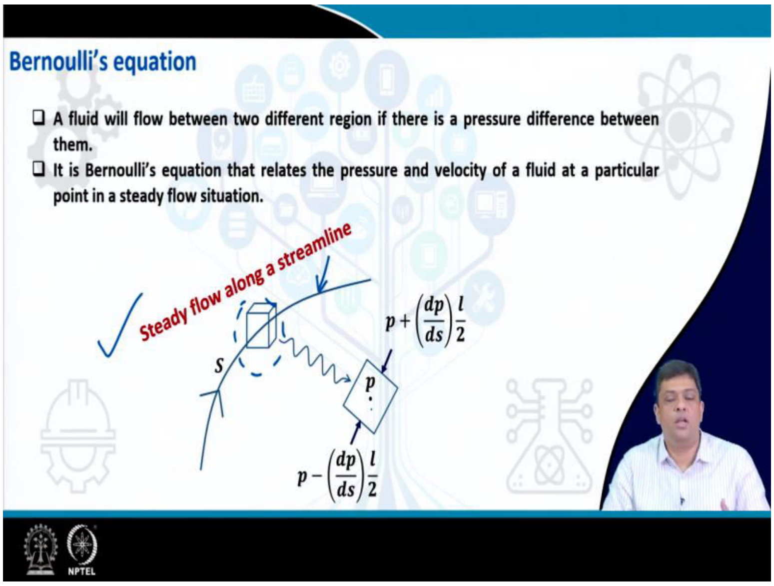

What other and the other is the famous Bernoulli's equation. I know most of you have studied but for our purpose. Let us very quickly also try to revise it, how do we derive Bernoulli's equation, let us consider a steady flow (around) along a stream line and along the stream line we will consider a small region and we will have this point. So, if the pressure is p at this point then above it and below it, what is the pressure. This is what we are trying to find out.

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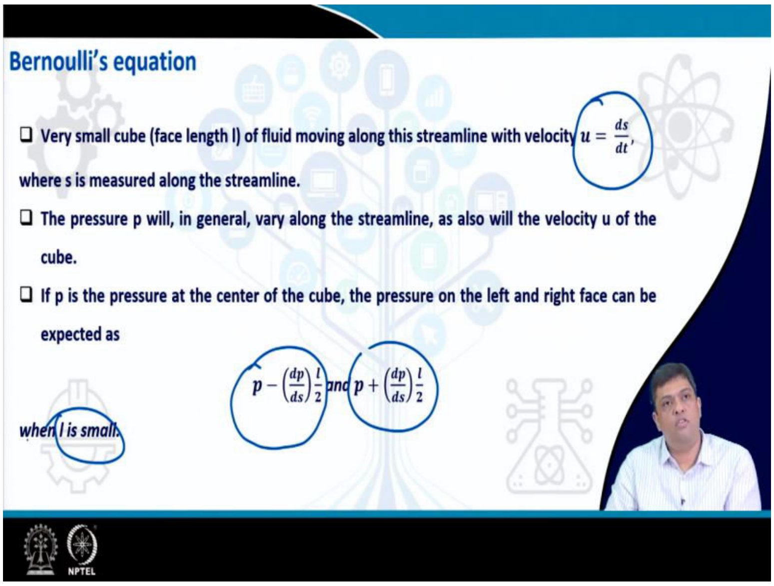

So, for that cubic cross area which we have chosen on a stream line, let the length be l and the velocity would then become equal to ds by dt simple, where s is measured along the streamline. So, that small section area which you are using you will be choosing at any point along that streamline.

Now, if you take the streamline you will find that the pressure in general it varies along that streamline and if pressure is changing, velocity of u of the cube will also be changing that is what can be expected, whether it happens, then why it happens and if it does not happen, then why does not happen is the question we should ask now. So, let us try to write the things which are known to us.

As of now, what we know that if p is the pressure at the center of the cube, the pressure to the left or right of the cube can be expected as p minus dp by ds l by 2 and p plus dp by ds l by 2. So, above and below the cube and we believe that l is small and we are asking what will happen to the flow along this streamline.

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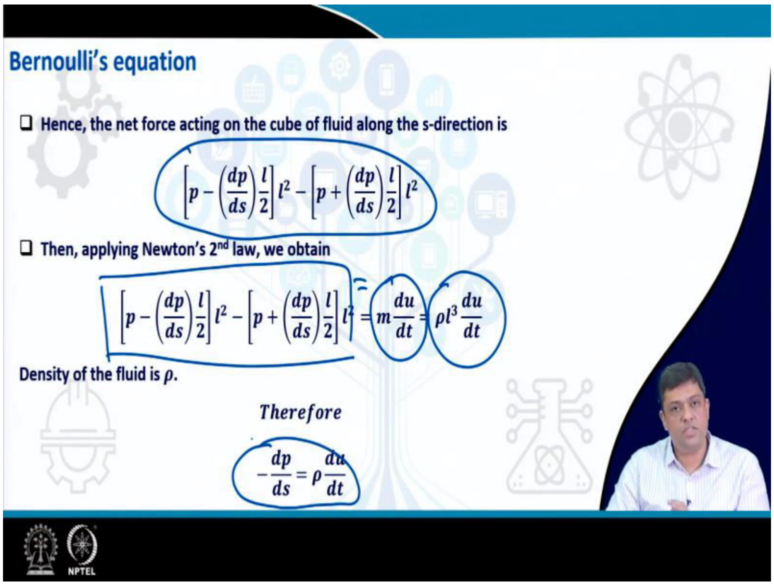

What is the net force acting on the cube along the x direction? It is simply given as the difference. As explained in the previous class while solving the fluid mechanics problem, we will be using Newton's second law. So, now, we know that what is the net force acting on the cube of the fluid and force is m du by dt. So, we equate, so we equate and if what is the value of mass we know then we can write rho l cube du by dt, where rho is the density of the fluid. And solving what do we get, we get minus dp by ds is equal to rho du by dt. This is what we obtained as one of the relations.

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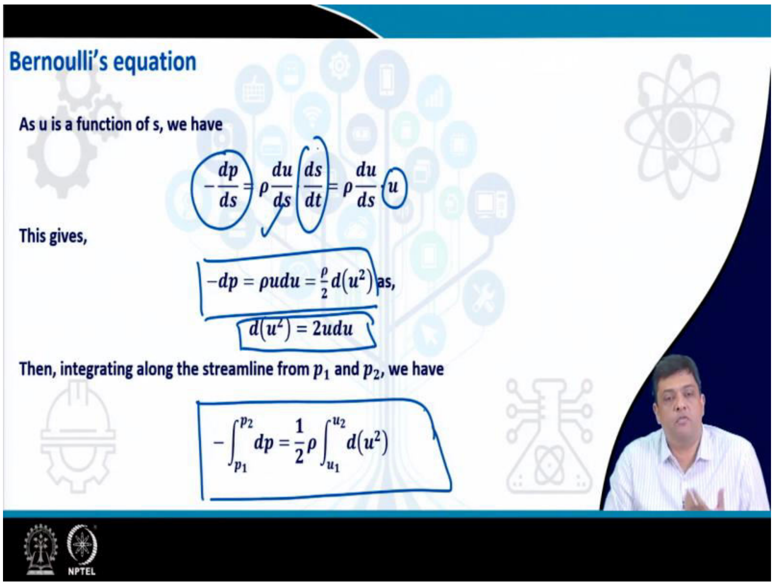

So, as u is a function of s, we can write minus dp by ds as rho du by ds and into ds by dt, ds by dt is known as u. So, what we get, minus dp is equal to rho u du that is equal to rho by 2 du square, why because d of u square is equal to 2u du. Thus, if we integrate along the streamline from p1 to p2 we should get a relationship between p and rho. So, this is the integration which we must perform to obtain the relationship between pressure and density and also the velocity or speed.

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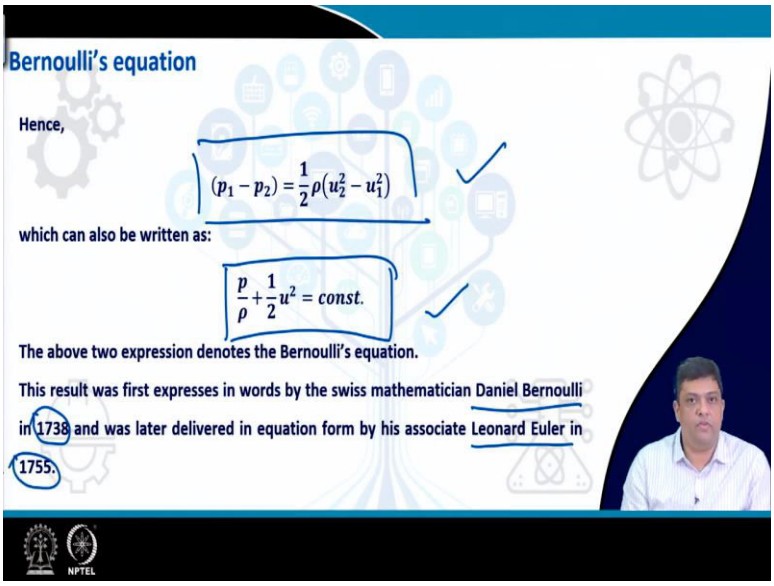

And by integrating we get p1 minus p2 is equal to half rho u2 square minus u1 square which can again be written as p by rho plus half u square is a constant. And these two expressions denote the Bernoulli's equations and this was first written or extracted from continuity equation by Daniel Bernoulli in 1738, which was later on explained further by his associate, Leonard Euler in 1755. So, math conceptually Bernoulli's had already explained everything and in terms of equation it was later written by Euler in 1755.

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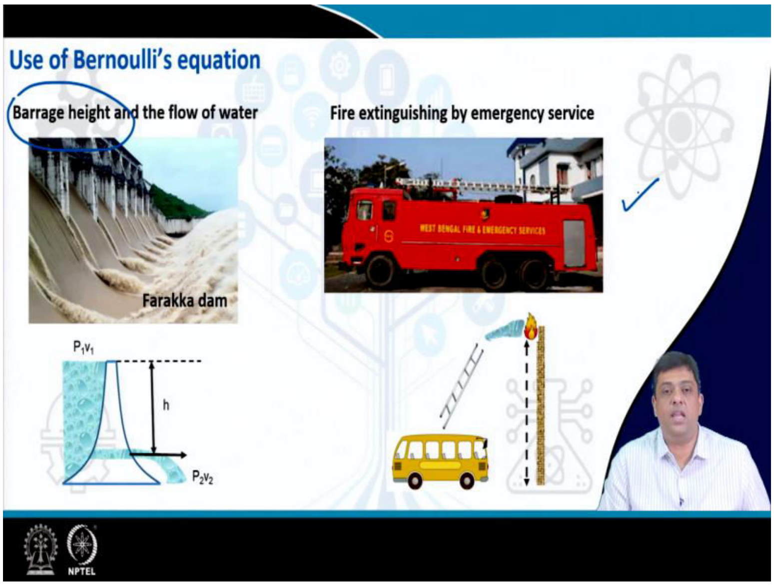

And using this Bernoulli's equation, we have large number of applications or concepts which are used for example, in defining the barrage height of the dams and defining what would be the flow of water in these dams or through these barrages, we can find out what should be the flow speeds coming in, coming out from the jets in fire engines, so that they can reach to the expected heights and you will see that this concept is also used in designing the wind turbine blades.

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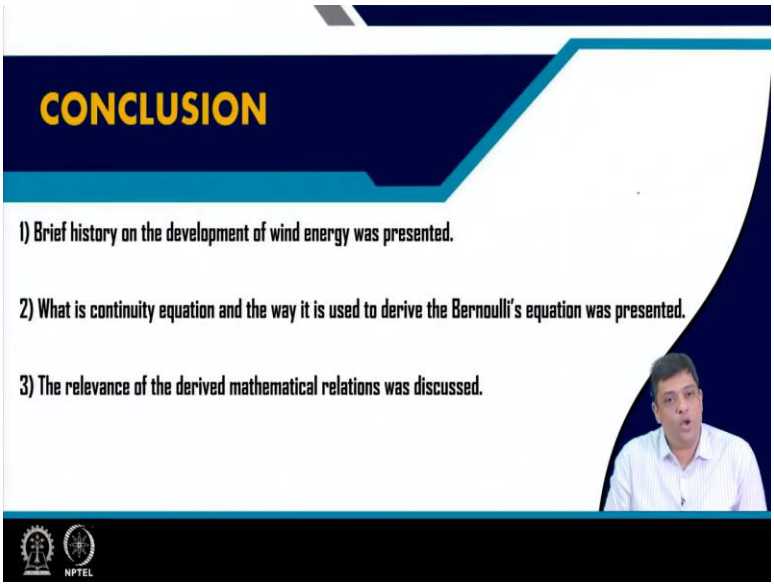

So, I hope one thing is clear to you that wind energy and extracting wind power is a concept which has been known to us for generations. And we are now entering an era where wind energy will become a part of our life. To develop next generation wind turbines or wind turbine blades, the concepts of continuity equation and Bernoulli's equations become very critical and that was discussed today.

And we had also discussed the formulation using certain mathematical relations, so that we can arrive to certain exact values while we engineer the wind blades, because there we need some values for engineers to work upon, when they construct the wind blades.

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These are the two references which we used in preparing today's lecture and in the next lecture, I will start with the Betz Criteria, which is extensively used to understand the concept of extraction of wind power and how does it limit the efficiency of wind based energy power stations. I thank you for attending today's lecture.